Please note that Quiz 3 will also cover topics covered on Quizzes 1 and 2. Please review those topics as well, even if they do not appear on this document.

1. Find the radius of the circle if an arc subtended by a central angle of $30°$ has a length of $20\,\text{cm}$.
2. Find the radius of the circle if a sector of $30°$ has an area of $20\,\text{cm}^2$.
3. Simplify each of the following.

   a) $\sin 30° \sin 45° \sin 60° + \cos 30° \cos 45° \cos 60°$

   b) $\tan 30° - \tan 45° + \tan 60°$

   c) $\cos^2 17° + \sin^2 17°$

   d) $\cos^2 30° - \sin^2 30°$

4. We are driving toward a tower. The angle of elevation is $35°$. Then we drive $50\,\text{ft}$ toward the tower. Now the angle of elevation is $40°$. How tall is the tower? Present your answer as an approximation, accurate up to three decimal places.
5. Prove each of the following identities.

   a) $1 + \tan^2 x = \sec^2 x$

   b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

   c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right) \cos x$

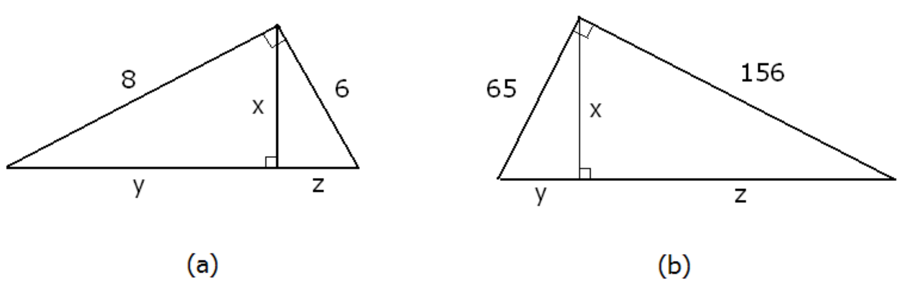
   d) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

   e) $\cos x\,(\sec x - \cos x) = \sin^2 x$

   f) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$

   g) $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

6. a) Solve the equation $9x^2 - 12x = 1$. b) Check your solution using exact values.
7. Find the exact value of x, y, and z, based on the figures shown below.



8. An arch is in the shape of a semicircle. At a point along the base 2 feet from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch.
9. Seattle, WA and Los Angeles, CA have approximately the same longitude. The latitude of Seattle is $47.6°$ and that of Los Angeles is $34.1°$. Find the distance to the nearest mile between the two cities. (The radius of the earth is approximately 3960 miles.)
10. A satellite is 1000 miles above the surface of the equator. The radius of Earth is 3960 miles. To the nearest miles per hour, how fast is the satellite moving if it can be seen at the same point in the sky?
11. Find the exact value of each of the following expressions.

    a) $3 \sin 30° \cos 60° - 2 \sin 60° \cos 30°$

    b) $2 \sin 60° + 2 \sin 30° - 2 \tan 45°$

    c) $\sin^2 16° + \cos^2 16°$

    d) $\dfrac{\cos 40°}{\sin 50°}$

    e) $\dfrac{\sec 45° \cos 30° - \csc 60°}{\sin 60°}$

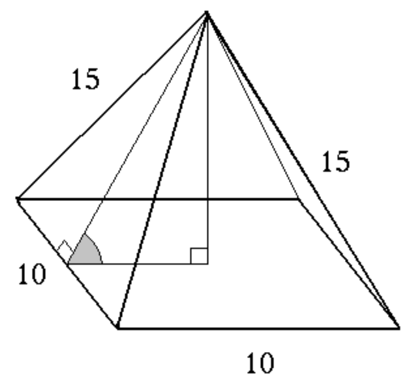
    f) $\dfrac{\cot 30° - \tan^2 60°}{\cot 30° + \tan^2 60°}$

    g) $\sin 30° \cos 60° - \cos 30° \sin 60°$

    h) $\cot 30° + \cot 45° + \cot 60°$

    i) $\sin^4 45° + \cos^4 45°$

12. The picture shows a straight pyramid with a square base. The sides of the base are 10 in long. All other edges are 15 in long.
a) Find the height of a triangular face.
b) Use part a) to find the height of the pyramid.
c) Find an approximate value of the angle formed between the base and a triangular face. (This angle is marked on the picture.)



