1. Simplify each of the following.

a) $\log_{25}\left(\frac{1}{125}\right)$ b) $3^{\log_9 5}$ c) $\ln\left(5e^2\right)$ d) $\cos\left(\frac{13\pi}{6}\right)$ e) $\csc\left(\frac{13\pi}{4}\right)$ f) $\cos(-105^\circ)$ g) $\tan 75^\circ$ h) $\tan 22.5^\circ$ i) $5^{\log_{25} A}$ j) $e^{-2\ln B}$

k) $\dfrac{\csc 30^\circ + \csc 60^\circ + \csc 90^\circ}{\sec 0^\circ + \sec 30^\circ + \sec 60^\circ}$

l) $\sec^2\beta - \tan^2\beta$

m) $\log_6 18 + \log_6 2$

n) $\log_{10} 2 + \log_{10} 50 - 1$

o) $2\log_2\left(6x^3\right) - \log_2\left(18x^5\right)$

p) $\log_3\left(\tan\left(\frac{\pi}{6}\right)\right)$

q) $\cos 48^\circ \cos 3^\circ + \sin 48^\circ \sin 3^\circ$

r) $\dfrac{\tan\frac{2\pi}{15} + \tan\frac{\pi}{5}}{1 - \left(\tan\frac{2\pi}{15}\right)\left(\tan\frac{\pi}{5}\right)}$

s) $\left(\sqrt{6+2\sqrt{5}} - \sqrt{6-2\sqrt{5}}\right)^2$

t) $\cos\left(-\frac{13\pi}{3}\right) + \sin\frac{13\pi}{4} - \tan\frac{9\pi}{4} + \cos\left(\frac{13\pi}{3}\right)$

u) $\sin 120^\circ \cos 150^\circ - 3\tan 330^\circ - \tan 495^\circ$

v) $\sin 20^\circ \cos(-70^\circ) + \cos 380^\circ \cos(-20^\circ)$

w) $\cos 68^\circ \sin 8^\circ - \sin 68^\circ \cos 8^\circ$

x) $\cos\left(-\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{3}\right) - \tan(3\pi)$

y) $\sin 143^\circ \cos 82^\circ + \cos 143^\circ \sin 82^\circ$

2. Simplify each of the following.

a) $\dfrac{\log_3 90}{\log_{30} 3} - \dfrac{\log_3 270}{\log_{10} 3}$

b) $\log_{\sec(\pi/4)}\left(\tan 15^\circ + \tan 60^\circ\right)$

c) $\log_2\left(\sin^{2015}\left(\frac{\pi}{4}\right)\right)$

d) $\log_3(\tan 45^\circ) \cdot \log_3(\tan 75^\circ)$

e) $\log_5(\tan 20^\circ) + \log_5(\tan 70^\circ)$

f) $1 + \log_2(\sin 15^\circ) + \log_2(\cos 15^\circ)$

3. Solve the equation $3x^2 - x - 1 = 0$ by completing the square. Use exact values Check your solution.

4. Find the domain of each of the following expressions.

a) $\log_5\left(-x^2 + 2x + 20\right)$

b) $\dfrac{1}{\sqrt{2x-6} - 6}$

c) $\dfrac{1}{\log_2(x-5) + \log_2(x+5)}$

d) $\dfrac{1}{x^2-4} + \ln(x+10)$

e) $\log_7\left(x^2+3\right)$

f) $\ln\left(x^2 + x - 30\right)$

g) $\ln(x-5) + \ln(x+6)$

h) $\dfrac{\ln(x+1)}{\ln(x-1)}$

j) $\dfrac{\cos x + 1}{2\sin x - 1}$

i) $\cot x$

5. Solve each of the following equations.

a) $\log_3(x-7)(2x+7) = 4$

b) $\dfrac{2\log_2(x+8) - 1}{3} = 1$

c) $3^{2x-5} + 1 = \dfrac{10}{9}$

d) $\sin 2x = -4\cos x$

e) $3 \cdot e^{3x-2} - 1 = 44$

f) $\tan^2 x = \tan x$

g) $1 - \sin x = 2\cos^2 x$

h) $\cos x = \cos 2x$

i) $\tan^2 x = 3$

j) $\sin^2 x = \sin x$

k) $3\cos x + 2\cos^2 x = 2$

l) $\sin x = 2\sin x \cos x$

m) $2\cos^2 x + 5\cos x = -2$

n) $2 + 3\sin x = \cos 2x$

o) $\dfrac{3}{5}\ln(2x-1) = 6$

p) $\log_2(3-x) + \log_2(-x-4) = 3$

q) $\log_3(x-7) + \log_3(2x+7) = 4$

r) $\log_3(x-5) - \log_3(2x-11) = -1$

s) $\log_2(2x-3) - \log_2(x+1) = -3$

6. Suppose that $\log_2 3 = x$. Express each of the following in terms of x.

a) $\log_2 6$ b) $\log_2 12$ c) $\log_2 18$ d) $\log_2 72$ e) $\log_2\left(\frac{3}{2}\right)$ f) $\log_3 2$ g) $\log_2\left(\frac{4}{27}\right)$ h) $\log_6 12$

