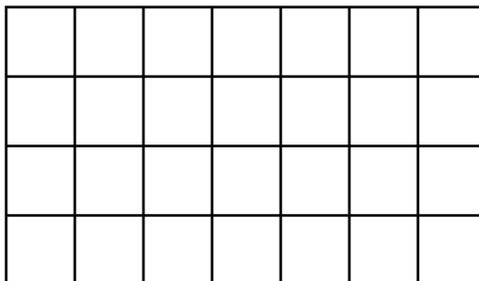


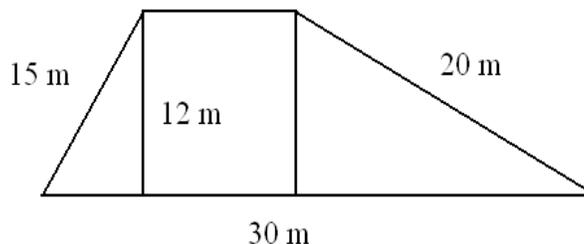
## Review Problems

1. Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{1, 4, 9\}$ ,  $B = \{1, 2, 3, 4, 5\}$ , and  $C = \{1, 3, 5, 7\}$ 
  - (a) Draw a Venn diagram depicting these sets.
  - (b) Find  $(B \cup C)'$  =
  - (c) Find  $(A \cap C)' \cup (A' \cap C) =$
  - (d) Find  $(A \cup B) \setminus C =$
  - (e) List all subsets of  $C$ .
  - (f) How many 5-element subsets does  $U$  have? (You don't have to list these sets.)
  - (g) Is it true that  $(A \cap B) \cap C' \subseteq (A \cup B) \cap C'$ ?
2. Find  $n(B)$  if we know that  $n(A) = 19$ ,  $n(A \cup B) = 37$ , and  $n(A \cap B) = 8$ .
3. A book went on a 15% sale. The sale price is \$ 40.80. Find the original price.
4. The population of a town has decreased from 120 000 to 104 400. What percent of a change does this represent?
5. We borrowed \$ 2400 for two years, with a simple annual interest rate of 10%. After 7 months, we make a partial payment of \$ 900. After an additional 6 months, we make another partial payment of \$ 1000. How much do we owe at the end of the two years?
6. We placed \$ 2000 into a bank account with an annual simple interest rate of 8%. How much money do we have after 15 years?
7. We placed \$ 2000 into a bank account with an annual compound interest rate of 8%. How much money do we have after 15 years if the bank compounds
  - (a) annually
  - (b) semi-annually
  - (c) monthly
  - (d) daily (use 1 year = 360 days)
  - (e) continuously
8. Find the sum  $\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6} =$
9. We asked 120 people if they listen to radio, watch TV or read books. 85 watch TV, 69 listen to radio, and 74 read books. 51 watch TV and listen to radio, 57 watch TV and read books, and 43 listen to radio and read books. 29 do all three.
  - (a) Draw a Venn diagram depicting the information given.
  - (b) How many people only listen to radio?
  - (c) How many people read books or watch TV?
  - (d) How many people do neither of these?

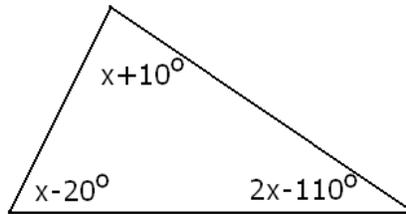
- (e) We randomly pick a person among the people listening to radio. What is the probability that he also reads books?
- (f) We randomly pick a person among the people reading books. What is the probability that he also listens to radio?
10. A club has 20 members.
- (a) In their annual election, they elect their president, vice president, and secretary. How many different outcomes are possible?
- (b) If everyone shakes hands with everyone in the room, how many handshakes took place?
11. Suppose that  $X$  is a set with  $n(X) = 12$ .
- (a) How many 6-element subsets does  $X$  have?
- (b) We randomly pick a subset  $A$  of  $X$ . What is the probability that  $n(A) = 4$ ?
12. Consider the picture shown below.



