

Please note that Quiz 4 will also cover topics covered on Quizzes 1-3 and Exam 1. Please review those topics as well, even if they do not appear on this document.

1. Convert to radians.

- a) 45° b) -120° c) 10° d) 135° e) 270°

2. Convert to degrees.

- a) $\frac{\pi}{6}$ b) $-\frac{\pi}{9}$ c) $\frac{5\pi}{12}$ d) -3π e) $\frac{7\pi}{4}$

3. Use the unit circle to find the exact value of each of the following expressions.

- a) $\sin 120^\circ$ c) $\tan \frac{7\pi}{4}$ e) $\cos(-90^\circ)$ g) $\sec\left(-\frac{5\pi}{3}\right)$ i) $\tan(-270^\circ)$
 b) $\cos 120^\circ$ d) $\sin \frac{5\pi}{6}$ f) $\cos \frac{5\pi}{6}$ h) $\sin\left(-\frac{\pi}{2}\right)$ j) $\sec\left(\frac{5\pi}{3}\right)$

4. The following are all equal to each other, except for one. Which one?

- A) $\cos 150^\circ$ B) $\sin(-60^\circ)$ C) $\cos 210^\circ$ D) $\cos(-120^\circ)$ E) $\sin 300^\circ$

5. Simplify each of the following expressions. Use exact values and rationalize denominators.

- a) $\frac{\tan 120^\circ - \tan 45^\circ}{1 + \tan 120^\circ \tan 45^\circ}$ b) $\frac{\sin(-240^\circ) \cos 135^\circ - \sin 30^\circ \sin 315^\circ \tan 300^\circ}{\sin 150^\circ \sec 225^\circ \tan(-30^\circ)}$ c) $\cot 300^\circ \sec 210^\circ \tan^2 240^\circ$

6. Which of the following angles are co-terminal? Group co-terminal angles together.

- -585° 630° 270° -225° -180° 660° 180° -540°
 135° 300° 540° -90° 495° -780° -60°

7. Find all angles coterminal with 120° . Express your answer

- a) in degrees b) in radians
 c) Find all coterminal angles α such that $-500^\circ < \alpha < 500^\circ$.

8. True or false?

- a) If an angle α is co-terminal with 40° , then 2α is co-terminal with 80° .
 b) If twice an angle 2β is co-terminal with 20° , then β is co-terminal with 10° .

9. a) Compute the exact value of all six trigonometric function values of α if α is an acute angle with $\cos \alpha = \frac{3}{7}$
 b) Compute the exact value of all six trigonometric function values of β if β is an acute angle with $\tan \beta = M$

10. Simplify each of the following.

- a) $\log_3(\tan 60^\circ) + \log_2(\sin 45^\circ) - \ln(\tan 45^\circ)$ c) $\frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$
 b) $\sec 30^\circ - \csc 45^\circ + \cot 60^\circ$ d) $\tan^2 30^\circ - \tan^2 60^\circ$

11. a) Find the radius if a circle if we know that the length of an arc subtended by a central angle of 85° is 28 cm.
 b) Find the radius if a circle if we know that the area of a sector defined by a central angle of 35° is 75 cm^2 .

12. Compute the exact value of each of the following.

a) $\sin 120^\circ \cos 225^\circ - \tan(-60^\circ)$

b) $\sec\left(\frac{\pi}{4}\right) - \csc\left(\frac{\pi}{6}\right) + \sin^2\left(\frac{\pi}{3}\right)$

c) $2 \sin 150^\circ \cos 240^\circ$

d) $\sin^4(-225^\circ) + \cos^4(-135^\circ)$

e) $\sin 28^\circ - \cos 62^\circ$

f)
$$\frac{\sin(-45^\circ) \cos 300^\circ - \cos(-45^\circ) \sin 300^\circ}{\tan 225^\circ \tan 330^\circ}$$

g)
$$\frac{\sin\left(\frac{7\pi}{4}\right) - \tan\left(\frac{7\pi}{6}\right)}{\sin\left(\frac{7\pi}{6}\right) + \tan\left(\frac{9\pi}{4}\right)}$$

h)
$$\frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(\frac{\pi}{3}\right) \tan\left(\frac{\pi}{4}\right)}$$

13. a) Compute the exact value of $\sin 30^\circ + \sin 30^\circ$

b) Compute the exact value of $\sin 60^\circ$

c) Based on your findings, determine whether the following statement is true or false:

For all angles α and β , $\sin(\alpha + \beta) = \sin \alpha + \sin \beta$.

d) True or false? *For all angles α , $\sin 2\alpha = 2 \sin \alpha$*

14. a) Compute the exact value of $\tan 30^\circ + \tan 30^\circ$

b) Compute the exact value of $\tan 60^\circ$

c) Based on your findings, determine whether the following statement is true or false:

For all angles α and β , $\tan(\alpha + \beta) = \tan \alpha + \tan \beta$.

d) True or false? *For all angles α , $\tan 2\alpha = 2 \tan \alpha$*

15. Suppose that C is a circle with radius 12 feet. Compute the exact and approximate values of each of the following.

a) The length of an arc subtended by a central angle of 42° .

b) The area of a sector determined by a central angle of 42° .

