Please note that Exam 2 will also cover topics covered on Quizzes 1-4 and Exam 1. Please review those topics as well, even if they do not appear in this document.

1. a) Suppose that α is an acute angle, i.e. $0 < \alpha < 90°$. Find the exact value of $\cos\alpha$ and $\tan\alpha$ if $\sin\alpha = \frac{1}{3}$. Rationalize the denominator in your answers.

   b) Suppose that β is an acute angle, i.e. $0 < \beta < 90°$. Find the value of $\sin\beta$ and $\cos\beta$ if $\tan\beta = 4$. Rationalize the denominator in your answer.

   c) Suppose that γ is an acute angle, i.e. $0 < \gamma < 90°$. Find the value of $\sin\gamma$, $\cos\gamma$, and $\tan\gamma$ if $\sec\gamma = \frac{5}{2}$. Rationalize the denominator in your answer.

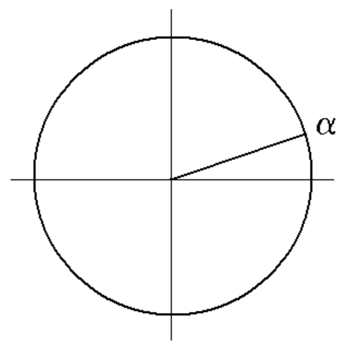
2. Simplify each of the following. (Write it in terms of $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$.

a) $\sin(\alpha + 360°)$	c) $\tan(\alpha - 180°)$	e) $\cos(-\alpha)$	g) $\sin(\alpha - 180°)$
b) $\cos(\alpha + 180°)$	d) $\sin(180° - \alpha)$	f) $\tan(-\alpha)$	h) $\tan(180° - \alpha)$

3. Consider the expression $\cos(180° - \alpha)$. All of the following expressions are equal to $\cos(180° - \alpha)$, except for one. Which one?

   A) $-\cos\alpha$ B) $-\sin(90° - \alpha)$ C) $\sin(\alpha - 90°)$ D) $\cos\alpha$ E) $-\cos(-\alpha)$

4. The picture shows an angle α on the unit circle. Draw each of the following angles in the same circle.
   a) $-\alpha$
   b) $\alpha - 180°$
   c) $\alpha + 90°$



5. a) Find the exact value of $\cos x$ if $\sin x = \frac{2}{5}$.

   b) Find the exact value of $\cos x$ if $\sin x = \frac{2}{5}$ and x is not in the first quadrant.

   c) Find the exact value of $\sin x$ if $\cot x = -2$ and x is an angle between $\frac{3\pi}{2}$ and 2π.

   d) Find the exact value of $\cos\theta$ given that $\tan\theta = M$.

6. Simplify each of the following. (i.e. write it in terms of trigonometric functions of α.)

a) $\sin(90° - \alpha)$	d) $\sin(\alpha + 180°)$	g) $\cos(-\alpha)$	j) $\tan(180° - \alpha)$
b) $\sin(180° - \alpha)$	e) $\cos(90° - \alpha)$	h) $\cos(\alpha + 180°)$	k) $\tan(-\alpha)$
c) $\sin(-\alpha)$	f) $\cos(180° - \alpha)$	i) $\tan(90° - \alpha)$	l) $\tan(\alpha + 180°)$

7. Simplify each of the following. Present exact values. Rationalize the denominator.

a) $\sin\left(\frac{5\pi}{3}\right) - 2\tan\left(-\frac{\pi}{6}\right) + \sin\left(\frac{7\pi}{6}\right)$

b) $\sin\left(\frac{5\pi}{2}\right) - \cos\left(\frac{7\pi}{3}\right) + \tan\left(\frac{3\pi}{4}\right) - \cos(7\pi)$

c) $\cos 420° - \tan 210° + \sec 240° + \cot 135°$

d) $\cos 1° + \cos 2° + \cos 3° + \ldots.. + \cos 179° + \cos 180°$

e) $\dfrac{\sin\left(\frac{3\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right)}{1 + 4\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{5\pi}{6}\right)}$

f) $\dfrac{\sin 30° \sin 45° \sin 60°}{\tan 30° - \tan 60°}$

8. Solve each of the following equations. You may present your answer in degrees.

a) $\sin x = -\frac{1}{\sqrt{2}}$

b) $\cos x = -\frac{\sqrt{3}}{2}$

c) $\tan x = -\sqrt{3}$

d) $\sin x = -\frac{3}{2}$

e) $\cos x = -1$

f) $\tan x = 0$

g) $\sin\alpha = -\frac{\sqrt{3}}{2}$

h) $\cos\beta = -1$

i) $\tan\gamma = \frac{2}{3}$

j) $\sin 3x = -1$

k) $\cos 5x = \frac{1}{\sqrt{2}}$

l) $\tan 2x = \sqrt{3}$

9. Circle C_1 has a radius 5 unit long. Circle C_2 has a radius 11 unit long. The centers are at a distance of 12 units from each other. We draw the common tangent lines drawn to the circles.

a) Find an approximation of the angle formed by the two tangent lines.

b) Compute the distance between the two points of tangency on one of the common tangent lines.