13. Compute the exact value of the area of an equilateral triangle with sides 12 meters.

14. Suppose that C_1 and C_2 are circles such that an arc subtended in C_1 by a central angle of $20°$ has the same length as an arc subtended by C_2 by a central angle of $50°$. What is the ratio between the areas of C_1 and C_2?

15. Simplify each of the following.

a) $(3\sqrt{7}-2)(\sqrt{7}+1)$

b) $\dfrac{3-\sqrt{5}}{\sqrt{5}-1}$

c) $(3\sqrt{2}-1)^2$

d) $(3\sqrt{2}-1)^3$

e) $\dfrac{24-\sqrt{40}}{2}$

f) $\dfrac{\sqrt{12}-\sqrt{75}}{\sqrt{3}}$

g) $2\sqrt{80}-5\sqrt{45}+\sqrt{500}$

h) $\dfrac{11}{3\sqrt{5}-1}$

i) Find the exact value of $-a^2-5a+8$ if $a=3\sqrt{5}-1$

16. Rationalize the denominator in $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

17. Find all real numbers that are exactly 1 greater than their own reciprocal.

18. Find the exact value of the longest line segment that can be drawn inside the rectangular prism with sides 5 cm, 8 cm, and 10 cm long.

19. Consider the right triangle ABC. Find all sides if $\gamma = 90°$, $\beta = 24°$ and $a = 17$. Present your answer as an approximation, accurate up to at least four decimal places.

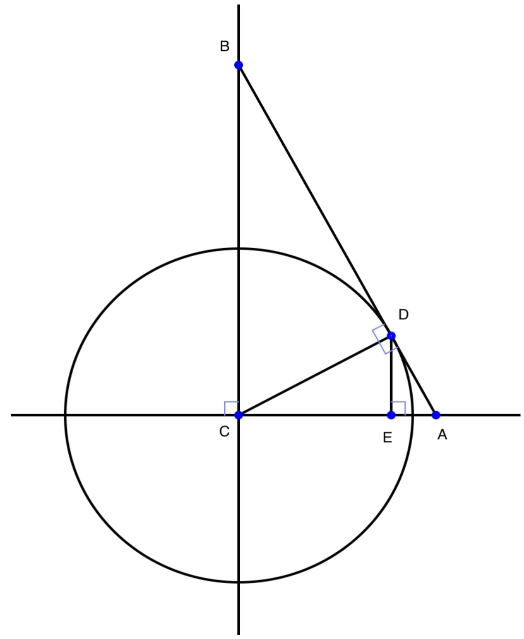
20. True or false? For all angles α, $\sin 2\alpha = 2\sin\alpha$.

21. Find the area of a regular 9−sided polygon written into a circle of radius 10 cm.

22. Consider the triangle with sides 6 m, 6 m, and 8 m long.
a) Compute the exact value of the area of the triangle.
b) Compute the exact value of the smallest angle in the triangle.
c) Compute an approximate value of the smallest angle in the triangle.

23. Suppose that C is a circle with radius 8 meters. Let P be a point at a distance of 20 meters from C. Find the approximate value of the angle that is formed between the two tangent lines drawn to the circle.

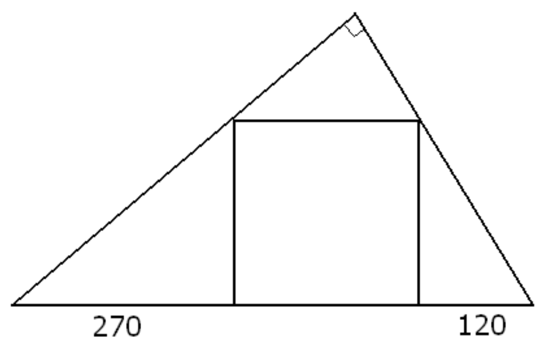
24. Consider the picture given. Line AB is tangent to the unit circle, where D is the point of tangency. Let α denote angle DCE. Match each of the six trigonometric functions with the length of each of the line segments given.
$\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha, \sec\alpha, \cot\alpha$
and
AC, AD, BC, BD, CE, DE