16. a) Find the speed of the satellite that orbits the Earth above the equator at a height of 1200 miles and appears at the same point in the sky at all times. Round your answer to the nearest miles per hour. (Assume that Earth is a sphere with radius 3960 mi)

b*) Find the speed of the satellite that orbits the Earth above Chicago at a height of 1200 miles and appears at the same point in the sky at all times. Assume that Earth is a sphere with radius 3960 mi and that the latitude of Chicago is $42^\circ N$. Round your answer to the nearest miles per hour.

17. How fast is the endpoint of the minute pointer moving on my watch if the pointer is 1.2 centimeters long? Express your answer in centimeters per minute.

18. Berlin, Germany and Copenhagen, Denmark have nearly the same longitude. Use this fact to find the distance between the two cities if the latitude of Berlin is $52.5^\circ N$ and the latitude of Copenhagen is $55.7^\circ N$. Assume that Earth is a sphere with radius 3960 mi. Round your answer to the nearest mile.

19. Compute the exact and approximate values for the perimeter and area of an 18-sided regular polygon that was written in a circle with radius 7 centimeters.

20. We are driving away from a tower. First, the angle of elevation from the car to the top of the tower is 52° . After we drove 300 feet away from the tower, the angle of elevation from the car to the top of the tower is 32° . How tall is the tower?

21. Find the approximate values of the angles in a right triangle where for angle β we know that $\sin \beta = 2 \cos \beta$.

22. Prove each of the following identities.

$$\text{a) } \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$$

$$\text{c) } \frac{1 - \cos x}{1 + \cos x} = \frac{\sec x - 1}{\sec x + 1}$$

$$\text{b) } \tan x - \csc x \sec x (1 - 2 \cos^2 x) = \cot x$$

$$\text{d) } \frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$$

23. Consider a circle with radius 10 m. Let P be a point 35 m away from the center of the circle. Compute the angle formed between the two tangent lines drawn from P to the circle.

24. Compute the area of a triangle with sides 15 m, 15 m, and 22 m.

25. (Enrichment) At noon, the minute and hour hands of the clock exactly overlap. At what time does that happen again for the first time after noon?

Answers

$$1. \text{ a) } \frac{\pi}{4} \quad \text{b) } -\frac{2\pi}{3} \quad \text{c) } \frac{\pi}{18} \quad \text{d) } \frac{3\pi}{4} \quad \text{e) } \frac{3\pi}{2}$$

$$2. \text{ a) } 30^\circ \quad \text{b) } -20^\circ \quad \text{c) } 75^\circ \quad \text{d) } -540^\circ \quad \text{e) } 315^\circ$$

$$3. \text{ a) } \frac{\sqrt{3}}{2} \quad \text{b) } -\frac{1}{2} \quad \text{c) } -1 \quad \text{d) } \frac{1}{2} \quad \text{e) } 0 \quad \text{f) } -\frac{\sqrt{3}}{2} \quad \text{g) } 2 \quad \text{h) } -1 \quad \text{i) } \text{undefined} \quad \text{j) } 2$$

4. D

$$5. \text{ a) } \sqrt{3} + 2 \quad \text{b) } -3 \quad \text{c) } 2$$

$$6. \text{ Group 1: } 135^\circ, 495^\circ, -225^\circ, -585^\circ \quad \text{Group 2: } -60^\circ, 300^\circ, 660^\circ, -780^\circ$$

$$\text{Group 3: } 180^\circ, -180^\circ, 540^\circ, -540^\circ \quad \text{Group 4: } -90^\circ, 270^\circ, 630^\circ$$

$$7. \text{ a) } 120^\circ + k \cdot 360^\circ \text{ where } k = 0, 1, -1, 2, -2, 3, -3, \dots \quad \text{c) } -240^\circ, 120^\circ, 480^\circ$$

$$\text{b) } \frac{2\pi}{3} + 2k\pi \text{ where } k = 0, 1, -1, 2, -2, 3, -3, \dots$$

8. True or false?

a) If an angle α is co-terminal with 40° , then 2α is co-terminal with 80° .

$$\text{True: } \alpha = 40^\circ + k \cdot 360^\circ \implies 2\alpha = 80^\circ + k \cdot 720^\circ$$

b) If twice an angle 2β is co-terminal with 20° , then β is co-terminal with 10° .