10. Seattle, WA and San Francisco, CA are located approximately on the same longitude. The latitude of these cities are 47.5° N and 37.4° N. Find the distance between the two cities assuming that the Earth is a sphere with radius 3960 miles. Round your answer to the nearest mile.

11. Consider the pyramid $ABCDE$ if its base is a square $ABCD$ with sides 6 m and side $AE = BE = CE = DE = 10$ m. Compute an approximate value for the angle that is formed between a triangular face and the base.

12. Prove each of the following identities.

a) $\tan x + \dfrac{\cos x}{1 + \sin x} = \sec x$

b) $\dfrac{\cos x}{1 - \sin x} = \sec x + \tan x$

c) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

d) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

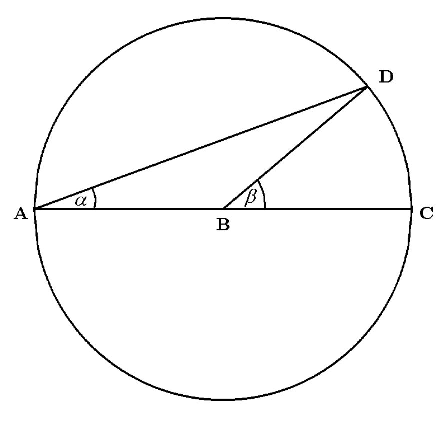
13. Compute an approximate value for each of the angles in a triangle with sides 7 cm, 7 cm, and 10 cm.

14. We drew an n-sided regular polygon into a circle with radius R. In terms of R and n, express

a) the perimeter of the polygon

b) the area of the polygon

15. Consider the picture given. Given that B is the center of the circle, prove that $\beta = 2\alpha$.



16. We are driving toward a tall building. At some time, the angle of elevation from the car to the top of the building is $41°$. After we drive 100 feet toward the building, the angle of elevation changes to $53°$. How tall is the building?

Answers

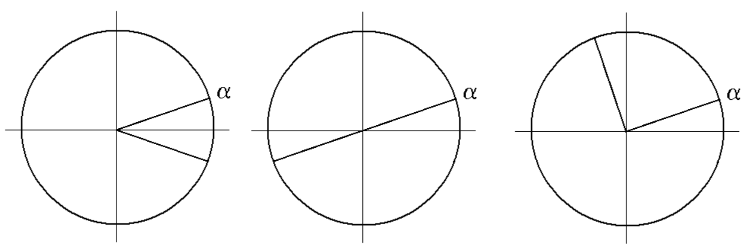
1. a) $\cos\alpha = \dfrac{\sqrt{8}}{3} = \dfrac{2\sqrt{2}}{3}$ $\tan\alpha = \dfrac{\sqrt{2}}{4}$ b) $\sin\beta = \dfrac{4\sqrt{17}}{17}$ and $\cos\beta = \dfrac{\sqrt{17}}{17}$

   c) $\sin\gamma = \dfrac{\sqrt{21}}{5}$, $\cos\gamma = \dfrac{2}{5}$, and $\tan\gamma = \dfrac{\sqrt{21}}{2}$

2. a) $\sin\alpha$ b) $-\cos\alpha$ c) $\tan\alpha$ d) $\sin\alpha$ e) $\cos\alpha$ f) $-\tan\alpha$ g) $-\sin\alpha$ h) $-\tan\alpha$

3. D

4. a) b) c)



5. a) $\pm\dfrac{\sqrt{21}}{5}$ b) $-\dfrac{\sqrt{21}}{5}$ c) $-\dfrac{\sqrt{5}}{5}$ d) $\pm\dfrac{1}{\sqrt{M^2+1}}$

6. a) $\cos\alpha$ b) $\sin\alpha$ c) $-\sin\alpha$ d) $-\sin\alpha$ e) $\sin\alpha$ f) $-\cos\alpha$ g) $\cos\alpha$ h) $-\cos\alpha$ i) $\cot\alpha$
   j) $-\tan\alpha$ k) $-\tan\alpha$ l) $\tan\alpha$

7. a) $\dfrac{\sqrt{3}-3}{6}$ b) $\dfrac{1}{2}$ c) $-\dfrac{\sqrt{3}}{3} - \dfrac{5}{2}$ d) -1 e) $-\dfrac{\sqrt{2}+\sqrt{6}}{2}$ f) $-\dfrac{3\sqrt{2}}{16}$

8. a) $x = -45° + k\cdot 360°$ or $x = -135° + k\cdot 360°$ where $k \in \mathbb{Z}$

