1. Re-write the repeating decimal $0.7\overline{16} = 0.7161616.....$ as a quotient of two integers.

2. Simplify each of the following.

   a) $\left(-2a^3b^2\right)^{-2}\left(-2a^5b^6\right)$

   b) $\dfrac{a^3b^{-2}}{a^{-1}b^{-1}}$

   c) $\dfrac{a^3-b^{-2}}{a^{-1}+b^{-1}}$

   d) $2^{-3}\left(1-3^{-2}\right)$

   e) $2^{-3}-\left(1-3^{-2}\right)$

   f) $\left(\left(2^{-2}\right)^{-2}\right)^{-2}$

   g) $\left(\dfrac{a^{-2}}{b^{-1}}\right)^{-3}$

   h) $\left(\dfrac{a^{-2}}{b^{-1}}\right)^{0}$

   i) $-2^2$

   j) $(-2)^{-2}$

   k) $32^{-2/5}$

   l) $-32^{-2/5}$

   m) $(-32)^{-2/5}$

   n) $\left(\dfrac{a^2b^0}{a^{-2}b^{12}}\right)^{1/2}$

   o) $\dfrac{a^{1/2}b^{2/3}}{a^{-1/2}b^{-1/3}}$

3. Let $k$ denote $2^{1000}$. Re-write each of the following in terms of $k$.

   a) $2^{1001}$ b) $2^{999}$ c) $2^{1002}$ d) $2^{3000}$ e) $4^{1000}$ f) $2^{500}$ g) $2^{1000}-2^{1001}+2^{1002}$

