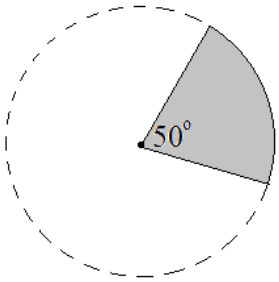
Please note that Quiz 5 will also cover topics covered on Quizzes 1-4 and Exam 2. Please review those topics as well, even if they do not appear on this document.

1. Graph each of the following finctions on $[-2\pi, 2\pi]$ and state domain, range, and basic properties for each function.

   a) $f(x) = \sin x$ b) $f(x) = \cos x$ c) $f(x) = \tan x$

2. Find the perimeter and area of the sector shown on the picture. We know that the circle has a radius of 12 cm.



3. A regular polygon of 12 sides is written into a circle of radius 5 ft. Find the area of the polygon.

4. Convert 3 radians to degrees. Present both exact value and approximations.

5. Prove each of the following identities.

   a) $1 - 2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$

   b) $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

   c) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

   d) $\dfrac{1 + 2\sin x \cos x}{\sin^2 x - \cos^2 x} = \dfrac{\tan x + 1}{\tan x - 1}$

6. a) Find the exact value of $\tan\theta$ if we know that $\cos\theta = \dfrac{1}{5}$.

   b) Find the exact value of $\cos\beta$ if we know that $\sin\beta = \dfrac{\sqrt{8}}{3}$ and β is not in the first quadrant.

7. True or false?

   a) For any angle, the angle and its opposite have the same cosine.

   b) An angle and its opposite have the same sine.

   c) If α is any angle in radians and k is any integer, then $\tan\alpha = \tan(\alpha + k\pi)$

   d) For all angles α and β, if 2α and 2β are coterminal, then so are α and β.

8. The following are all equivalent except for one. Which one?

   A) $\cos\theta$ B) $\sin\left(\dfrac{\pi}{2} - \theta\right)$ C) $-\cos(\pi - \theta)$ D) $\sin(\theta - \pi)$ E) $\cot\theta\sin\theta$

9. Find the exact value of each of the following expressions.

   a) $\cos 660° - \sin 675°$

   b) $\cos\left(-\dfrac{13\pi}{6}\right) + \sin\dfrac{13\pi}{4} - \cot\dfrac{9\pi}{2}$

   c) $\dfrac{\cos 60°\cos 30° - \sin 60°\sin 30°}{\tan 45°\sec 60°}$

   d) $\dfrac{\tan\left(\dfrac{7\pi}{6}\right) - \tan\left(\dfrac{7\pi}{4}\right)}{1 + \tan\left(\dfrac{7\pi}{6}\right)\left(\tan\left(\dfrac{7\pi}{4}\right)\right)}$

   e) $\tan 3\pi + \cos^2 9\pi - 3\tan\dfrac{7\pi}{3}$

   f) $\sin^2\left(\dfrac{18\pi}{7}\right) + \cos^2\left(\dfrac{18\pi}{7}\right)$

10. Solve each of the following equations.

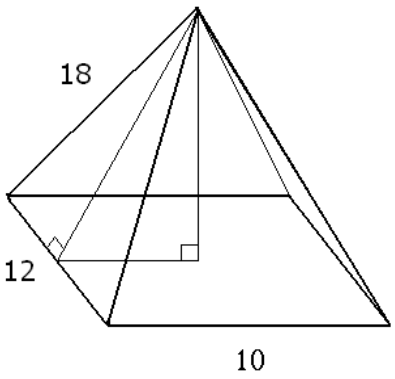
    a) $\cos x = -1$ b) $\sin \alpha = \dfrac{\sqrt{3}}{2}$ c) $\sin 2x = 1$

11. Find all angles α between 0° and 180° with the property that $\sin 4\alpha = \dfrac{1}{2}$.

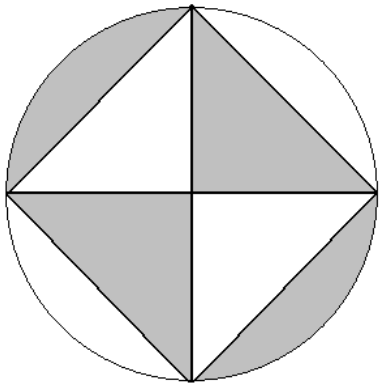
12. a) An object is traveling around a circle of radius 18 m. It completes a cycle in every 5 seconds. Find the speed of the object in meters per second.
    b) An object is traveling around a circle of radius 9 m. It completes a cycle in every 5 seconds. Find the speed of the object in meters per second.