7. Suppose that $\log_2 3 = a$ and $\log_2 5 = b$. Express each of the following in terms of a.and b.

   a) $\log_2 15$ b) $\log_2 30$ c) $\log_2 240$ d) $\log_3 15$ e) $\log_6 30$

8. a) Suppose that $a = \log_5 4$ and $b = \log_2 6$. Express $\log_3 5$ in terms of a and b.

   b) Suppose that $\log_3 6 = x$ and $\log_5 9 = y$. Express $\log_{18} 100$ in terms of x and y.

9. Compute the area of a regular $n-$sided polygon that is written inside a circle with radius R.

10. a) Find the smallest value of $a^2 + b^2$ if $a + 3b = 50$.
    b) Find the smallest possible value of $2a^2 + 3b^2$ if $a + b = 20$.

11. Find an equation for the tangent line drawn to $14y + x^2 + y^2 + 13 = 4x$ at the point $P(8, -5)$.

12. Simplify $(\sin 15°)(2\cos 15°)(2\cos 30°)(2\cos 60°)(2\cos 120°)\ldots(2\cos 960°)(2\cos 1920°)$

13. Prove that $\log_{a/b}\left(\frac{c}{d}\right) = \log_{b/a}\left(\frac{d}{c}\right)$

14. Write each of the following in terms of $\sin x$, $\cos x$, or $\tan x$.

    a) $\sin(180° - x)$ b) $\tan(-x)$ c) $\cos(x + 180°)$ d) $\sin(x - 180°)$ e) $\cos(90° - x)$ f) $\tan(x + 180°)$

15. Consider the line $y = \frac{12}{5}x$. Find an equation of the line that bisects the smaller angle that is formed between the line and the positive part of the $x-$axis.

16. Find the first element and common difference in the arithmetic sequence $\{a_n\}$ if we know that the sum of the first four terms is -14 and the sum of the square of the first four terms is 94.

17. Graph each of the following relations.

    a) $y = -\frac{1}{2}x^2 - 3x + \frac{7}{2}$ b) $y = \sqrt[3]{x}$ c) $f(x) = |x|$ d) $y = 1.5^x$

    e) $f(x) = \log_2 x$ f) $f(x) = \log_{1/2} x$ g) $f(x) = -\frac{1}{x}$ h) $2y = 4x + x^2 + y^2$

18. Compute the exact value of each of the following.

    a) $\ln(\tan 45°)$ b) $\frac{\sqrt{2}\sqrt[3]{2}}{\sqrt[4]{2}}$ c) $\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2\right)$

19. Suppose that we draw both tangent lines to $(x-6)^2 + (y+3)^2 = 25$ from the point $P(-2, 2)$.
    a) Find the exact value of the sine of the angle formed by the two tangent lines.
    b) Find an aproximate value for the angle formed by the two tangent lines.

20. Compute each of the following sums.

    a) $17 + 23 + 29 + \ldots + 1511$ b) $42 + 47 + 52 + \ldots + 937$ c) $12 + 19 + 26 + \ldots + 1405$

21. Prove each of the following identities.

    a) $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = 4\tan x \sec x$

    b) $\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$

    c) $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

    d) $\cos 2x = \frac{\cot x - \tan x}{\cot x + \tan x}$

22. Suppose that $\sin\alpha = \dfrac{3}{5}$ and α is not in the first quadrant; $\cos\beta = -\dfrac{7}{25}$ and β is not in the third quadrant. Find the exact value for each of the following.

    a) $\sin 2\beta$ c) $\sin 2\alpha$ e) $\sin 3\alpha$ g) $\tan(\alpha+\beta)$

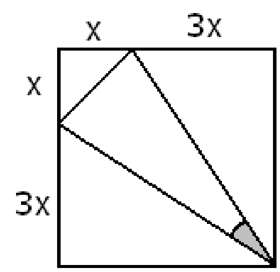
    b) $\cos 2\beta$ d) $\cos 2\alpha$ f) $\cos(\alpha-\beta)$ h) $\tan(\alpha-\beta)$