4. Let $f(x)=\dfrac{x-2}{x+5}$ and $g(x)=\dfrac{5x+2}{-x+1}$. Compute each of the following.

   a) $f(0)$

   b) $g(0)$

   c) $g(1)$

   d) $g\left(\sqrt{2}\right)$

   e) $f\left(\sqrt{20}\right)$

   f) $f(g(0))$

   g) $g(f(3))$

   h) $f(1+3)$

   i) $f(1)+f(3)$

   j) $2f(5)$

   k) $f(2\cdot 5)$

5. Simplify each of the following. Assume that $x$ is positive.

   a) $(-16)^{-3/4}$

   b) $-16^{-3/4}$

   c) $\left(x^{2/3}\right)^{3/4}\left(\dfrac{1}{\sqrt{x}}\right)$

   d) $\dfrac{2}{\left(\sqrt{5}-1\right)^2}$

   e) $\left(1+\sqrt{3}\right)^3\left(1-\sqrt{3}\right)^3$

   f) $\left(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right)^2$

   g) $\dfrac{3+\sqrt{3}}{3-\sqrt{3}}$

6. Simplify each of the following. Assume that $a$ is positive.

   a) $\log_4\left(\dfrac{1}{2}\right)$

   b) $\log_4 8$

   c) $\log_{10} 1000$

   d) $\log_{100} 1000$

   e) $\log_2\left(\dfrac{1}{8}\right)$

   f) $\log_8 2$

   g) $\log_{\sqrt{2}} 8$

   h) $\log_4 8$

   i) $\log_4(-8)$

   j) $\log_8\left(\dfrac{1}{4}\right)$

   k) $\log_{(1/5)} 125$

   l) $\log_{0.1} 100\,000$

   m) $\log_5 1$

   n) $\log_1 5$

   o) $\log_a\left(a^{17}\right)$

   p) $\log_{(-5)} 25$

   q) $\log_9\left(\dfrac{1}{\sqrt{27}}\right)$

   r) $\log_a\left(\sqrt[7]{a^2}\right)$

   s) $\log_{\sqrt{3}}(81)$

   t) $\log_2\left(16^2\right)$

   u) $\log_2\left(8^x\right)$

   v) $\log_8\left(2^x\right)$

7. Simplify each of the following. Assume that $x$ represents a positive number.

   a) $\left(\sqrt{3}\right)^6$

   b) $\left(\sqrt[7]{x}\right)^{21}$

   c) $\sqrt{\sqrt{\sqrt{x}}}$

   d) $\left(\sqrt[12]{x}\right)^4$

   e) $\sqrt[2]{\sqrt[3]{x}}$

   f) $(\sec 45^\circ)^{10}$

   g) $\log_3(\tan 60^\circ)$

   h) $\left(\dfrac{1}{\sqrt[3]{x}}\right)^{-12}$

   i) $\dfrac{\sqrt{x}}{\sqrt[3]{\sqrt{x}}\cdot\sqrt{\sqrt{x}}}$

   j) $\log_3\left(\dfrac{\sqrt[3]{9}}{\sqrt{3}}\right)$

8. Simplify each of the following. Use exact values.

   a) $\cos 30^\circ$

   b) $\csc 45^\circ$

   c) $\sin 60^\circ$

   d) $\cot 45^\circ$

   e) $\tan 30^\circ$

   f) $\sec 60^\circ$

   g) $\csc 30^\circ$

   h) $\sin 45^\circ$

   i) $\sec 30^\circ$

   j) $\cot 60^\circ$

9. Suppose that $a$ and $b$ are real numbers so that $a$ is eight less than three times $b$. Find the greatest possible value of $5b^2 - a^2$.

10. Suppose that $\alpha$ is the smallest angle in a right triangle with sides 8 cm, 15 cm, and 17 cm long. Compute the exact value of each of the following.

    a) $\sin\alpha$ b) $\cos\alpha$ c) $\tan\alpha$ d) $\sin^2\alpha + \cos^2\alpha$ e) $\sec^2 x - \tan^2 x$

11. Find the exact value of each of the following expressions and simplify.

    a) $\sin 30^\circ - \cos 45^\circ \tan 60^\circ$

    b) $\sin 45^\circ \cdot \cos 45^\circ - \cos 30^\circ \cdot \sin 30^\circ$

    c) $\dfrac{\sin 10^\circ}{\cos 80^\circ}$

    d) $\sin 60^\circ \sin 30^\circ + \cos 60^\circ \cos 30^\circ$

    e) $(\sin 45^\circ + \cos 45^\circ)^2$

    f) $\sin^2 45^\circ + \cos^2 45^\circ$

    g) $(\sin 45^\circ + \cos 45^\circ)^4$

    h) $\sin^4 45^\circ + \cos^4 45^\circ$

    i) $\log_3(\tan 60^\circ)$

    j) $\log_2(\sin 45^\circ)$

    k) $\dfrac{\sec 45^\circ \cos 30^\circ - \csc 60^\circ}{\sin 60^\circ}$

    l) $\dfrac{\cot 30^\circ - \tan^2 60^\circ}{\cot 30^\circ + \tan^2 60^\circ}$

12. Approximate each of the following by placing them between two consecutive integers.

    a) $\log_2 100$ b) $\log_2 200$ c) $\log_{10} 2014$ d) $\log_5 2014$ e) $\log_2\left(\dfrac{1}{3}\right)$

13. Re-write each of the following as an exponential statement.

    a) $x = \log_7(2y-1)$

    b) $3 = \log_B(A-1)$

    c) $p = \log_q T$

    d) $x + 2 = \log_a\left(x^2+1\right)$

    e) $2 = \log_x 20$

14. Re-write each of the following as a logarithmic statement. Assume that all variables represent positive numbers.

    a) $3^{x-2} = 60$ b) $10^{2x-5} = 2012$ c) $\left(\dfrac{1}{3}\right)^{a+1} = b - 2$ d) $A^B = C$

15. Solve each of the following equations. Make sure to check your solutions.

    a) $\dfrac{2x+1}{5} - \dfrac{x-1}{3} = 2x + 16$

    b) $(2x-1)^2 - (x-2)^2 = x + 1$

    c) $\sqrt{x+6} + \sqrt{11-x} = 5$

    d) $\sqrt{3x+10} - \sqrt{x+4} = 2$

    e) $\sqrt{x-3} + 1 = \sqrt{x+2}$

    f) $\sqrt{x-1} - 1 = \sqrt{2x-9}$

    g) $2x^5 = 32x^4$

    h) $2x^5 = 32x^3$

    i) $2x^5 = 32x$

    j) $x^2 + x = 1$

16. Solve each of the following inequalities.

    a) $3x^2 \geq 15x$ b) $28x + 88 \leq -2x^2$ c) $\dfrac{1}{3}x^2 < 4x - 12$ d) $\dfrac{1}{3}x^2 \leq 4x - 12$

