Please note that Exam 1 will also cover all material for the reviews for Quiz 1, 2, and 3. Please review those as well for Exam 1.

1. a) The radius of a circle is 40 cm. Find the central angle that subtends an arc of length 12 cm. Present the exact value and an approximation for the answer.

   b) Find the radius of the circle if we know that a sector determined by a central angle of 70° has an area of 18 m. Present the exact value and an approximation for the answer.

2. Find the height of a tree if the angle of elevation of its top changes from 10° to 14° as the observer advances 60 ft toward its base.

3. Find the exact value for each of the following expressions.

   a) $\dfrac{4\cos 45^\circ - 3\sin 60^\circ \sin 30^\circ}{\tan 60^\circ}$ b) $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

4. Prove each of the following identities.

   a) $\tan x + \dfrac{\cos x}{1+\sin x} = \dfrac{1}{\cos x}$

   b) $\tan^2 x + 1 = \sec^2 x$

   c) $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$

   d) $\tan x + \cot x = \sec x \csc x$

   e) $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

   f) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

5. a) Find the exact values for $\sin\alpha$ and $\cos\alpha$ if we know α is an acute angle with $\tan\alpha = \dfrac{3}{8}$.

   b) Find the exact values for $\sin\beta$ and $\cos\beta$ if we know β is an acute angle with $\cot\beta = M$.

   c) Re-write the results of the previous problem as a trigonometric identities.

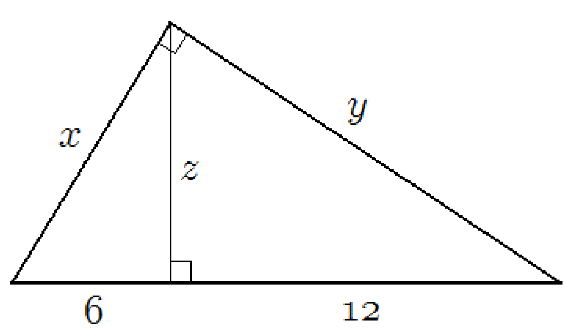
   d) Prove the identities you just wrote.

6. A satellite can be seen over the same point over the Equator. It is 200 miles above the surface. Find the speed of the satellite in miles per hour. (The radius of the earth is 3960 miles).

7. Find the angle that is formed by the positive part of the x-axis and the line $2y - 5x = -3$. Present the exact value and an approximation for the answer.

8. a) Find the exact values of x, y, and z based on the picture given.

   b) Find the exact and approximate values of the measure of the smallest angle in the triangle.



9. Find the area and perimeter of a 20−sided polygon that is written into a circle with radius 7 feet.

Answers

1. a) $\left(\dfrac{54}{\pi}\right)^\circ \approx 17.188\,733\,854^\circ$ b) $18\sqrt{\dfrac{2}{7\pi}}\,\mathrm{m} \approx 5.428\,3\,\mathrm{m}$

2. $36.133\,675\,39\,\mathrm{ft}$

3. a) $\dfrac{2}{3}\sqrt{6}-\dfrac{3}{4}$ b) 0

4. a) $\tan x + \dfrac{\cos x}{1+\sin x} = \dfrac{1}{\cos x}$

$$\text{LHS} = \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x}$$

bring to common denominator:

$$\begin{aligned}\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} &= \frac{\sin x\,(1+\sin x)}{\cos x\,(1+\sin x)} + \frac{\cos x\,(\cos x)}{(1+\sin x)\,(\cos x)} = \frac{\sin x\,(1+\sin x)+\cos^2 x}{\cos x\,(1+\sin x)}\\ &= \frac{\sin x+\sin^2 x+\cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x+1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \text{RHS}\end{aligned}$$

b) $\tan^2 x + 1 = \sec^2 x$

$$\text{LHS} = \tan^2 x + 1 = \left(\frac{\sin x}{\cos x}\right)^2 + 1 = \frac{\sin^2 x}{\cos^2 x} + 1$$

bring to common denominator

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

c) $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$

$$\begin{aligned}\text{LHS} &= \frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{1+\sin x}{(1-\sin x)\,(1+\sin x)} - \frac{1-\sin x}{(1+\sin x)\,(1-\sin x)}\\ &= \frac{1+\sin x-(1-\sin x)}{(1-\sin x)\,(1+\sin x)} = \frac{1+\sin x-1+\sin x}{1-\sin^2 x} = \frac{2\sin x}{\cos^2 x} = 2\cdot\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\\ &= 2\tan x\sec x = \text{RHS}\end{aligned}$$

d) $\tan x + \cot x = \sec x \csc x$

$$\begin{aligned}\text{LHS} &= \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x}{\sin x\cos x} + \frac{\cos^2 x}{\sin x\cos x} = \frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\\ &= \frac{1}{\sin x\cos x} = \frac{1}{\sin x}\cdot\frac{1}{\cos x} = \sec x\csc x = \text{RHS}\end{aligned}$$

