

Review Problems

1. Simplify each of the following.

$$\text{a) } \frac{5^{-1} + \left(-\frac{1}{2}\right)^2}{5^{-1} + 2^{-1}} - 2^{-1}$$

$$\text{d) } \frac{a^{-3} (b^{-4})^2}{(ab^2)^{-3}}$$

$$\text{g) } \frac{(m^{2/3})^{-3/5}}{\sqrt[5]{m^3}}$$

$$\text{b) } 5\sqrt{18} - 7\sqrt{32} + 4\sqrt{50}$$

$$\text{e) } (3x - 1)(3x + 9x^2 + 1)$$

$$\text{h) } e^{-\ln 7}$$

$$\text{c) } \frac{x^3 (-2x^3 y^{-2})^{-4} 2yx^{-1}}{x^{-3} y x^3 y^0 (-3x^5 y^{-6})^{-2} y^{-3}}$$

$$\text{f) } \frac{2}{(\sqrt{3} - 1)^2}$$

2. Simplify each of the following.

$$\text{a) } \log_{25} \left(\frac{1}{125}\right)$$

$$\text{d) } \log_6 18 + \log_6 2$$

$$\text{g) } \left(169^{\frac{3}{2}}\right)^{\log_{13} 2}$$

$$\text{j) } \log_4 \left(\frac{1}{\sqrt{8}}\right)$$

$$\text{b) } 3^{\log_9 5}$$

$$\text{e) } \log_{10} 2 + \log_{10} 50 - 1$$

$$\text{h) } \log_9 \sqrt{3} - \log_4 \left(\frac{1}{\sqrt{2}}\right)$$

$$\text{k) } \log_{a^2} \left(\frac{1}{\sqrt[3]{a}}\right)$$

$$\text{c) } \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$$

$$\text{f) } \ln(5e^2)$$

$$\text{i) } \log_a(4a) + \log_a(25a^3) - 2\log_a(10a)$$

3. Solve each of the following inequalities.

$$\text{a) } 12x - 2x^2 > 20 \quad \text{b) } 12x + x^2 \geq 45 \quad \text{c) } \frac{1}{3}x^2 - 4x \leq -12 \quad \text{d) } 8 \geq 2x^2 - 4x \quad \text{e) } 5x^2 + 65 \geq 20x$$