- (a) How many rectangles are there on the picture?
- (b) How many squares are there on the picture?
13. The shorter sides of a right triangle are 12 ft and 35 ft long.
- (a) Find the perimeter of the triangle. Include units in your computation and answer.
- (b) Find the area of the triangle. Include units in your computation and answer.
14. Find the distance between the points  $(2, -6)$  and  $(-6, 9)$ .
15. The supplement of an angle is  $24^\circ$  less than twice the angle. Find this angle.
16. Find the area and perimeter of the trapezoid shown on the picture below. Include units in your computation and answer.



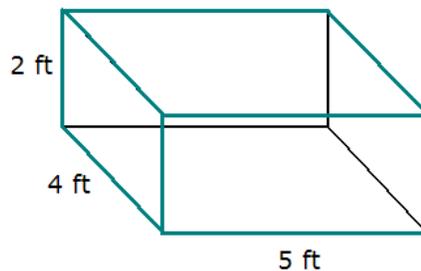
17. Find the measure of an inner angle in a regular polygon of 16 sides.
18. Find  $x$  based on the picture below.



19. Find the perimeter and area of the parallelogram shown below.

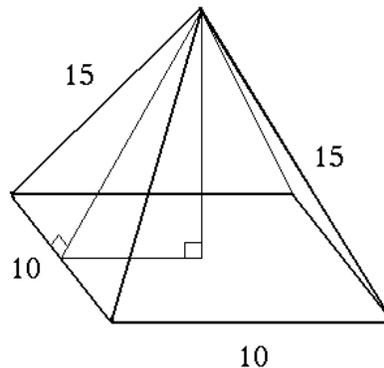


20. Find the volume of the rectangular prism shown on the picture below.



21. Volumes.
- (a) Find the volume of a cylinder with a base of radius 8 cm and height 15 cm. Include units in your computation and answer.
- (b) Find the volume of a cone with a base of radius 8 cm and height 15 cm. Include units in your computation and answer.

22. Consider the pyramid shown on the picture below.



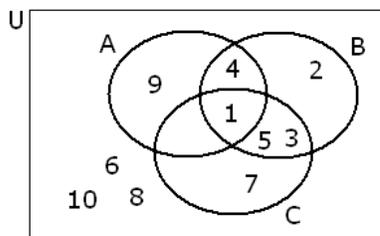
- (a) Compute the height of the pyramid.
- (b) Compute the volume of the pyramid.
23. We throw a die twice in a row. Find each of the following probabilities.
- (a) The second number rolled is larger than the first number rolled.
- (b) The sum of the two numbers rolled is 6.
- (c) The product of the two numbers rolled is 6.
- (d) The sum of the two numbers rolled is odd.
- (e) The product of the two numbers rolled is odd.
24. We toss a coin 10 times in a row.
- (a) What is the total number of outcomes?
- (b) Find the probability of 3 heads and 7 tails.
- (c) Find the probability of 5 heads and 5 tails.
- (d) Find the probability of the number of tails being 3 or more.
25. Susan is a member of a 16-member club. The club is to elect a four-person committee. What is the probability that Susan will be selected to be on the committee?
26. We have a bag with 10 marbles, 6 red, 3 blue, and 1 yellow. We randomly pull two marbles, with replacement. (Replacement means that before pulling again, we put back what we pulled first.) Find each of the following probabilities.
- (a) We pull two red marbles.
- (b) We pull two yellow marbles.
- (c) The two marbles pulled are of the same color.
- (d) The two marbles pulled are of different color.
- (e) We pull a blue and a yellow marble. (Careful! Either a blue first and then a yellow, or the other way around.)

- (f) Neither of the two marbles is yellow.
27. We have a bag with 10 marbles, 6 red, 3 blue, and 1 yellow. We randomly pull two marbles, **without replacement**. (No replacement means that before pulling again, we do NOT put back what we pulled first.) Find each of the following probabilities.
- We pull two red marbles.
  - We pull two yellow marbles.
  - The two marbles pulled are of the same color.
  - The two marbles pulled are of different color.
  - We pull a blue and a yellow marble. (Careful! Either a blue first and then a yellow, or the other way around.)
  - Neither of the two marbles is yellow.