13. Aberdeen, Scotland and Barcelona, Spain have nearly the same longitude. Use this fact to find the distance between the two cities if the latitude of Aberdeen is 57.2° N and the latitude of Barcelona is 41.4° N. Assume that Earth is a sphere with radius 3960 mi. Round your answer to the nearest mile.

14. a) Find the exact value of the height of the pyramid shown on the picture.
    b) Find the angle that is formed between the base and a triangular face.



15. Find the area of the shaded region on the picture given. The area of the circle is $4\,\text{m}^2$.

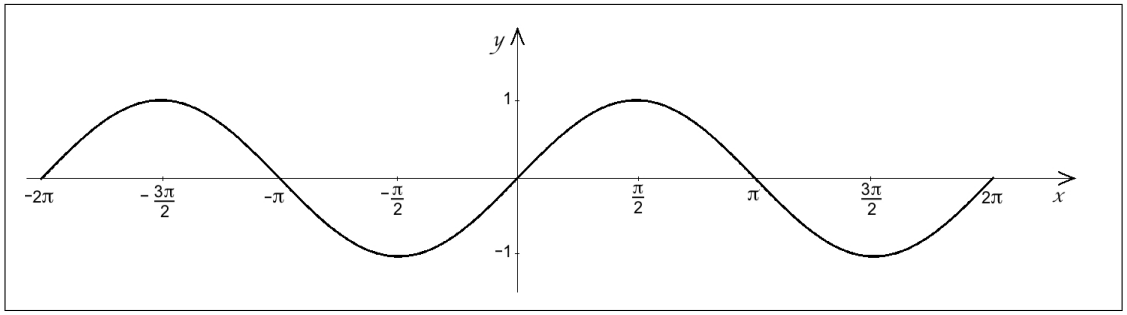


Answers

1. a) $f(x) = \sin x$

domain: $\mathbb{R}$
range: $[-1, 1]$

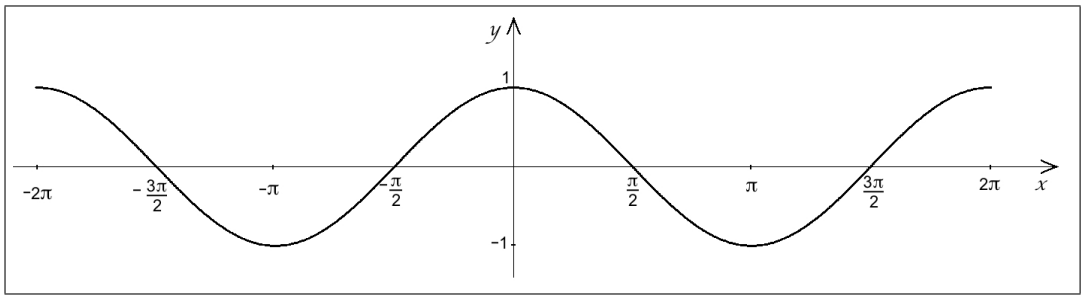
periodic with period 2π: for all x, $\sin(x + 2\pi) = \sin x$
odd function: for all x, $\sin(-x) = -\sin x$



b) $f(x) = \cos x$

domain: $\mathbb{R}$
range: $[-1, 1]$

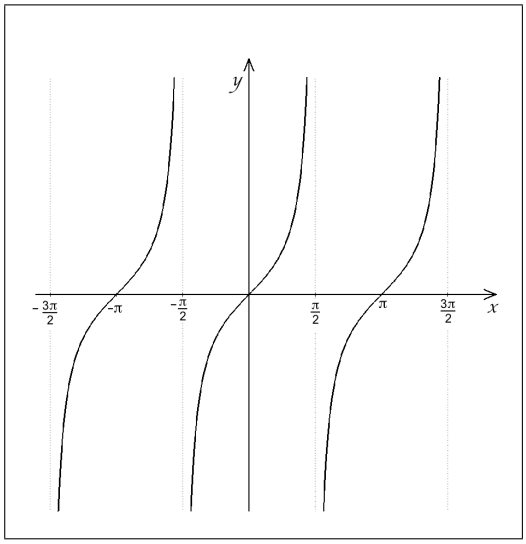
periodic with period 2π: for all x, $\cos(x + 2\pi) = \cos x$
even function: for all x, $\cos(-x) = \cos x$



c) $f(x) = \tan x$

domain: $x \neq \dfrac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$
range: $\mathbb{R}$
periodic with period π: for all x, $\tan(x + \pi) = \tan x$

vertical asymptotes at $x = \dfrac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$
odd function: for all x, $\tan(-x) = -\tan x$