4. Solve the equation $3x^2 - x - 1 = 0$ by completing the square. Use exact values. Check your solution.

5. Find the domain of each of the following expressions.

$$\text{a) } \log_5(-x^2 + 2x + 20)$$

$$\text{d) } \sqrt{5x - x^2}$$

$$\text{g) } \ln(x - 5) - \ln(x + 6)$$

$$\text{j) } \frac{x^2 - 1}{\log_2(x - 7) - 2}$$

$$\text{b) } \frac{1}{\sqrt{2x - 6} - 6}$$

$$\text{e) } \log_7(x^2 + 3)$$

$$\text{h) } \frac{\ln(x + 1)}{\ln(x - 1)}$$

$$\text{c) } \frac{1}{\log_2(x - 5) + \log_2(x + 5)}$$

$$\text{f) } \ln\left(\frac{x - 5}{x + 6}\right)$$

$$\text{i) } \frac{x - 3}{x^2 + 1}$$

6. Solve each of the following equations.

$$\text{a) } \sqrt{x + 3} + \sqrt{2x + 5} = 2$$

$$\text{e) } \log_2(2x - 3) - \log_2(x + 1) = -3$$

$$\text{i) } 5^{2x-1} = 2^{x+3}$$

$$\text{b) } \sqrt{x + 5} = 1 + \sqrt{x - 2}$$

$$\text{f) } \log_2(3 - x) + \log_2(-x - 4) = 3$$

$$\text{j) } 9^{x+1} - 28 \cdot 3^x = -3$$

$$\text{c) } 5x - 3(x - 1)^2 = -3(x - 2)^2$$

$$\text{g) } \log_3(x - 5) - \log_3(2x - 11) = -1$$

$$\text{d) } \log_3(x - 7) + \log_3(2x + 7) = 4$$

$$\text{h) } 2^{x+1} \cdot 5 = 3^{x-1}$$

7. Suppose that $\log_2 3 = x$. Express each of the following in terms of x .

$$\text{a) } \log_2 6$$

$$\text{c) } \log_2 18$$

$$\text{e) } \log_2 \left(\frac{3}{2}\right)$$

$$\text{g) } \log_2 \left(\frac{4}{27}\right)$$

$$\text{b) } \log_2 12$$

$$\text{d) } \log_2 72$$

$$\text{f) } \log_3 2$$

$$\text{h) } \log_6 12$$

8. Suppose that $\log_2 3 = a$ and $\log_2 5 = b$. Express each of the following in terms of a and b .

$$\text{a) } \log_2 15$$

$$\text{b) } \log_2 30$$

$$\text{c) } \log_2 240$$

$$\text{d) } \log_3 15$$

$$\text{e) } \log_6 30$$

9. a) Suppose that $a = \log_5 4$ and $b = \log_2 6$. Express $\log_3 5$ in terms of a and b .
 b) Suppose that $\log_3 6 = x$ and $\log_5 9 = y$. Express $\log_{18} 100$ in terms of x and y .
10. Two numbers p and q are such that $p - 3q = 50$. Find
 a) the minimal value of pq . b) the minimal value of $p^2 + q^2$.
11. Divide the polynomials as indicated.
 a) $x^4 - 5x^2 + 1$ by $x^2 - x + 2$ b) $2x - 7$ by $x + 4$ c) $32x^5 - 1$ by $2x - 1$
12. Find the smallest value of $2a^2 + 3b^2$ if $a + b = 15$.
13. Find the coordinates of all points of intersection for the equations given.
 a) $(x + 1)^2 + (y - 4)^2 = 5$ and $(x - 5)^2 + (y - 2)^2 = 25$
 b) $(x + 3)^2 + y^2 = 10$ and $(x - 1)^2 + (y - 8)^2 = 10$
 c) $(x + 1)^2 + (y + 6)^2 = 25$ and $(x - 5)^2 + (y - 2)^2 = 25$
 d) $(x - 4)^2 + (y - 2)^2 = 13$ and $(x - 8)^2 + (y + 2)^2 = 53$
14. Find an equation of the tangent line drawn to the graph of $6y - 8x + x^2 + y^2 = -5$ at the point $(2, -7)$.
15. We place \$3000 into a bank account with an annual compound interest rate of 8%, compounded annually. How long does it take for the money to double in the account?
16. If we take Q amount of a certain medication, the amount of it in our system, t hours after intake is

$$A(t) = Q \left(\frac{4}{5} \right)^{0.6t}$$

- a) Approximately what percent of the medication is in our system 2 hours after taking it?
 b) How long until we have only 50% left in our system?
 c) How long until we have only 1% left in our system?
17. Graph each of the following functions.
 a) $f(x) = \sqrt{x}$ b) $f(x) = \log_2 x$ c) $f(x) = \frac{1}{x}$ d) $f(x) = \log_{1/2} x$
 e) $f(x) = 2^x$ f) $f(x) = \left(\frac{1}{2} \right)^x$ g) $f(x) = \frac{1}{x^2}$ h) $f(x) = -|x|$
18. We placed \$3000 into a bank account with an annual compound interest rate of 4%, compounded annually. How long until the account reaches
 a) \$10 000 b) \$20 000 c) \$30 000 d) \$40 000 e) \$50 000 f) \$60 000

Answers

- 1.) a) $\frac{1}{7}$ b) $7\sqrt{2}$ c) $\frac{9}{8y}$ d) $\frac{1}{b^2}$ e) $27x^3 - 1$ f) $\sqrt{3} + 2$ g) $\frac{1}{m}$ h) $\frac{1}{7}$
- 2.) a) $-\frac{3}{2}$ b) $\sqrt{5}$ c) $\sqrt{3} + 2$ d) 2 e) 1 f) $2 + \ln 5$ g) 8 h) $\frac{1}{2}$ i) 2 j) $-\frac{3}{4}$ k) $-\frac{1}{6}$
- 3.) a) no solution b) $(-\infty, -15] \cup [3, \infty)$ c) 6 d) $[1 - \sqrt{5}, 1 + \sqrt{5}]$ e) \mathbb{R} 4.) $\frac{1 + \sqrt{13}}{6}, \frac{1 - \sqrt{13}}{6}$

- 5.) a) $1 - \sqrt{21} < x < 1 + \sqrt{21}$ b) $x \geq 3, x \neq 21$ c) $x > 5, x \neq \sqrt{26}$ d) $0 \leq x \leq 5$ e) \mathbb{R}
 f) $-6 < x$ or $x > 5$ g) $x > 5$ h) $x > 1$ and $x \neq 2$ i) \mathbb{R} j) $x > 7$ but $x \neq 11$

- 6.) a) -2 b) 11 c) 9 d) 10 e) $\frac{5}{3}$ f) -5 g) no solution h) $\log_{3/2} 30 = \frac{\ln 30}{\ln 3 - \ln 2}$

i) $\log_{25/2} 40 = \frac{\ln 40}{\ln 25 - \ln 2}$ j) $1, -2$

- 7.) a) $x + 1$ b) $x + 2$ c) $2x + 1$ d) $2x + 3$ e) $x - 1$ f) $\frac{1}{x}$ g) $2 - 3x$ h) $\frac{x + 2}{x + 1}$