17. Find the domain of each of the following functions.

    a) $f(x) = \dfrac{1}{5x+3}$

    b) $f(x) = \sqrt{5x+3}$

    c) $f(x) = \dfrac{1}{x^2 - 10x + 24}$

    d) $f(x) = \sqrt{x^2 - 10x + 24}$

    e) $f(x) = \sqrt{x-3} - 2 + \dfrac{1}{x-10}$

    f) $f(x) = \dfrac{1}{\sqrt{x-3} - 2}$

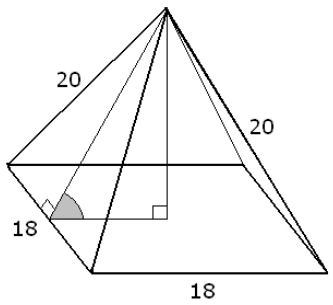
    g) $f(x) = \dfrac{3x-1}{x^2+4}$

    h) $f(x) = \sqrt{9-x^2} - \sqrt{x+1}$

18. The sum of $a$ and three times $b$ is 100. What is the smallest possible value of $a^2 - (2b)^2$?

19. Find the height of a buiding if the angle of elevation from the ground to its top changes from 15° to 25° as the observer advances 120 ft toward it.

20. A straight pyramid has a square base with sides 18 units long. All other edges are 20 units long. Compute the angle formed by the base and a triangular face.




21. Compute the area and perimeter of a regular 12−sided polygon written into a circle with radius 9 cm. Present the exact value and approximation of your answer.

22. Find all points of intersections of the circles given.
    a) $(x+4)^2+(y-4)^2=25$ and $(x-10)^2+(y-2)^2=125$
    b) $x^2+(y-4)^2=25$ and $(x-6)^2+(y-1)^2=10$
    c) $(x+2)^2+(y+2)^2=5$ and $(x-2)^2+y^2=5$
    d) $(x+5)^2+(y-3)^2=13$ and $(x-4)^2+(y+3)^2=52$

23. Consider the circle $x^2-20x+y^2=21+4y$. Find the equation of the tangent line drawn to the circle at $(0,7)$.

24. A circle has a radius of 10 units. A point $P$ is 17 units away from the center of the circle. Find the measure of the angle formed by the tangent lines drawn from $P$ to the circle.

25. A person is standing 35 feet away from a street light that is 28 ft tall. How long is his shadow if he is 5.6 feet tall?

26. Compute each of the following sums.
    a) $210+223+236+...+1302$ b) $100+103+106+...+847$ c) $1010+1000+990+...+120$

27. Paul is starting at his new job today. His starting salary is \$2000 a month (after all deductions and taxes). If this amount is expected to increase by \$100 after each year, how much money in total would he make at this company during the next 10 years?

28. Suppose that $A=\{2,3,5,8,9,10\}$.
    a) How many subsets does the set $A$ have?
    b) List all two-element subsets of $A$.
    c) How many different ways can we list the elements of $A$? (Don't list them.)

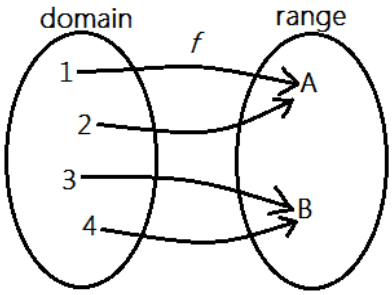
29. Find an equation for the line that
    a) is parallel to $5x+3y=-15$ and passes through the point $(-6,2)$.
    b) is perpendicular to $5x+3y=-15$ and passes through the point $(-6,2)$.
    c) passes through the points $(3,8)$ and $(-1,2)$.

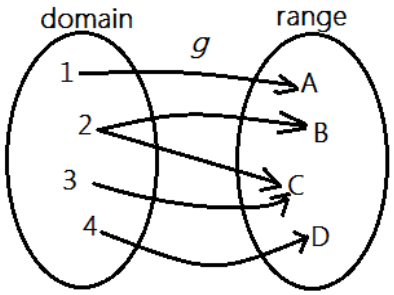
30. An arch is in the shape of a semicircle. At a point along the base 2 feet from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch.