### Review Problems - Answers

1. Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{1, 4, 9\}$ ,  $B = \{1, 2, 3, 4, 5\}$ , and  $C = \{1, 3, 5, 7\}$

- (a) Draw a Venn diagram depicting these sets.



- (b) Find  $(B \cup C)' = \{7\}$
- (c) Find  $(A \cap C)' \cup (A' \cap C) = \{1, 2, 3, 5, 6, 7, 8, 10\}$
- (d) Find  $(A \cup B) \setminus C = \{1, 7\}$
- (e) List all subsets of  $C$ .

$\emptyset$

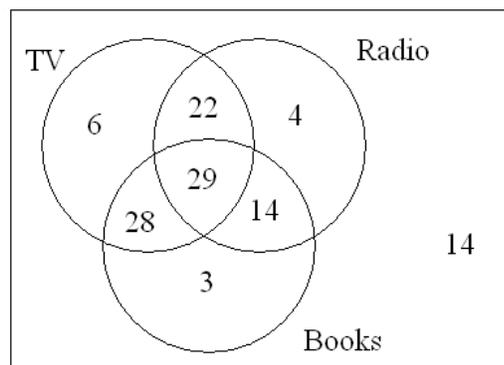
$\{1\}$        $\{3\}$        $\{5\}$        $\{7\}$   
 $\{1, 3\}$      $\{1, 5\}$      $\{1, 7\}$      $\{3, 5\}$      $\{3, 7\}$      $\{5, 7\}$   
 $\{1, 3, 5\}$     $\{1, 3, 7\}$     $\{1, 5, 7\}$     $\{3, 5, 7\}$   
 $\{1, 3, 5, 7\}$

- (a) How many 5-element subsets does  $U$  have? (You don't have to list these sets.)    **252**
- (b) Is it true that  $(A \cap B) \cap C' \subseteq (A \cup B) \cap C'$ ?    **yes**

2. Find  $n(B)$  if we know that  $n(A) = 19$ ,  $n(A \cup B) = 37$ , and  $n(A \cap B) = 8$ .    **26**

3. A book went on a 15% sale. The sale price is \$ 40.80. Find the original price.    **\$ 48**

4. The population of a town has decreased from 120 000 to 104 400. What percent of a change does this represent? **13% decrease**
5. We borrowed \$ 2400 for two years, with a simple annual interest rate of 10%. After 7 months, we make a partial payment of \$ 900. After an additional 6 months, we make another partial payment of \$ 1000. How much do we owe at the end of the two years? **\$ 788.18**
6. We placed \$ 2000 into a bank account with an annual simple interest rate of 8%. How much money do we have after 15 years? **\$ 4400**
7. We placed \$ 2000 into a bank account with an annual compound interest rate of 8%. How much money do we have after 15 years if the bank compounds
- annually **\$ 6344.34**
  - semi-annually **\$ 6486.80**
  - monthly **\$ 6613.84**
  - daily (use 1 year = 360 days) **\$6639.35**
  - continuously **\$ 6640.23**
8. Find the sum  $\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6} =$  **32**
9. We asked 120 people if they listen to radio, watch TV or read books. 85 watch TV, 69 listen to radio, and 74 read books. 51 watch TV and listen to radio, 57 watch TV and read books, and 43 listen to radio and read books. 29 do all three.
- Draw a Venn diagram depicting the information given.



- How many people only listen to radio? **4**
- How many people read books or watch TV? **102**
- How many people do neither of these? **14**
- We randomly pick a person among the people listening to radio. What is the probability that he also reads books?  $\frac{43}{69}$
- We randomly pick a person among the people reading books. What is the probability that he also listens to radio?  $\frac{43}{74}$

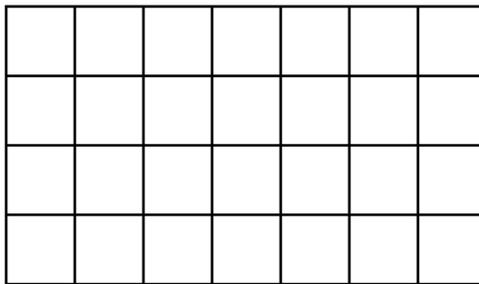
10. A club has 20 members.