2. $P = \left(24 + \frac{10}{3}\pi\right)\text{ cm} \approx 34.471\,975\,512\text{ cm} \qquad A = 20\pi\text{ cm}^2 \approx 62.831\,853\,0718\text{ cm}^2$

3. 75 ft^2

4. $\frac{540}{\pi} = 171.887°$

5. a) $1 - 2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$

Solution:

$$\begin{aligned}\text{LHS} &= \frac{\tan^2 x - 1}{\tan^2 x + 1} = \frac{\frac{\sin^2 x}{\cos^2 x} - 1}{\frac{\sin^2 x}{\cos^2 x} + 1} = \frac{\frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}} = \frac{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}} \\ &= \frac{\sin^2 x - \cos^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x - \cos^2 x}{1} \\ &= \sin^2 x - \cos^2 x = \left(1 - \cos^2 x\right) - \cos^2 x = 1 - 2\cos^2 x = \text{RHS}\end{aligned}$$

b) $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

Solution:

$$\begin{aligned}\text{RHS} &= \frac{\cos x}{1 - \sin x} = \frac{\cos x}{1 - \sin x} \cdot 1 = \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x\,(1 + \sin x)}{(1 - \sin x)\,(1 + \sin x)} \\ &= \frac{\cos x\,(1 + \sin x)}{1 - \sin^2 x} = \frac{\cos x\,(1 + \sin x)}{\cos^2 x} = \frac{1 + \sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \text{LHS}\end{aligned}$$

c) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

Solution:

$$\begin{aligned}\text{LHS} &= \sin^4 x - \cos^4 x = \left(\sin^2 x\right)^2 - \left(\cos^2 x\right)^2 = \left(\sin^2 x + \cos^2 x\right)\left(\sin^2 x - \cos^2 x\right) \\ &= 1 \cdot \left(\sin^2 x - \cos^2 x\right) = \left(1 - \cos^2 x\right) - \cos^2 x = 1 - 2\cos^2 x = \text{RHS}\end{aligned}$$

d) $\dfrac{1 + 2\sin x \cos x}{\sin^2 x - \cos^2 x} = \dfrac{\tan x + 1}{\tan x - 1}$

$$\begin{aligned}\text{RHS} &= \frac{\tan x + 1}{\tan x - 1} = \frac{\frac{\sin x}{\cos x} + 1}{\frac{\sin x}{\cos x} - 1} = \frac{\frac{\sin x}{\cos x} + 1}{\frac{\sin x}{\cos x} - 1} \cdot \frac{\cos x}{\cos x} = \frac{\sin x + \cos x}{\sin x - \cos x} = \frac{\sin x + \cos x}{\sin x - \cos x} \cdot \frac{\sin x + \cos x}{\sin x + \cos x} \\ &= \frac{(\sin x + \cos x)^2}{\sin^2 x - \cos^2 x} = \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x}{\sin^2 x - \cos^2 x} = \frac{1 + 2\sin x \cos x}{\sin^2 x - \cos^2 x} = \text{LHS}\end{aligned}$$

6. a) $\pm 2\sqrt{6}$ b) $-\frac{1}{3}$ 7. a) true b) false c) true d) false 8.) D

9. a) $\frac{\sqrt{2}}{2} + \frac{1}{2}$ b) $\frac{\sqrt{3} - \sqrt{2}}{2}$ c) 0 d) $\sqrt{3} + 2$ e) $1 - 3\sqrt{3}$ f) 1

10. a) $\pi + 2k\pi, \quad k \in \mathbb{Z}$ b) $\frac{\pi}{3} + 2k\pi, \quad \frac{2\pi}{3} + 2l\pi \quad k, l \in \mathbb{Z}$ c) $\frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}$

11. $7.5°,\ 37.5°,\ 97.5°,\ 127.5°$ 12. a) $7.2\pi\frac{\text{m}}{\text{s}} \approx 22.619\,5\frac{\text{m}}{\text{s}}$ b) $3.6\pi\frac{\text{m}}{\text{s}} \approx 11.309\,75\frac{\text{m}}{\text{s}}$ 13. 1092 miles

14. a) $\sqrt{263}$ b) $\cos^{-1}\left(\frac{5}{12\sqrt{2}}\right) \approx 72.864\,778\,5°$ 15. 2 m^2