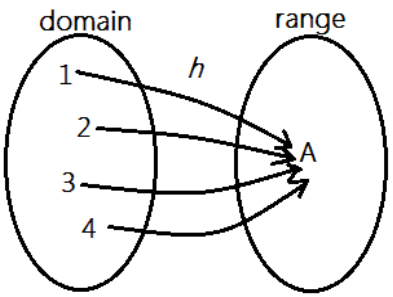
31. The shortest side of a right triangle is 12 cm. The difference between the other two sides is 2 cm. Find an approximate value for the smallest angle in the triangle.

32. Prove that if $\alpha$ is an acute angle, then $\sin\alpha+\cos\alpha>1$.

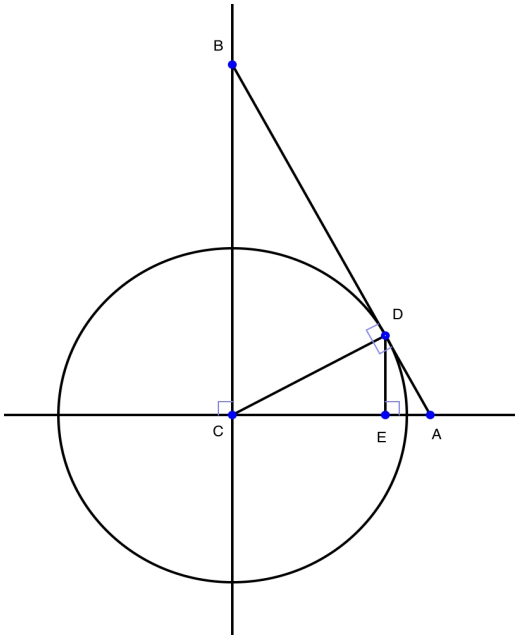
33. Which of the following relations are functions?




34. Consider the picture shown. Line $AB$ is tangent to the unit circle, where $D$ is the point of tangency. Let $\alpha$ denote angle $DCE$. Match each of the six trigonometric functions with the length of each of the line segments given.
$\sin\alpha,\ \cos\alpha,\ \tan\alpha,\ \csc\alpha, \sec\alpha, \cot\alpha$
and
$AC,\ AD,\ BC,\ BD,\ CE,\ DE$



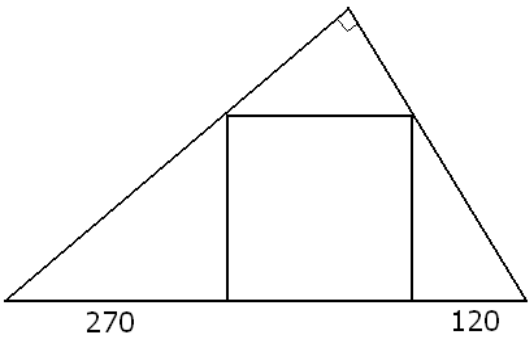

35. Graph each of the following.

a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$ c) $(x-2)^2 + (y-1)^2 = 25$ d) $f(x) = \left(\frac{1}{2}\right)^x$

36. a) Find the exact and approximate values of the smaller angle that is formed between the positive part of the $x-$axis and the line $y = 2x - 1$.

b) Find an equation for the straight line that passes through the point $P(4, -2)$ and forms a $60^\circ$ angle with the positive part of the $x-$axis.

37. The picture shows a square within a right triangle.
a) Find the length of the sides in the square.
b) Compute the angles in the triangle.



# Answers

1. $\frac{709}{990}$ 2. a) $-\frac{b^2}{2a}$ b) $\frac{a^4}{b}$ c) $\frac{a^4b^2-a}{ab+b^2}$
d) $\frac{1}{9}$ e) $-\frac{55}{72}$ f) $\frac{1}{256}$ g) $\frac{a^6}{b^3}$
h) 1 i) $-4$ j) $\frac{1}{4}$ k) $\frac{1}{4}$ l) $-\frac{1}{4}$
m) undefined n) $\frac{a^2}{b^6}$ o) $ab$