23. Find the exact value of $\tan\alpha$ if α is the acute angle formed by the lines $2x+3y=-12$ and $2x-7y=1$.

24. Find the exact value of $\tan x$ if $\tan y = \dfrac{2}{3}$ and $\tan(x+y)=4$.

25. Supose that x is an angle with $\cot x = -3$. Compute the exact value of each of the following.
    a) $\cos x$ b) $\sin 2x$ c) $\cos 2x$

26. Compute the exact value of the cosine of the angle shaded on the picture.



27. Two circles, of radii 10 cm and 14 cm are placed so that their centers are 20 cm apart. Compute the exact value of the sine of the angle formed by the common tangent lines drawn to the circles.

28. Find the coordinates of all points where the circles $(x+2)^2+(y+1)^2=5$ and $(x-4)^2+(y-1)^2=25$ intersect each other.

29. a) Prove that the area of any triangle can be computed as $A=\dfrac{1}{2}ab\sin\gamma$.

    b) Find the value of x so that the area of the triangle with sides 12 cm, 12 cm and x, is the greatest?

30. A person is standing 35 feet away from a street light that is 28 ft tall. How long is his shadow if he is 5.6feet tall?

31. Graph each of the following.

    a) $f(x)=\sin x$ on $[-2\pi,2\pi]$ b) $f(x)=\cos x$ on $[-2\pi,2\pi]$ c) $f(x)=\tan x$ on $[-\pi,\pi]$

32. We placed \$3000 into a bank account with an annual compound interest rate of 4%, compounded annually. How long until the account reaches

    a) \$10 000 b) \$20 000 c) \$30 000 d) \$40 000 e) \$50 000 f) \$60 000

33. We draw tangent lines to a circle from a point P outside of the circle. The line segment between P and a point of tangency is 3 units long. The line segment connecting P and the center of the circle intersects the circle in point Q. Line segment PQ is $\sqrt{3}$ units long. Compute the angle formed between the two tangent lines.

34. Compute the exact value of $\log_2\left(\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}}\right)$

Answers

1. a) $-\frac{3}{2}$ b) $\sqrt{5}$ c) $2+\ln 5$ d) $\frac{\sqrt{3}}{2}$ e) $-\sqrt{2}$ f) $\frac{\sqrt{2}-\sqrt{6}}{4}$ g) $\sqrt{3}+2$ h) $\sqrt{2}-1$
i) $\sqrt{A}$ j) $\frac{1}{B^2}$ k) 1 l) 1 m) 2 n) 1 o) $1+\log_2 x$ p) $-\frac{1}{2}$ q) $\frac{1}{\sqrt{2}}$ r) $\sqrt{3}$
s) 4 t) $-\frac{\sqrt{2}}{2}$ u) $\sqrt{3}+\frac{1}{4}$ v) 1 w) $-\frac{\sqrt{3}}{2}$ x) $\sqrt{3}$ y) $-\frac{1}{\sqrt{2}}$

2. a) 2 b) 2 c) -1007.5 d) 0 e) 0 f) -1 3. $\frac{1+\sqrt{13}}{6}, \frac{1-\sqrt{13}}{6}$

4. a) $1-\sqrt{21} < x < 1+\sqrt{21}$ b) $x \geq 3$, $x \neq 21$ c) $x > 5$, $x \neq \sqrt{26}$ d) $x > -10$ and $x \neq \pm 2$
e) $\mathbb{R}$ f) $x < -6$ or $x > 5$ g) $x > 5$ h) $x > 1$ and $x \neq 2$ i) $x \neq k\pi$ $k \in \mathbb{Z}$
j) $x \neq \frac{\pi}{6} + 2k\pi$, $x \neq \frac{5\pi}{6} + 2k\pi$

5. a) $-\frac{13}{2}, 10$ b) -4 c) $\frac{3}{2}$ d) $\frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$ e) $\frac{2+\ln 15}{3}$ f) $k\pi$, $\frac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$
g) $\frac{\pi}{2} + 2k\pi$, $-\frac{\pi}{6} + 2k\pi$, $-\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ h) $2k\pi, \pm\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$
i) $\pm\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$ j) $\frac{\pi}{2} + 2k\pi$, $k\pi$ where $k \in \mathbb{Z}$ k) $\pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$
l) $k\pi, \pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ m) $\pm\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$
n) $x = -\frac{\pi}{2} + 2k_1\pi$, $-\frac{\pi}{6} + 2k_2\pi$, $-\frac{5\pi}{6} + 2k_3\pi$ where $k_1, k_2, k_3 \in \mathbb{Z}$ o) $\frac{1}{2}e^{10} + \frac{1}{2}$ p) -5
q) 10 r) no solution s) $\frac{5}{3}$