   $x = -\dfrac{\pi}{4} + 2k\pi$ or $x = -\dfrac{3\pi}{4} + 2k\pi$ where $k \in \mathbb{Z}$

   b) $x = \pm 150° + k\cdot 360°$ where $k \in \mathbb{Z}$ $x = \pm\dfrac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

   c) $x = -60° + k\cdot 180°$ where $k \in \mathbb{Z}$ $x = -\dfrac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$

   d) no solution e) $x = 180° + k\cdot 360°$ where $k \in \mathbb{Z}$ $x = \pi + 2k\pi$ where $k \in \mathbb{Z}$

   f) $x = k\cdot 180°$ where $k \in \mathbb{Z}$ $x = k\pi$ where $k \in \mathbb{Z}$

   g) $\alpha = -60° + k\cdot 360°$ or $\alpha = -120° + k\cdot 360°$ where $k \in \mathbb{Z}$

   h) $\beta = 180° + k\cdot 360°$ where $k \in \mathbb{Z}$ i) $\gamma = 33.690\,07° + k\cdot 180°$ where $k \in \mathbb{Z}$

   j) $-30° + k\cdot 120°$ where $k \in \mathbb{Z}$ $-\dfrac{\pi}{6} + \dfrac{2}{3}k\pi$ where $k \in \mathbb{Z}$

   k) $\pm 9° + k\cdot 72°$ where $k \in \mathbb{Z}$ $\pm\dfrac{\pi}{20} + \dfrac{2}{5}k\pi$ where $k \in \mathbb{Z}$

   l) $30° + k\cdot 90°$ where $k \in \mathbb{Z}$ $\dfrac{\pi}{6} + \dfrac{1}{2}k\pi$ where $k \in \mathbb{Z}$

9. a) 60° b) $\sqrt{108} = 6\sqrt{3}$

10. 698 miles

11. $71.670\,246\,2^\circ$

12. a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\begin{aligned} \text{RHS} &= \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x\,(1+\sin x) + \cos^2 x}{\cos x\,(1+\sin x)} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x + 1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \sec x = \text{LHS} \end{aligned}$$

b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} \\ &= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = \text{RHS} \end{aligned}$$

c) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

$$\text{LHS} = \frac{\csc^2 x - 1}{\csc^2 x} = \frac{\dfrac{1}{\sin^2 x} - 1}{\dfrac{1}{\sin^2 x}} = \frac{\dfrac{1}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}} = \frac{\dfrac{1-\sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}}$$

To divide is to multiply by the reciprocal:

$$\frac{\dfrac{1-\sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}} = \frac{1-\sin^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1} = 1 - \sin^2 x = \cos^2 x = \text{RHS}$$

d) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1-\tan x}{1+\tan x}$

Solution:

$$\text{LHS} = \frac{\cot x - 1}{\cot x + 1} = \frac{\dfrac{\cos x}{\sin x} - 1}{\dfrac{\cos x}{\sin x} + 1}$$

Multiply numerator and denominatior by $\sin x$

$$\frac{\dfrac{\cos x}{\sin x} - 1}{\dfrac{\cos x}{\sin x} + 1} = \frac{\cos x - \sin x}{\cos x + \sin x}$$

Divide numerator and denominator by $\cos x$

$$\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\dfrac{\cos x - \sin x}{\cos x}}{\dfrac{\cos x + \sin x}{\cos x}} = \frac{\dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\cos x} + \dfrac{\sin x}{\cos x}} = \frac{1-\tan x}{1+\tan x} = \text{RHS}$$

13. $44.415\,3^\circ$, $44.415\,3^\circ$, and $91.169\,4^\circ$

14. a) $2nR\sin\left(\dfrac{180^\circ}{n}\right)$ b) $nR\sin\left(\dfrac{180^\circ}{n}\right)R\cos\left(\dfrac{180^\circ}{n}\right) = nR^2\sin\left(\dfrac{180^\circ}{n}\right)\cos\left(\dfrac{180^\circ}{n}\right)$

15. Line segments AB and BD are both radii in the circle, and so they are equal. So ABD triangle is isosceles and so the angles opposite AB and BD are also equal to each other. Thus $\measuredangle ADB = \alpha$. The third angle in triangle ADB is $180^\circ - 2\alpha$. Angles ABD and DBC are supplementary beause together they form a straight angle. Thus

$$\begin{aligned} \measuredangle ABD + \measuredangle DBC &= 180^\circ \\ 180^\circ - 2\alpha + \beta &= 180^\circ && \text{subtract } 180^\circ \\ -2\alpha + \beta &= 0 && \text{add } 2\alpha \\ \beta &= 2\alpha \end{aligned}$$

16. $\dfrac{100\tan 53^\circ \tan 41^\circ}{\tan 53^\circ - \tan 41^\circ}\text{ ft} \approx 252.007\,011\text{ ft}$

Last revised: February 25, 2015