

Review Problems

- Let f be the function defined by $f(x) = x^2 - 4x + 9$.
 - Compute $f(3)$.
 - Is it true that $f(2+3) = f(2) + f(3)$?
 - Simplify $f(2a)$
- Let f be the function defined by $f(x) = \sqrt{x-5}$.
 - Compute $f(3)$.
 - Compute $f(9) + f(21)$.
 - Compute $f(9+21)$.
- Given that $f(x) = 2x - 3$ and $g(x) = x^2 + 5$ compute each of the following.
 - $(f+g)(5)$
 - $\left(\frac{f}{g}\right)(4)$
 - $g(f(2))$
 - $g(f(0))$
 - $(f+g)(-2)$
 - $f(g(2))$
 - $f(g(0))$
- Compute each of the following limits. Show computation.
 - $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^2)$
 - $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^2)$
 - $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^6)$
 - $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^6)$
 - $\lim_{x \rightarrow \infty} \log_2 x$
 - $\lim_{x \rightarrow -\infty} \log_2 x$
 - $\lim_{x \rightarrow -\infty} 1.02^x$
 - $\lim_{x \rightarrow \infty} 1.02^x$
 - $\lim_{x \rightarrow \infty} \log_5 x$
 - $\lim_{x \rightarrow \infty} \left(\frac{3}{5}\right)^x$
 - $\lim_{x \rightarrow \infty} \frac{2^{x+5}}{4^{x-1}}$
 - $\lim_{x \rightarrow \infty} \frac{3^{x+1} \cdot \left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$
 - $\lim_{x \rightarrow \infty} \frac{5^{x+3}}{3^{2x-1}}$
- Solve the following system of equations.

$$\begin{aligned} 2^x &= 8^{y+1} \\ 9^y &= 3^{x-9} \end{aligned}$$
- Compute the exact value of each of the following expressions.

$$A = (4\sqrt{18} - 5\sqrt{50} + 3\sqrt{98})(2\sqrt{2}) \quad C = 2 \log_{10} 2 + \log_{10} 18 + 6 \log_{10} \sqrt{5} - 2 \log_{10} 3$$

$$B = 6^{-2} \cdot 16^{-1/2} \cdot 36^2 \cdot 4^{-3/2}$$
- Find the domain for each of the following functions.
 - $f(x) = \sqrt{25x - x^3}$
 - $f(x) = \sqrt{\frac{x}{25 - x^2}}$
- Find the exact value of $\cos \alpha$ where α is the acute angle formed by the lines $y = x$ and $y = 3x$.
- If $\cot A = 2$, then what is the exact value of $\sin 2A$?
- Let l be the line $y = \frac{3}{4}x$. Find an equation for the line that bisects the angle formed between l and the positive part of the x -axis.

11. Let x be an angle such that $\cos x < 0$ and $3 \sin^2 x = 5 \sin x + 2$. Find the exact value of $\sin 2x$.
12. Write each of the following as a sum or difference.
a) $\sin 13A \sin 7A$ b) $\cos 6x \sin 8x$ c) $\cos 5\alpha \cos \alpha$ d) $\sin 4y \cos 10y$
13. Write each of the following as a product.
a) $\sin 13A + \sin 7A$ b) $\cos 6x + \cos 8x$ c) $\sin 5m - \sin 11m$ d) $\cos 4y - \cos 18y$
14. A right triangle has hypotenuse 25 and one leg 7. Find the exact value of the length of the altitude to the hypotenuse.
15. An equilateral triangle is inscribed in a circle. What percentage of the area of the circle is covered by the triangle? Round to the nearest whole percent.
16. Which of the following is NOT equivalent to $\cos(x + 270^\circ)$?
A) $\sin x$ B) $\cos(x - 90^\circ)$ C) $\cos(90^\circ - x)$ D) $\sin(360^\circ - x)$ E) $\sin(180^\circ - x)$
17. A geometric sequence has a first term of $\tan t$ and a common ratio of $2 \cos t$. Which of the following is the third term?
A) $4 \sin t$ B) $2 \sin 2t$ C) $4 \sec t$ D) $4 \cos 2t$ E) $4 \cos^2 t$
18. A ship is 800 miles from land when its engine breaks. In the first hour, the ship covers 75 miles. After that, the engine loses 5% of its capacity in every hour.
a) How long would it take for the ship to travel 500 miles with the broken engine?
b) What is the greatest distance the ship could cover with the broken engine?
19. Write an equation for all straight lines that are parallel to $y = x + 17$ and tangent to the circle $x^2 + y^2 - 6x - 6y - 14 = 0$.
20. Suppose that three numbers are consecutive elements of a geometric sequence. The product of these numbers is 216. If we decrease the third number by 3, we obtain consecutive elements of an arithmetic sequence. Find the first element and common ratio in the geometric sequence.
21. Suppose that three numbers are consecutive elements of a geometric sequence. The product of these numbers is 125. These numbers, in the same order, are also the first, third, and sixth elements in an arithmetic sequence. Find the first element and common ratio in the geometric sequence.
22. Let a, b, c, d all be different positive numbers. Is it true that $\log_{a/b} \left(\frac{c}{d} \right) = \log_{b/a} \left(\frac{d}{c} \right)$