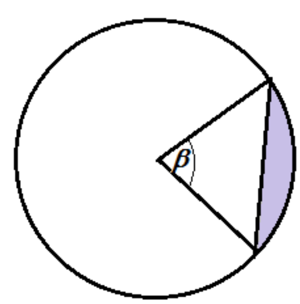
25. a) Find the exact and approximate values of the smaller angle that is formed between the positive part of the $x-$axis and the line $y = 2x - 1$.

b) Find an equation for the straight line that passes through the point $P(4, -2)$ and forms a $60°$ angle with the positive part of the $x-$axis.

26. The picture shows a square within a right triangle.
a) Find the length of the sides in the square.
b) Compute the angles in the triangle.



27. Compute the area of the shaded region shown on the picture below. The radius of the circle is 3 meters and $\beta = 80°$. Present the exact value and approximate value of the answer.



28. (Enrichment) Let ABC be a right triangle with hypotenuse AB. Let h denote the altitude drawn to the hypotenuse. Let D denote the point where h intersects the hypotenuse. Let x denote line segment AD and y denote line segment BD. Prove that $h = \sqrt{xy}$.

29. (Enrichment) A satellite is 1000 miles above the surface over Chicago. The radius of Earth is 3960 miles, and the latitude of Chicago is 42°N. To the nearest miles per hour, how fast is the satellite moving if it can be seen at the same point in the sky?

Answers

1. $r = \dfrac{120}{\pi}\,\text{cm} \approx 38.197\,186\,342\,\text{cm}$

2. $r = \sqrt{\dfrac{240}{\pi}}\,\text{cm} \approx 8.740\,387\ \text{cm}$

3. a) $\dfrac{\sqrt{6}}{4}$ b) $\dfrac{4\sqrt{3}}{3} - 1$ c) 1 d) $\dfrac{1}{2}$

4. $211.510\,92\,\text{ft}$

5. a) $1 + \tan^2 x = \sec^2 x$

$$\text{LHS} = 1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

$$\text{LHS} = \frac{\cot x - 1}{\cot x + 1} = \frac{\dfrac{1}{\tan x} - 1}{\dfrac{1}{\tan x} + 1} \cdot \frac{\tan x}{\tan x} = \frac{1 - \tan x}{1 + \tan x} = \text{RHS}$$

c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$

$$\text{RHS} = \left(\sin^2 x - \sin^4 x\right)\cos x = \sin^2 x\left(1 - \sin^2 x\right)\cos x = \sin^2 x \cos^2 x \cos x = \text{LHS}$$

d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

$$\text{LHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + 2\sin x \cos x = \text{RHS}$$

e) $\cos x\,(\sec x - \cos x) = \sin^2 x$

$$\text{LHS} = \cos x\,(\sec x - \cos x) = \cos x\left(\frac{1}{\cos x} - \cos x\right) = 1 - \cos^2 x = \sin^2 x = \text{RHS}$$

f) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$

$$\text{LHS} = \frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\sin x\,(1 + \cos x)}{1 - \cos^2 x} = \frac{\sin x\,(1 + \cos x)}{\sin^2 x} = \frac{1 + \cos x}{\sin x} = \text{RHS}$$

g) $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 4\tan x \sec x$

$$\begin{aligned}\text{LHS} &= \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{(1 + \sin x)^2 - (1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{\left(1 + \sin^2 x + 2\sin x\right) - \left(1 + \sin^2 x - 2\sin x\right)}{1 - \sin^2 x} \\ &= \frac{1 + \sin^2 x + 2\sin x - 1 - \sin^2 x + 2\sin x}{\cos^2 x} = \frac{4\sin x}{\cos^2 x} = 4\frac{\sin x}{\cos x}\frac{1}{\cos x} = 4\tan x \sec x = \text{RHS}\end{aligned}$$