False: Let $\beta = 190^\circ$. Clearly 10° and 190° are not co-terminal, but $2\beta = 380^\circ$ which is co-terminal with 20° .

$$9. \text{ a) } \sin \alpha = \frac{2\sqrt{10}}{7} \quad \cos \alpha = \frac{3}{7} \quad \tan \alpha = \frac{2\sqrt{10}}{3} \quad \csc \alpha = \frac{7\sqrt{10}}{20} \quad \sec \alpha = \frac{7}{3} \quad \cot \alpha = \frac{3\sqrt{10}}{20}$$

$$\text{b) } \sin \beta = \frac{M}{\sqrt{1+M^2}} \quad \cos \beta = \frac{1}{\sqrt{1+M^2}} \quad \tan \beta = M \quad \csc \beta = \frac{\sqrt{1+M^2}}{M} \quad \sec \beta = \sqrt{1+M^2} \quad \cot \beta = \frac{1}{M}$$

$$10. \text{ a) } 0 \quad \text{b) } \sqrt{3} - \sqrt{2} \quad \text{c) } 2 - \sqrt{3} \quad \text{d) } -\frac{8}{3}$$

$$11. \text{ a) } \frac{1008}{17\pi} \text{ cm} \approx 18.87390384 \text{ cm} \quad \text{b) } \frac{30}{7} \sqrt{\frac{42}{\pi}} \text{ cm} \approx 15.6701417 \text{ cm}$$

12. a) $\sqrt{3} - \frac{\sqrt{6}}{4}$ b) $\sqrt{2} - \frac{5}{4}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$ e) 0 f) $\frac{\sqrt{6} - 3\sqrt{2}}{4}$ g) $-\sqrt{2} - \frac{2}{3}\sqrt{3}$ h) $2 - \sqrt{3}$

13. a) 1 b) $\frac{\sqrt{3}}{2}$ c) False, it clearly failed for $\alpha = \beta = 30^\circ$. d) False, it clearly failed for $\alpha = 30^\circ$

14. a) $\frac{2\sqrt{3}}{3}$ b) $\sqrt{3}$ c) False, it clearly failed for $\alpha = \beta = 30^\circ$. d) False, it clearly failed for $\alpha = 30^\circ$

15. a) $\frac{14}{5}\pi$ ft ≈ 8.79646 ft b) $\frac{84}{5}\pi$ ft ≈ 52.77875658 ft

16. a) $1351 \frac{\text{mi}}{\text{h}}$ b*) $1004 \frac{\text{mi}}{\text{h}}$

17. $0.04\pi \frac{\text{cm}}{\text{min}} \approx 0.1256637 \frac{\text{cm}}{\text{min}}$

18. 221 mi

19. $P = 252 \sin 10^\circ \text{ cm} \approx 43.759341 \text{ cm}$ $A = 882 \sin 10^\circ \cos 10^\circ \text{ cm}^2 \approx 150.830883 \text{ cm}^2$

20. 366.2785 ft

21. $\beta = \tan^{-1} 2 \approx 63.43495^\circ$ and $\alpha = 90^\circ - \beta \approx 26.56505^\circ$

22. a) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

$$\text{LHS} = \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\left(\frac{1}{\sin x}\right)} + \frac{\cos x}{\left(\frac{1}{\cos x}\right)} = \sin^2 x + \cos^2 x = 1 = \text{RHS}$$

b) $\tan x - \csc x \sec x (1 - 2 \cos^2 x) = \cot x$

$$\begin{aligned} \text{LHS} &= \tan x - \csc x \sec x (1 - 2 \cos^2 x) = \frac{\sin x}{\cos x} - \frac{1}{\sin x} \left(\frac{1}{\cos x}\right) (1 - 2 \cos^2 x) \\ &= \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos x}{\sin x} \\ &= \frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x - 1 + 2 \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x - 1 + \cos^2 x}{\sin x \cos x} \\ &= \frac{1 - 1 + \cos^2 x}{\sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x = \text{RHS} \end{aligned}$$

c) $\frac{1 - \cos x}{1 + \cos x} = \frac{\sec x - 1}{\sec x + 1}$

$$\text{RHS} = \frac{\sec x - 1}{\sec x + 1} = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1} = \frac{\frac{1}{\cos x} - \frac{\cos x}{\cos x}}{\frac{1}{\cos x} + \frac{\cos x}{\cos x}} = \frac{\frac{1 - \cos x}{\cos x}}{\frac{1 + \cos x}{\cos x}} = \frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} = \text{LHS}$$

d) $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

$$\begin{aligned} \text{LHS} &= \frac{\sec x + \csc x}{\tan x + \cot x} = \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{\sin x}{\cos x \sin x} + \frac{\cos x}{\sin x \cos x}}{\frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\sin x \cos x}} = \frac{\frac{\sin x + \cos x}{\cos x \sin x}}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}} = \frac{\sin x + \cos x}{\frac{\cos x \sin x}{1}} \\ &= \frac{\sin x + \cos x}{\cos x \sin x} \cdot \frac{\cos x \sin x}{1} = \sin x + \cos x = \text{RHS} \end{aligned}$$

23. $2 \sin^{-1} \left(\frac{10}{35} \right) \approx 33.2031^\circ$

24. $A = 22\sqrt{26} \text{ m}^2 \approx 112.17843 \text{ m}^2$

25. $1\frac{1}{11}$ hour after noon, that is approximately 1 hour, 5 minutes, and 27 seconds after 12:00 noon.

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