6. a) $x+1$ b) $x+2$ c) $2x+1$ d) $2x+3$ e) $x-1$ f) $\frac{1}{x}$ g) $2-3x$ h) $\frac{x+2}{x+1}$

7. a) $a+b$ b) $a+b+1$ c) $a+b+4$ d) $\frac{a+b}{a}$ e) $\frac{a+b+1}{a+1}$ 8. a) $\frac{2}{a(b-1)}$ b) $\frac{2x-2+\frac{4}{y}}{x+1}$

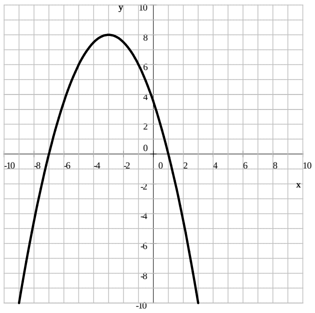
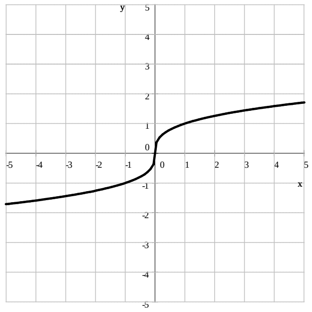
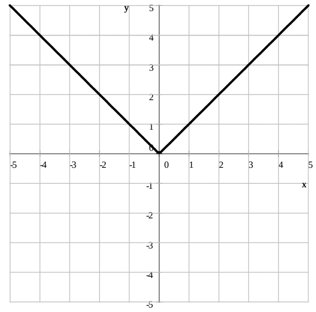
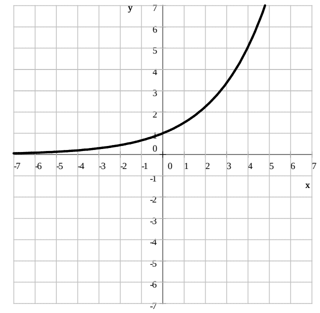
9. $A = 2nR^2 \sin\left(\frac{180°}{n}\right) \cos\left(\frac{180°}{n}\right)$ 10. a) 250 b) 480 11. $y = -3x + 19$ 12. $-\frac{\sqrt{3}}{2}$

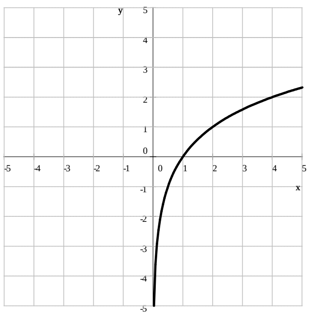
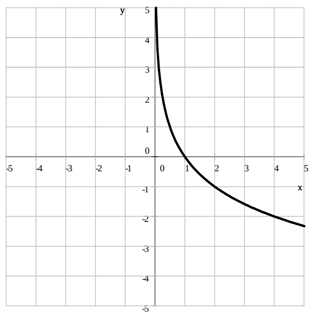
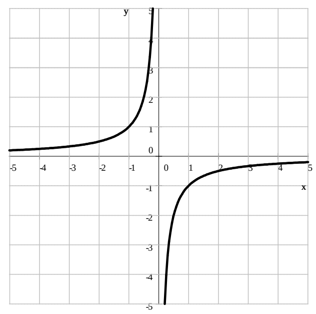
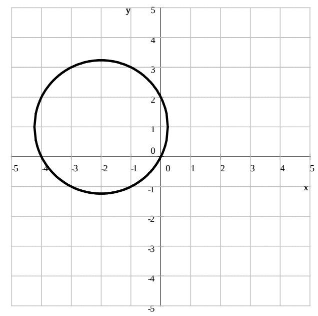
13. $$\log_{a/b}\left(\frac{c}{d}\right) = \frac{\ln\left(\frac{c}{d}\right)}{\ln\left(\frac{a}{b}\right)} = \frac{\ln c - \ln d}{\ln a - \ln b} = \frac{-1(\ln c - \ln d)}{-1(\ln a - \ln b)} = \frac{\ln d - \ln c}{\ln b - \ln a} = \frac{\ln\left(\frac{d}{c}\right)}{\ln\left(\frac{b}{a}\right)} = \log_{b/a}\left(\frac{d}{c}\right)$$

