Sample Midterm Exam

1. Find the exact value for each of the following expressions.

   (a) $\cos 75^\circ =$

   (b) $\tan 22.5^\circ =$

   (c) $\cos 68^\circ \sin 8^\circ - \sin 68^\circ \cos 8^\circ =$

   (d) $\dfrac{\tan \dfrac{2\pi}{15} + \tan \dfrac{\pi}{5}}{1 - \left(\tan \dfrac{2\pi}{15}\right)\left(\tan \dfrac{\pi}{5}\right)} =$

2. Prove each of the following identities.

   (a) $\cot 2x = \dfrac{\cot^2 x - 1}{2 \cot x}$

   (b) $4 \sin^4 x = 1 - 2 \cos 2x + \cos^2 2x$

   (c) $\cos 3x = 4 \cos^3 x - 3 \cos x$

   (d) $\tan 3x = \tan x \dfrac{2 \cos 2x + 1}{2 \cos 2x - 1}$

3. Find the exact value of all solutions for each of the following equations. Present your answer in radians.

   (a) $2 + 3 \sin x = \cos 2x$

   (b) $\sin 2x = 2 \cos x$

   (c) $\cos 3x = -\dfrac{\sqrt{3}}{2}$

4. Suppose that $\sin \alpha = -\dfrac{8}{17}$ and α is not in the fourth quadrant; $\cos \beta = \dfrac{12}{13}$ and β is not in the first quadrant. Find the exact value for each of the following.

   (a) $\tan (\alpha - \beta) =$

   (b) $\cos (\alpha + \beta) =$

   (c) $\sin 2\alpha =$

   (d) $\sin \dfrac{\alpha}{2} =$

5. Find the exact value of $\tan \alpha$ if α is the acute angle formed by the lines $2x - 3y = 5$ and $5x + 3y = 1$.

6. Find the area of a regular polygon of 10 sides, inscribed in a circle of radius 5 m.