3. a) $2k$ b) $\frac{k}{2}$ c) $4k$ d) $k^3$ e) $k^2$
f) $\sqrt{k}$ g) $3k$

4. a) $-\frac{2}{5}$ b) 2 c) undefined
d) $-7\sqrt{2}-12$ e) $\frac{14}{5}\sqrt{5}-6$ f) 0
g) 3 h) $\frac{2}{9}$ i) $-\frac{1}{24}$ j) $\frac{3}{5}$ k) $\frac{8}{15}$

5. a) undefined b) $-\frac{1}{8}$ c) 1
d) $\frac{\sqrt{5}+3}{4}$ e) $-8$ f) 2 g) $\sqrt{3}+2$

6. a) $-\frac{1}{2}$ b) $\frac{3}{2}$ c) 3 d) $\frac{3}{2}$ e) $-3$
f) $\frac{1}{3}$ g) 6 h) $\frac{3}{2}$ i) undefined
j) $-\frac{2}{3}$ k) $-3$ l) $-5$ m) 0
n) undefined o) 17 p) undefined
q) $-\frac{3}{4}$ r) $\frac{2}{7}$ s) 8 t) 8 u) $3x$ v) $\frac{x}{3}$

7. a) 27 b) $x^3$ c) $\sqrt[8]{x}$ d) $\sqrt[3]{x}$ e) $\sqrt[6]{x}$
f) 32 g) $\frac{1}{2}$ h) $x^4$ i) $\sqrt[12]{x}$ j) $\frac{1}{6}$

8. a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1 e) $\frac{\sqrt{3}}{3}$ f) 2
g) 2 h) $\frac{\sqrt{2}}{2}$ i) $\frac{2\sqrt{3}}{3}$ j) $\frac{\sqrt{3}}{3}$ 9. 80

10. a) $\frac{8}{17}$ b) $\frac{15}{17}$ c) $\frac{8}{15}$ d) 1 e) 1

11. a) $\frac{1}{2}(1-\sqrt{6})$ b) $\frac{1}{2}-\frac{1}{4}\sqrt{3}$ c) 1 d) $\frac{\sqrt{3}}{2}$
e) 2 f) 1 g) 4 h) $\frac{1}{2}$ i) $\frac{1}{2}$ j) $-\frac{1}{2}$
k) $\sqrt{2}-\frac{4}{3}$ l) $\sqrt{3}-2$

12. a) $6 < \log_2 100 < 7$ b) $7 < \log_2 200 < 8$
c) $3 < \log_{10} 2014 < 4$ d) $4 < \log_5 2014 < 5$
e) $-2 < \log_2\left(\frac{1}{3}\right) < -1$

13. a) $7^x = 2y-1$ b) $B^3 = A-1$
c) $q^p = T$ d) $a^{x+2} = x^2+1$ e) $x^2 = 20$

14. a) $x-2 = \log_3 60$ b) $2x-5 = \log_{10} 2012$
c) $a+1 = \log_{1/3}(b-2)$ d) $B = \log_A C$

15. a) $-8$ b) $-1, \frac{4}{3}$ c) $-5, 10$
d) 5 ($-3$ extreneous) e) 7 f) 5 (17 extreneous)
g) $0, 16$ h) $-4, 0, 4$ i) $-2, 0, 2$ j) $\frac{-1\pm\sqrt{5}}{2}$

16. a) $(-\infty, 0] \cup [5, \infty)$ b) $\left[-7-\sqrt{5}, -7+\sqrt{5}\right]$
c) no solution d) 6

17. a) $x \neq -\frac{3}{5}$ b) $x \geq -\frac{3}{5}$ c) $x \neq 4, 6$
d) $x \leq 4$ or $x \geq 6$ e) $x \geq 3$ and $x \neq 10$
f) $x \geq 3$ and $x \neq 7$ g) $\mathbb{R}$ h) $-1 < x \leq 3$

18. $-8000$ 19. 75.588 461 ft

20. $\cos^{-1}\left(\frac{9}{\sqrt{319}}\right) \approx 59.74142°$

21. $P = 216\sin 15°$ cm $\approx 55.904\,91$ cm
$A = 972\sin 15°\cos 15°$ cm$^2 \approx 243$ cm$^2$

22. a) $(0, 7)$ and $(-1, 0)$ b) $(5, 4)$ and $(3, 0)$
c) $(0, -1)$ d) $(-2, 1)$ 23. $y = 2x+7$