- (a) In their annual election, they elect their president, vice president, and secretary. How many different outcome is possible? **6840**
- (b) If everyone shakes hands with everyone in the room, how many handshakes took place? **190**

11. Suppose that  $X$  is a set with  $n(X) = 12$ .

- (a) How many 6–element subsets does  $X$  have? **924**
- (b) We randomly pick a subset  $A$  of  $X$ . What is the probability that  $n(A) = 4$ ?  **$\frac{\binom{12}{4}}{2^{12}} = \frac{495}{4096}$**

12. Consider the picture shown below.



- (a) How many rectangles are there on the picture? **280**
- (b) How many squares are there on the picture? **60**

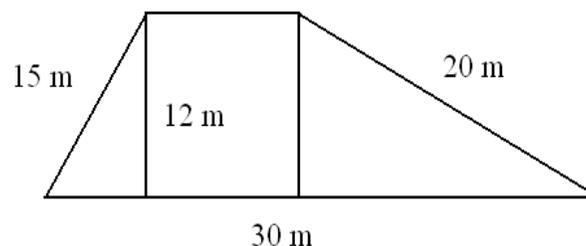
13. The shorter sides of a right triangle are 12 ft and 35 ft long.

- (a) Find the perimeter of the triangle. Include units in your computation and answer.  **$P = 84$  ft**
- (b) Find the area of the triangle. Include units in your computation and answer.  **$A = 210$  ft<sup>2</sup>**

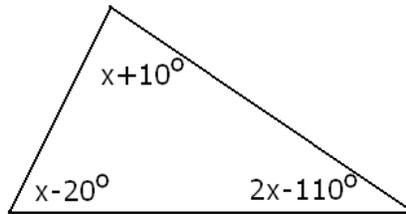
14. Find the distance between the points  $(2, -6)$  and  $(-6, 9)$ . **17 units**

15. The supplement of an angle is  $24^\circ$  less than twice the angle. Find this angle.  **$68^\circ$**

16. Find the area and perimeter of the trapezoid shown on the picture below. Include units in your computation and answer.  **$P = 70$  m,  $A = 210$  m<sup>2</sup>**



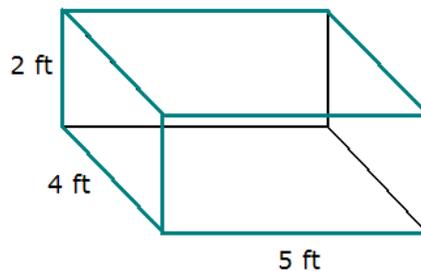
17. Find the measure of an inner angle in a regular polygon of 16 sides.  $157.5^\circ$
18. Find  $x$  based on the picture below.  $75^\circ$



19. Find the perimeter and area of the parallelogram shown below.  $P = 22$  unit,  $A = 21$  unit<sup>2</sup>



20. Find the volume of the rectangular prism shown on the picture below.  $40$  ft<sup>3</sup>

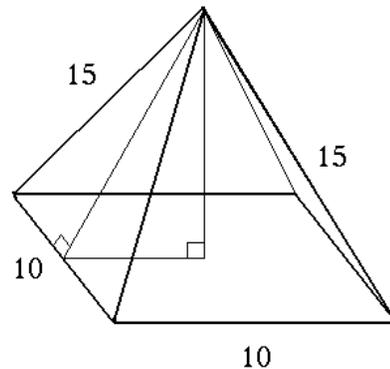


21. Volumes.

- (a) Find the volume of a cylinder with a base of radius 8 cm and height 15 cm. Include units in your computation and answer.  $V = Bh = \pi r^2 h = \pi (8 \text{ cm})^2 (15 \text{ cm}) = 960\pi \text{ cm}^3 \cong 3015.92894 \text{ cm}^3$
- (b) Find the volume of a cone with a base of radius 8 cm and height 15 cm. Include units in your computation and answer.  $V = \frac{Bh}{3} = \frac{\pi r^2 h}{3} = \frac{\pi (8 \text{ cm})^2 (15 \text{ cm})}{3} = 320\pi \text{ cm}^3 \cong 1005.3096 \text{ cm}^3$

