Please note that Quiz 10 will also cover material from reviews from previous material. Please review those topics even if they do not appear in this document.

1. Simplify each of the following.

   a) $\dfrac{2\cos^2\dfrac{\pi}{8}-1}{1+8\sin^2\dfrac{\pi}{8}\cos^2\dfrac{\pi}{8}}$ b) $\cos\dfrac{23\pi}{4}-\sin\dfrac{15\pi}{4}$ c) $\dfrac{1-\tan 75^\circ}{1+\tan 75^\circ}$

2. Prove each of the following.

   a) $\sin 35^\circ+\sin 25^\circ=\cos 5^\circ$ b) $\cos 12^\circ-\cos 48^\circ=\sin 18^\circ$ c) $1+\tan\alpha\tan\beta=\dfrac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta}$

3. Solve each of the following.

   a) $\sin 2x-\cos x=0$ b) $\sin 2x=\dfrac{\sqrt{3}}{2}$ c) $\sin x-\sqrt{3}\cos x=-1$ d) $\sin 5x-\sqrt{3}\cos 5x=-1$

4. Find the exact value of the cosine of the largest angle in a triangle with sides 10 cm, 8 cm, and 7 cm long.

5. Compute each of the following.

   a) $\sin(37.5^\circ)$ (Hint: 37.5° is half of what angle?) c) $\cos\left(\dfrac{225^\circ}{2}\right)$

   b) $\tan\alpha$ if $\sin 2\alpha=\dfrac{2}{3}$ and 2α is in the first quadrant.

6. For each of the following functions given, sketch its graph and state its domain and range.

   a) $f(x)=\sin^{-1}x$ b) $g(x)=\cos^{-1}x$ c) $h(x)=\tan^{-1}x$

7. Simplify each of the following.

   a) $\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$

   b) $\sin^{-1}(-1)$

   c) $\sin\left(\cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)\right)$

   d) $\cos^{-1}\left(\sin\left(\dfrac{3\pi}{4}\right)\right)$

   e) $\cos\left(\cos^{-1}\left(\dfrac{1}{2}\right)\right)$

   f) $\cos^{-1}\left(\cos\left(-\dfrac{\pi}{3}\right)\right)$

   g) $\sin\left(\cos^{-1}\left(-\dfrac{2}{3}\right)\right)$

   h) $\cos\left(\tan^{-1}\left(-\dfrac{1}{2}\right)\right)$

   i) $\sin\left(\tan^{-1}\left(-\dfrac{1}{2}\right)\right)$

   j) $\tan\left(\sin^{-1}\left(-\dfrac{3}{5}\right)\right)$

   k) $\sin\left(2\cos^{-1}\left(\dfrac{2}{5}\right)\right)$

   l) $\cos\left(2\tan^{-1}(-3)\right)$

   m) $\tan\left(2\tan^{-1}(-3)\right)$

   n) $\sin\left(\sin^{-1}\left(-\dfrac{3}{5}\right)+\cos^{-1}\left(-\dfrac{12}{13}\right)\right)$

   o) $\sin\left(\dfrac{1}{2}\cos^{-1}\left(\dfrac{2}{3}\right)\right)$

   p) $\cos\left(\dfrac{1}{2}\sin^{-1}\left(-\dfrac{8}{17}\right)\right)$

   q) $\tan\left(\dfrac{1}{2}\cos^{-1}\left(-\dfrac{1}{3}\right)\right)$

   r) $\sin\left(\cos^{-1}x\right)$

   s) $\cos\left(\tan^{-1}x\right)$

   t) $\sin\left(\tan^{-1}x\right)$

8. Find $\tan\beta$ if we know that $\tan\alpha=\dfrac{1}{2}$ and $\tan(\alpha+\beta)=\dfrac{13}{11}$.

9. Two sides of a triangle are 8 ft and 15 ft long. Find the exact value of the third side if we know that the area of the triangle is $48\,\text{ft}^2$.

10. Solve each of the following triangles.

    a) $a=6\,\text{m}$, $c=5\,\text{m}$, $\gamma=38^\circ$ b) $a=7\,\text{m}$, $c=3\,\text{m}$, $\gamma=28^\circ$ c) $a=6\,\text{m}$, $c=5\,\text{m}$, $\beta=38^\circ$

11. Suppose that $\underline{v} = 3\underline{i} - 2\underline{j}$, $\underline{w} = \underline{i} + 3\underline{j}$, and $\underline{u} = -\underline{i} + 4\underline{j}$. Compute each of the following.
    a) $\underline{v} + \underline{w} + \underline{u}$ b) $\underline{v} - 2\underline{w} + 3\underline{u}$
    c) Find real numbers A, B, and C, not all zero, so that $A\underline{v} + B\underline{w} + C\underline{u} = \underline{0}$.