Review Problems- Answers

- 1.) a) $f(3) = 6$ b) $f(5) = 14$ and $f(2) + f(3) = 11$ c) $f(2a) = 4a^2 - 8a + 9$
- 2.) a) $f(3) = \text{undefined}$ b) $f(9) + f(21) = 6$ c) $f(30) = 5$
- 3.) a) 37 b) 2 c) $\frac{5}{21}$ d) 15 e) 6 f) 7 g) 14
- 4.) a) ∞ b) $-\infty$ c) ∞ d) ∞ e) ∞ f) undefined g) 0 h) ∞ i) ∞ j) 0
k) 0 l) 3 m) 0
- 5.) $x = 21, y = 6$ 6.) $A = 32, B = \frac{9}{8}, C = 3$ 7.) a) $(-\infty, -5] \cup [0, 5]$ b) $(-\infty, -5) \cup [0, 5)$
- 8.) $\frac{2}{\sqrt{5}}$ 9.) $\frac{4}{5}$ 10.) $y = \frac{1}{3}x$ 11.) $\frac{4\sqrt{2}}{9}$
- 12.) a) $\frac{1}{2}(\cos 6A - \cos 20A)$ b) $\frac{1}{2}(\sin 2x + \sin 14x)$ c) $\frac{1}{2}(\cos 4\alpha + \cos 6\alpha)$ d) $\frac{1}{2}(\sin 14y - \sin 6y)$
- 13.) a) $2 \sin 10A \cos 3A$ b) $2 \cos x \cos 7x$ c) $-2 \cos 8m \sin 3m$ d) $2 \sin 11y \sin 7y$
- 14.) $\frac{168}{25} = 6.72$ 15.) $\frac{3\sqrt{3}}{4\pi} \approx 0.41349667 \approx 41\%$ 16.) D 17.) B
- 18.) a) $\frac{\ln\left(\frac{2}{3}\right)}{\ln 0.95} \approx 7.90484$ hours b) 1500 miles 19.) $y = x + 8$ and $y = x - 8$
- 20.) $a = 3, r = 2$ or $a = 12, r = \frac{1}{2}$ 21.) $a = 5, r = 1$ or $a = \frac{10}{3}, r = \frac{3}{2}$
- 22.) $\text{LHS} = \log_{a/b} \left(\frac{c}{d} \right) = \frac{\ln \left(\frac{c}{d} \right)}{\ln \left(\frac{a}{b} \right)} = \frac{\ln c - \ln d}{\ln a - \ln b} = \frac{(\ln c - \ln d)(-1)}{(\ln a - \ln b)(-1)} = \frac{\ln d - \ln c}{\ln b - \ln a} = \frac{\ln \left(\frac{d}{c} \right)}{\ln \left(\frac{b}{a} \right)} = \log_{b/a} \left(\frac{d}{c} \right) = \text{RHS}$