e) $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

$$\begin{aligned}\text{LHS} &= \frac{1+\tan^2 x}{1-\tan^2 x} = \frac{1+\left(\dfrac{\sin x}{\cos x}\right)^2}{1-\left(\dfrac{\sin x}{\cos x}\right)^2} = \frac{1+\dfrac{\sin^2 x}{\cos^2 x}}{1-\dfrac{\sin^2 x}{\cos^2 x}} = \frac{\dfrac{\cos^2 x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}}{\dfrac{\cos^2 x}{\cos^2 x}-\dfrac{\sin^2 x}{\cos^2 x}}\\ &= \frac{\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x}}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}} = \frac{\dfrac{1}{\cos^2 x}}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}\end{aligned}$$

To divide is to multiply by the reciprocal:

$$\frac{\dfrac{1}{\cos^2 x}}{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}} = \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\cos^2 x - \sin^2 x} = \frac{1}{\cos^2 x - \sin^2 x} = \text{RHS}$$

f) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$$\begin{aligned} \text{LHS} &= \tan^2 x - \sin^2 x = \left(\frac{\sin x}{\cos x}\right)^2 - \sin^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x}{1} \\ &= \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x} = \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} \end{aligned}$$

We factor out $\sin^2 x$. Because $\sin^2 x + \cos^2 x = 1$, we also have that $1 - \cos^2 x = \sin^2 x$

$$\frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \frac{\sin^2 x \left(1 - \cos^2 x\right)}{\cos^2 x} = \frac{\sin^2 x \left(\sin^2 x\right)}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \left(\sin^2 x\right) = \tan^2 x \sin^2 x = \text{RHS}$$

5. a) $\sin\alpha = \dfrac{3}{\sqrt{73}}$ and $\cos\alpha = \dfrac{8}{\sqrt{73}}$ b) $\sin\beta = \dfrac{1}{\sqrt{1+M^2}}$ and $\cos\beta = \dfrac{M}{\sqrt{1+M^2}}$

c) $\sin\beta = \dfrac{1}{\sqrt{1+\cot^2\beta}}$ and $\cos\beta = \dfrac{\cot\beta}{\sqrt{1+\cot^2\beta}}$

d)

$$\begin{aligned} \text{RHS} &= \frac{1}{\sqrt{1+\cot^2\beta}} = \frac{1}{\sqrt{1+\dfrac{\cos^2\beta}{\sin^2\beta}}} = \frac{1}{\sqrt{\dfrac{\sin^2\beta}{\sin^2\beta}+\dfrac{\cos^2\beta}{\sin^2\beta}}} = \frac{1}{\sqrt{\dfrac{\sin^2\beta+\cos^2\beta}{\sin^2\beta}}} = \frac{1}{\sqrt{\dfrac{1}{\sin^2\beta}}} \\ &= \frac{1}{\dfrac{1}{\sin\beta}} = 1 \cdot \sin\beta = \sin\beta = \text{LHS} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{\cot\beta}{\sqrt{1+\cot^2\beta}} = \frac{\cot\beta}{\sqrt{1+\dfrac{\cos^2\beta}{\sin^2\beta}}} = \frac{\cot\beta}{\sqrt{\dfrac{\sin^2\beta}{\sin^2\beta}+\dfrac{\cos^2\beta}{\sin^2\beta}}} = \frac{\cot\beta}{\sqrt{\dfrac{\sin^2\beta+\cos^2\beta}{\sin^2\beta}}} = \frac{\cot\beta}{\sqrt{\dfrac{1}{\sin^2\beta}}} \\ &= \frac{\dfrac{\cos\beta}{\sin\beta}}{\dfrac{1}{\sin\beta}} = \frac{\cos\beta}{\sin\beta} \cdot \frac{\sin\beta}{1} = \cos\beta = \text{LHS} \end{aligned}$$

6. $1089.085 \dfrac{\text{mi}}{\text{h}}$ 7. $\tan^{-1}\left(\dfrac{5}{2}\right) \approx 68.198\,59^\circ$

8. a) $x = 6\sqrt{3}$, $y = 6\sqrt{6}$ and $z = \sqrt{72} = 6\sqrt{2}$ b) $\sin^{-1}\left(\dfrac{\sqrt{3}}{3}\right) \approx 35.264\,39^\circ$

9.) $P = 280 \sin 9^\circ \text{ ft} \approx 43.801\,65 \text{ ft}$ $A = 980 \sin 9^\circ \cos 9^\circ \text{ ft}^2 \approx 151.418\,327 \text{ ft}^2$

Last revised: February 4, 2015