A complete square is the square of a sum or a difference. For example, $(2a-5)^2$ and $(5x+3y)^2$ are complete squares. Completing the square is an extremely powerful technique, with many applications. One of them is factoring by completing the square.
Example 1. Factor $224-2x^2-12x$ by completing the square.

1. We first rearrange the polynomial by degree, starting with the highest degree term.

$$224-2x^2-12x=-2x^2-12x+224$$

2. We now factor out -2. Completing the square is easiest if the leading coefficient is 1.

$$-2x^2-12x+224=-2\left(x^2+6x-112\right)$$

3. We now obtain the "magic number" that is half of the coefficient (includes sign!) $\dfrac{+6}{2}=+3$

4. We write x in front of the magic number, place them in parentheses and square. We obtain the complete square $(x+3)^2$.

5. We multiply out the complete square.

$$(x+3)^2=(x+3)(x+3)=x^2+3x+3x+9=x^2+6x+9$$

6. We have found the only complete square that begins with the same first two terms, x^2+6x as our expression to be factored. We will complete the square by smuggling in the last term, 9. The smuggling step that involves first writing down the expression as it is, with a gap in front of the last term.

$$\begin{aligned} 224-2x^2-12x &= -2x^2-12x+224 \qquad\qquad (x+3)^2=x^2+6x+9 \\ &= -2\left(x^2+6x-112\right) \\ &= -2\left(x^2+6x \qquad\qquad -112\right) \end{aligned}$$

and then adding zero by adding and then immediately subtracting 9.

$$=-2\left(x^2+6x+9-9-112\right)$$

7. The expression now has five terms. The first three form the complete(d) square, the last two will simply be combined.

$$\begin{aligned} &= -2\left(\underbrace{x^2+6x+9}\underbrace{-9-112}\right) \\ &= -2\left((x+3)^2-121\right) \end{aligned}$$

8. We re-write 121 as a square and factor via the difference of squares theorem.

$$\begin{aligned} &= -2\left((x+3)^2-11^2\right) \\ &= -2\left((x+3+11)(x+3-11)\right) \end{aligned}$$

9. We simplify the expression by dropping the unnecessary parentheses and by combining like terms.

$$= -2(x+14)(x-8)$$

10. We check by multiplication.

$$-2(x+14)(x-8) = -2\left(x^2-8x+14x-112\right) = -2\left(x^2+6x-112\right) = -2x^2-12x+224$$

Example 2. Factor $-70x+5x^2+165$ by completing the square.
Solution:

$$\begin{aligned} -70x+5x^2+165 &= && \text{rearrange terms by degree} \\ 5x^2-70x+165 &= && \text{factor out the greatest common factor, 5} \\ &= 5\left(x^2-14x+33\right) \end{aligned}$$

Half of the linear coefficient is $\dfrac{-14}{2} = -7$, and so the complete square to work toward is $(x-7)^2$.

$$\begin{aligned} 5\left(x^2-14x+33\right) &= && (x-7)^2 = x^2-14x+49 \quad \text{we smuggle } 49 \\ 5\left(\underbrace{x^2-14x+49}-49+33\right) &= && \text{realize the complete square, combine the last 2 terms} \\ 5\left((x-7)^2-16\right) &= && \text{re-write 16 as } 4^2 \\ 5\left((x-7)^2-4^2\right) &= && \text{factor via the difference of squares theorem} \\ 5(x-7+4)(x-7-4) &= && \text{combine like terms} \\ &= 5(x-3)(x-11) \end{aligned}$$

We check:

$$5(x-11)(x-3) = 5\left(x^2-3x-11x+33\right) = 5\left(x^2-14x+33\right) = 5x^2-70x+165$$

Example 3. Factor $x^2-40x+336$ by completing the square.
Solution: There is no GCF, and the leading coefficient is 1, thus we can start completing the square
Half of the linear coefficient is $\dfrac{-40}{2} = -20$, and so the complete square to work toward is $(x-20)^2$.

$$\begin{aligned} x^2-40x+336 &= && (x-20)^2 = x^2-40x+400 \quad \text{smuggle in } 400 \\ \underbrace{x^2-40x+400}-400+336 &= && \text{realize the complete square, combine the last 2 terms} \\ (x-20)^2-64 &= && \text{re-write 64 as } 8^2 \\ (x-20)^2-8^2 &= && \text{factor via the difference of squares theorem} \\ (x-20+8)(x-20-8) &= && \text{combine like terms} \\ &= (x-12)(x-28) \end{aligned}$$

We check:

$$(x-12)(x-28) = x^2-12x-28x+336 = x^2-40x+336$$

Example 4. Factor $12x - 6x^2 + 1008$ by completing the square.
Solution:

$$\begin{aligned}
12x - 6x^2 + 1008 &= && \text{rearrange terms} \\
-6x^2 + 12x + 1008 &= && \text{factor out } -6 \\
-6\left(x^2 - 2x - 168\right) &= && (x-1)^2 = x^2 - 2x + 1 \\
-6\left(\underbrace{x^2 - 2x + 1}{} - 1 - 168\right) &= && \text{realize the complete square, combine last terms} \\
-6\left((x-1)^2 - 169\right) &= && \text{re-write square} \\
-6\left((x-1)^2 - 13^2\right) &= && \text{factor via the difference of squares theorem} \\
-6(x - 1 + 13)(x - 1 - 13) &= -6(x+12)(x-14)
\end{aligned}$$

We check:

$$-6(x+12)(x-14) = -6\left(x^2 - 14x + 12x - 168\right) = -6\left(x^2 - 2x - 168\right) = -6x^2 + 12x + 1008$$

Example 5. Factor $x^2 - 10x + 24$ by completing the square.
Solution:

$$\begin{aligned}
x^2 - 10x + 24 &= && (x-5)^2 = x^2 - 10x + 25 \\
\underbrace{x^2 - 10x + 25}{} - 25 + 24 &= && \text{realize the complete square, combine last terms} \\
(x-5)^2 - 1 &= && \text{re-write square} \\
(x-5)^2 - 1^2 &= && \text{factor via the difference of squares theorem} \\
(x - 5 + 1)(x - 5 - 1) &= (x-4)(x-6)
\end{aligned}$$

We check:

$$(x-4)(x-6) = x^2 - 6x - 4x + 24 = x^2 - 10x + 24$$