24. $2\sin^{-1}\left(\frac{10}{17}\right) \approx 72.063\,758°$ 25. 8.75 ft

26. a) 64 260 b) 118 375 c) 50 850

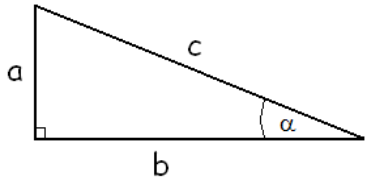
27. \$294 000 28. a) 64 b) see below c) 720

$\{1,2\}$
$\{1,3\}$ $\{2,3\}$
$\{1,4\}$ $\{2,4\}$ $\{3,4\}$
$\{1,5\}$ $\{2,5\}$ $\{3,5\}$ $\{4,5\}$
$\{1,6\}$ $\{2,6\}$ $\{3,6\}$ $\{4,6\}$ $\{5,6\}$

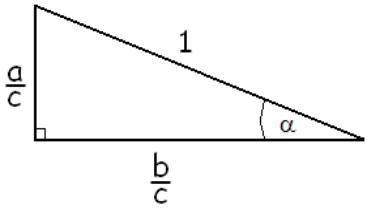
29. a) $y = -\frac{5}{3}x-8$ b) $y = \frac{3}{5}x+\frac{28}{5}$
c) $y = \frac{3}{2}x+\frac{7}{2}$ 30. 17 feet

31. $\sin^{-1}\left(\frac{12}{37}\right) \approx 18.924\,64°$

32. Let $\alpha$ be an acute angle and triangle $ABC$ a rigth triangle containing $\alpha$.




With this notation, $\sin\alpha = \frac{a}{c}$ and $\cos\alpha = \frac{b}{c}$.
Let us replace this triangle with a similar triangle that has hypotenuse 1. Then the shorter sides become $\frac{a}{c}$ and $\frac{b}{c}$.

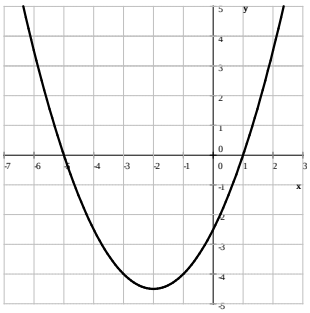


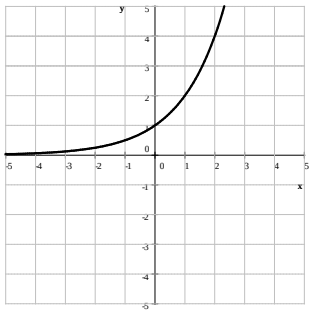

By the triangle inequality, $\frac{a}{c} + \frac{b}{c} > 1$. But that is the same as $\sin\alpha + \cos\alpha = 1$.

33. $f$ and $h$

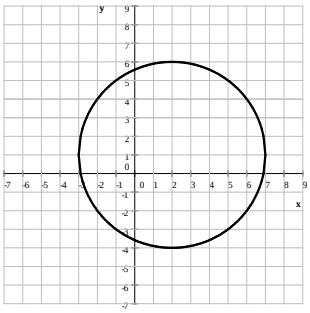
34. $\sin\alpha = DE$ $\cos\alpha = CE$ $\tan\alpha = AD$
$\csc\alpha = BC$ $\sec\alpha = AC$ $\cot\alpha = BD$

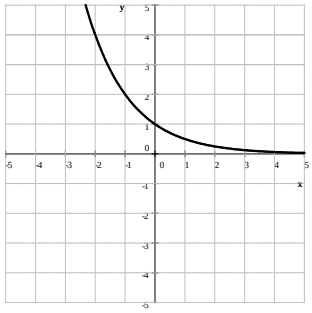
35. a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$





c) $(x-2)^2 + (y-1)^2 = 25$ d) $f(x) = \left(\frac{1}{2}\right)^x$





36. a) $\tan^{-1} 2 \approx 63.434949$ b) $y + 2 = \sqrt{3}(x-4)$

37. a) 180 b) $\alpha = \tan^{-1}\left(\frac{180}{270}\right) \approx 33.6900675°$
$\beta = \tan^{-1}\left(\frac{180}{120}\right) \approx 56.3099325°$

Last revised: October 12, 2017