12. (Enrichment) Suppose that $ABCD$ is a trapezoid with parallel sides $AB = a$ and $CD = b$, $a > b$. The angles at A and D are right angles. Also suppose that the bisector of angle CBA intersects side AD at its midpoint. Express side AD of this trapezoid in terms of a and b.

Answers

1. a) $\frac{\sqrt{2}}{4}$ b) $\sqrt{2}$ c) $-\frac{\sqrt{3}}{3}$

2. a) $\sin 35^\circ + \sin 25^\circ = \cos 5^\circ$

$$\begin{aligned} \sin 35^\circ + \sin 25^\circ &= \sin(30^\circ + 5^\circ) + \sin(30^\circ - 5^\circ) \\ &= \sin 30^\circ \cos 5^\circ + \cos 30^\circ \sin 5^\circ + \sin 30^\circ \cos 5^\circ - \cos 30^\circ \sin 5^\circ \\ &= 2 \sin 30^\circ \cos 5^\circ = 2\left(\frac{1}{2}\right)\cos 5^\circ = \cos 5^\circ \end{aligned}$$

   b) $\cos 12^\circ - \cos 48^\circ = \sin 18^\circ$

$$\begin{aligned} \cos 12^\circ - \cos 48^\circ &= \cos(30^\circ - 18^\circ) - \cos(30^\circ + 18^\circ) \\ &= \cos 30^\circ \cos 18^\circ + \sin 30^\circ \sin 18^\circ - (\cos 30^\circ \cos 18^\circ - \sin 30^\circ \sin 18^\circ) \\ &= \cos 30^\circ \cos 18^\circ + \sin 30^\circ \sin 18^\circ - \cos 30^\circ \cos 18^\circ + \sin 30^\circ \sin 18^\circ \\ &= 2 \sin 30^\circ \sin 18^\circ = 2\left(\frac{1}{2}\right)\sin 18^\circ = \sin 18^\circ \end{aligned}$$

   c) $1 + \tan\alpha \tan\beta = \dfrac{\cos(\alpha - \beta)}{\cos\alpha \cos\beta}$

$$\begin{aligned} \text{LHS} &= 1 + \tan\alpha\tan\beta = 1 + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta} \\ &= \frac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = \text{RHS} \end{aligned}$$

3. a) $\frac{\pi}{2} + k\pi$ $\frac{\pi}{6} + 2k\pi$ $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ b) $x = \frac{\pi}{6} + k\pi$ or $x = \frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$
   c) $-\frac{\pi}{2} + 2k\pi$ or $\frac{\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ d) $-\frac{\pi}{10} + \frac{2k\pi}{5}$ or $\frac{\pi}{30} + \frac{2k\pi}{5}$ where $k \in \mathbb{Z}$

4. $\frac{13}{112}$

5. a) $\sqrt{\frac{\sqrt{2} + 4 - \sqrt{6}}{8}}$ b) $\frac{3 - \sqrt{5}}{2}$ c) $-\frac{1}{2}\sqrt{2 - \sqrt{2}}$

6. see handout

7. a) $-\frac{\pi}{6}$ b) $-\frac{\pi}{2}$ c) $\frac{\sqrt{2}}{2}$ d) $\frac{\pi}{4}$ e) $\frac{1}{2}$ f) $\frac{\pi}{3}$ g) $\frac{\sqrt{5}}{3}$ h) $\frac{2\sqrt{5}}{5}$ i) $-\frac{\sqrt{5}}{5}$
   j) $-\frac{3}{4}$ k) $\frac{4\sqrt{21}}{25}$ l) $-\frac{4}{5}$ m) $\frac{3}{4}$ n) $\frac{56}{65}$ o) $\frac{\sqrt{6}}{6}$ p) $\frac{4\sqrt{17}}{17}$ q) $\sqrt{2}$
   r) $\sqrt{1-x^2}$ s) $\frac{1}{\sqrt{x^2+1}}$ t) $\frac{x}{\sqrt{x^2+1}}$

8. $\frac{3}{7}$

9. $\sqrt{145}\,\text{ft}$ or $\sqrt{433}\,\text{ft}$

10. a) $\alpha_1 \approx 47.628\,764°$, $\beta_1 \approx 94.371\,236°$, $b_1 \approx 8.097\,722\,\text{m}$
    $\alpha_2 \approx 132.371\,236°$, $\beta_2 \approx 9.628\,764°$, $b_2 \approx 1.358\,41\,\text{m}$
    b) no solution c) $\alpha \approx 85.789\,71°$, $b \approx 3.703\,965\,\text{m}$, $\gamma \approx 56.210\,29°$

11. a) $3\underline{i}+5\underline{j}$ b) $-2\underline{i}+4\underline{j}$ c) (answers may vary) $A=7$, $B=-10$, $C=11$ works

12. $2\sqrt{ab}$

Last revised: April 22, 2015