22. Consider the pyramid shown on the picture below.

- (a) Compute the height of the pyramid.  $\sqrt{175}$
- (b) Compute the volume of the pyramid.  $V = \frac{Bh}{3} = \frac{(10)^2 \sqrt{175}}{3} \cong 440.95855$



23. We throw a die twice in a row. Find each of the following probabilities.

(a) The second number rolled is larger than the first number rolled.  $\frac{15}{36} = \frac{5}{12}$

(b) The sum of the two numbers rolled is 6.  $\frac{5}{36}$

(c) The product of the two numbers rolled is 6.  $\frac{4}{36} = \frac{1}{9}$

(d) The sum of the two numbers rolled is odd.  $\frac{1}{2}$

(e) The product of the two numbers rolled is odd.  $\frac{1}{4}$

24. We toss a coin 10 times in a row.

(a) What is the total number of outcomes? 1024

(b) Find the probability of 3 heads and 7 tails.  $\frac{\binom{10}{3}}{2^{10}} = \frac{15}{128}$

(c) Find the probability of 5 heads and 5 tails.  $\frac{\binom{10}{5}}{2^{10}} = \frac{63}{256}$

(d) Find the probability of the number of tails being 3 or more.  $\frac{2^{10} - \left( \binom{10}{0} + \binom{10}{1} + \binom{10}{2} \right)}{2^{10}} = \frac{121}{128}$

25. Susan is a member of a 16-member club. The club is to elect a four-person committee. What is

the probability that Susan will be selected to be on the committee?  $\frac{\binom{15}{3}}{\binom{16}{4}} = \frac{1}{4}$

26. We have a bag with 10 marbles, 6 red, 3 blue, and 1 yellow. We randomly pull two marbles, with replacement. (Replacement means that before pulling again, we put back what we pulled first.) Find each of the following probabilities.

- (a) We pull two red marbles.  $\left(\frac{6}{10}\right)^2 = \frac{9}{25}$
- (b) We pull two yellow marbles.  $\left(\frac{1}{10}\right)^2 = \frac{1}{100}$
- (c) The two marbles pulled are of the same color.  $\left(\frac{6}{10}\right)^2 + \left(\frac{3}{10}\right)^2 + \left(\frac{1}{10}\right)^2 = \frac{23}{50}$
- (d) The two marbles pulled are of different color.  $1 - \frac{23}{50} = \frac{27}{50}$
- (e) We pull a blue and a yellow marble. (Careful! Either a blue first and then a yellow, or the other way around.)  $\frac{3}{10} \cdot \frac{1}{10} + \frac{1}{10} \cdot \frac{3}{10} = \frac{3}{50}$
- (f) Neither of the two marbles is yellow.  $\frac{9}{10} \cdot \frac{9}{10} = \frac{81}{100}$

27. We have a bag with 10 marbles, 6 red, 3 blue, and 1 yellow. We randomly pull two marbles, **without replacement**. (No replacement means that before pulling again, we do NOT put back what we pulled first.) Find each of the following probabilities.

- (a) We pull two red marbles.  $\frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$
- (b) We pull two yellow marbles.  $\frac{1}{10} \cdot \frac{0}{9} = 0$
- (c) The two marbles pulled are of the same color.  $\frac{6}{10} \cdot \frac{5}{9} + \frac{3}{10} \cdot \frac{2}{9} + 0 = \frac{2}{5}$
- (d) The two marbles pulled are of different color.  $1 - \frac{2}{5} = \frac{3}{5}$
- (e) We pull a blue and a yellow marble. (Careful! Either a blue first and then a yellow, or the other way around.)  $\frac{3}{10} \cdot \frac{1}{9} + \frac{1}{10} \cdot \frac{3}{9} = \frac{1}{15}$
- (f) Neither of the two marbles is yellow.  $\frac{9}{10} \cdot \frac{8}{9} = \frac{4}{5}$