6. a) $\dfrac{2 \pm \sqrt{5}}{3}$ b) If $x = \dfrac{2 + \sqrt{5}}{3}$, then

$$\begin{aligned}\text{LHS} &= 9\left(\frac{2 + \sqrt{5}}{3}\right)^2 - 12\left(\frac{2 + \sqrt{5}}{3}\right) = 9\frac{\left(2 + \sqrt{5}\right)^2}{3^2} - 4\left(2 + \sqrt{5}\right) \\ &= \left(2 + \sqrt{5}\right)^2 - 4\left(2 + \sqrt{5}\right) = 4 + 5 + 4\sqrt{5} - 8 - 4\sqrt{5} = 1 = \text{RHS}\end{aligned}$$

and if $x = \dfrac{2-\sqrt{5}}{3}$, then

$$\begin{aligned} \text{LHS} &= 9\left(\frac{2-\sqrt{5}}{3}\right)^2 - 12\left(\frac{2-\sqrt{5}}{3}\right) = 9\frac{\left(2-\sqrt{5}\right)^2}{3^2} - 4\left(2-\sqrt{5}\right) \\ &= \left(2-\sqrt{5}\right)^2 - 4\left(2-\sqrt{5}\right) = 4+5-4\sqrt{5}-8+4\sqrt{5} = 1 = \text{RHS} \end{aligned}$$

7. a) $x = \dfrac{24}{5} = 4.8$, $y = \dfrac{32}{5} = 6.4$, $z = \dfrac{18}{5} = 3.6$ b) $x = 60$, $y = 25$, $z = 144$

8. 17 feet

9. 933 miles

10. $1299 \dfrac{\text{mi}}{\text{h}}$

11. a) $-\dfrac{3}{4}$ b) $\sqrt{3}-1$ c) 1 d) 1 e) $\sqrt{2}-\dfrac{4}{3}$ f) $\sqrt{3}-2$ g) $-\dfrac{1}{2}$ h) $\dfrac{4\sqrt{3}}{3}+1$ i) $\dfrac{1}{2}$

12. a) $10\sqrt{2}\,\text{in} \approx 14.142\,136\,\text{in}$ b) $5\sqrt{7}\,\text{in} \approx 13.228\,757$ in c) $69.295\,189°$

13. $36\sqrt{3}\,\text{m}^2$

14. $\dfrac{A_1}{A_2} = \dfrac{25}{4}$

15. a) $19+\sqrt{7}$ b) $\dfrac{\sqrt{5}-1}{2}$ c) $19-6\sqrt{2}$ d) $-55+63\sqrt{2}$ e) $12-\sqrt{10}$ f) -3

g) $3\sqrt{5}$ h) $\dfrac{3\sqrt{5}+1}{4}$ i) $-33-9\sqrt{5}$

16. $\dfrac{7+2\sqrt{10}}{3}$

17. $x_1 = \dfrac{1+\sqrt{5}}{2}$ and $x_2 = \dfrac{1-\sqrt{5}}{2}$

18. $\sqrt{189}\,\text{cm} = 3\sqrt{21}\,\text{cm}$

19. $c \approx 18.608\,816\,7$ $b \approx 7.568\,888$

20. False. For example, consider $\alpha = 30°$. Then $\sin 2\alpha = \sin 60° = \dfrac{\sqrt{3}}{2}$ and $2\sin\alpha = 2\sin 30° = 2\left(\dfrac{1}{2}\right) = 1$.

21. $900\cos 20° \sin 20°\,\text{cm}^2 \approx 289.254\,424\,36\,\text{cm}^2$

22. a) $8\sqrt{5}\,\text{m}^2$ b) $\cos^{-1}\left(\dfrac{4}{6}\right)$ c) $48.1897°$

23. $47.156\,357°$

24. $\sin\alpha = DE$ $\cos\alpha = CE$ $\tan\alpha = AD$ $\csc\alpha = BC$ $\sec\alpha = AC$ $\cot\alpha = BD$

25. a) $\tan^{-1} 2 \approx 63.434949$ b) $y+2 = \sqrt{3}\,(x-4)$

26. a) 180 b) $\alpha = \tan^{-1}\left(\dfrac{180}{270}\right) \approx 33.6900675°$ $\beta = \tan^{-1}\left(\dfrac{180}{120}\right) \approx 56.3099325°$

27. $(2\pi - 9\sin 40° \cos 40°)\ \text{m}^2 \approx 1.851\,55\,\text{m}^2$