- 8.) a) $a + b$ b) $a + b + 1$ c) $a + b + 4$ d) $\frac{a + b}{a}$ e) $\frac{a + b + 1}{a + 1}$ 9.) a) $\frac{2}{a(b - 1)}$ b) $\frac{2x - 2 + \frac{4}{y}}{x + 1}$

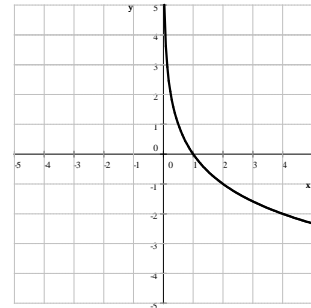
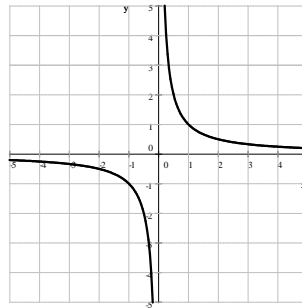
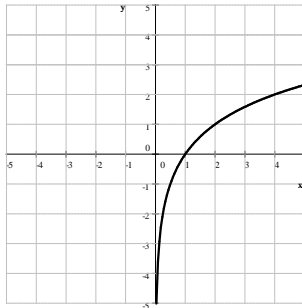
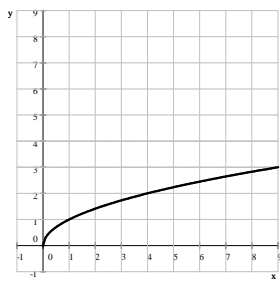
- 10.) a) $-\frac{625}{3}$ b) 250 11.) a) $x^2 + x - 6$ R $-8x + 13$ b) 2 R -15 c) $16x^4 + 8x^3 + 4x^2 + 2x + 1$ 12.) 270

- 13.) a) $(0, 2)$ and $(1, 5)$ b) no intersection point c) $(2, -2)$ d) $(1, 0)$ and $(6, 5)$

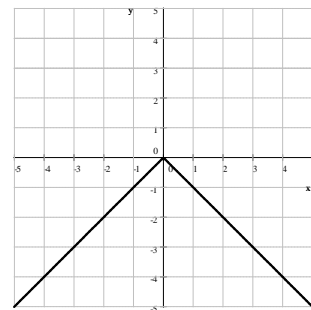
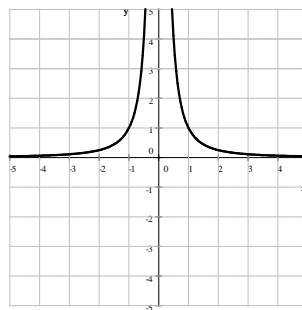
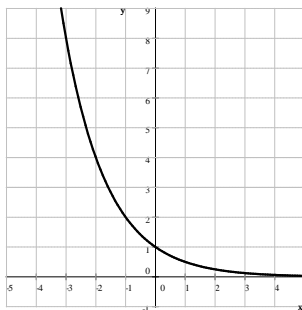
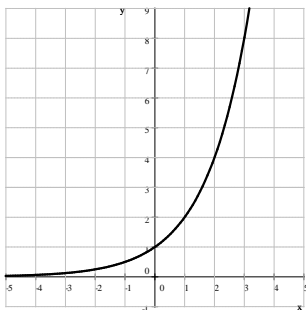
- 14.) $-\frac{1}{2}(x - 2) = y + 7$ or $y = -\frac{1}{2}x - 6$ 15.) $t = \frac{\ln 2}{\ln 1.08} \approx 9.00646834200060$ - a little bit more than 9 years

- 16.) a) 76.51% b) 5.18 hours c) 34.40 hours

- 17.) a) $f(x) = \sqrt{x}$ b) $f(x) = \log_2 x$ c) $f(x) = \frac{1}{x}$ d) $f(x) = \log_{1/2} x$



- e) $f(x) = 2^x$ f) $f(x) = \left(\frac{1}{2}\right)^x$ g) $f(x) = \frac{1}{x^2}$ h) $f(x) = -|x|$



- 18.) a) $\frac{\ln 10 - \ln 3}{\ln 1.04} \approx 30.6974$ during the 31st year b) $\frac{\ln 20 - \ln 3}{\ln 1.04} \approx 48.37036$ during the 49th year

- c) $\frac{\ln 10}{\ln 1.04} \approx 58.7084$ during the 59th year d) $\frac{\ln 40 - \ln 3}{\ln 1.04} \approx 66.043347$ during the 67th year

- e) $\frac{\ln 50 - \ln 3}{\ln 1.04} \approx 71.732778$ during the 72nd year f) $\frac{\ln 20}{\ln 1.04} \approx 76.3814$ during the 77th year