14. a) $\sin x$ b) $-\tan x$ c) $-\cos x$ d) $-\sin x$ e) $\sin x$ f) $\tan x$ 15. $y = \frac{2}{3}x$

16. There are two solutions: $a = 1, d = -3$, and $a = -8, d = 3$

17. a) $y=-\frac{1}{2}x^2-3x+\frac{7}{2}$ b) $y=\sqrt[3]{x}$ c) $f(x)=|x|$ d) $y=1.5^x$

e) $f(x)=\log_2 x$ f) $f(x)=\log_{1/2} x$ g) $f(x)=-\frac{1}{x}$ h) $2y=4x+x^2+y^2$

18. a) 0 b) $\sqrt[12]{x^7}$ c) 1 19. a) $\frac{80}{89}$ b) $64.010\,766\,4^\circ$ 20. a) $191\,000$ b) $88\,110$ c) $141\,700$

21. a) $\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}=4\tan x\sec x$

Solution: We bring fractions to the common denominator:

$$\begin{aligned}\text{LHS} &= \frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}=\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}-\frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}=\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}=\\ &= \frac{\left(1+\sin^2 x+2\sin x\right)-\left(1+\sin^2 x-2\sin x\right)}{1-\sin^2 x}=\frac{1+\sin^2 x+2\sin x-1-\sin^2 x+2\sin x}{\cos^2 x}=\frac{4\sin x}{\cos^2 x}=\\ &= 4\cdot\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}=4\tan x\sec x=\text{RHS}\end{aligned}$$

b) $\frac{1+\tan x}{1-\tan x}=\frac{\cos x+\sin x}{\cos x-\sin x}$

$$\text{LHS}=\frac{1+\tan x}{1-\tan x}=\frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}=\frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}=\frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}=\frac{\cos x+\sin x}{\cos x}\cdot\frac{\cos x}{\cos x-\sin x}=\frac{\cos x+\sin x}{\cos x-\sin x}=\text{RHS}$$

c) $\sin 2x=\frac{2\tan x}{1+\tan^2 x}$

$$\text{RHS}=\frac{2\tan x}{1+\tan^2 x}=\frac{2\frac{\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}}=\frac{\left(2\frac{\sin x}{\cos x}\right)\cos^2 x}{\left(1+\frac{\sin^2 x}{\cos^2 x}\right)\cos^2 x}=\frac{2\sin x\cos x}{\cos^2 x+\sin^2 x}=\frac{\sin 2x}{1}=\text{LHS}$$

d) $\cos 2x = \dfrac{\cot x - \tan x}{\cot x + \tan x}$

$$\text{RHS} = \frac{\cot x - \tan x}{\cot x + \tan x} = \frac{\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}} = \frac{\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \cdot \frac{\sin x \cos x}{\cos^2 x + \sin^2 x}$$

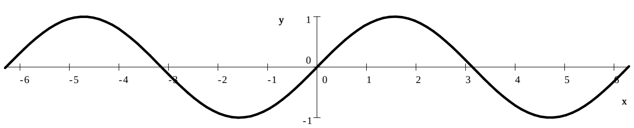
$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos 2x}{1} = \text{LHS}$$

22. a) $-\dfrac{336}{625}$ b) $-\dfrac{527}{625}$ c) $-\dfrac{24}{25}$ d) $\dfrac{7}{25}$ e) $\dfrac{117}{125}$ f) $\dfrac{4}{5}$ g) $\dfrac{117}{44}$ h) $\dfrac{3}{4}$ 23. $\dfrac{20}{17}$ 24. $\dfrac{10}{11}$

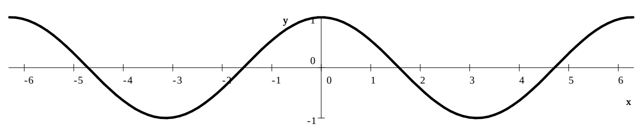
25. a) $\pm\dfrac{3\sqrt{10}}{10}$ b) $-\dfrac{3}{5}$ c) $\dfrac{4}{5}$ 26. $\dfrac{24}{25}$ 27. $\dfrac{4\sqrt{6}}{25}$ 28. $(-1, 1)$ and $(0, -2)$

29. a) $A = \dfrac{1}{2}ah$ and $h = b\sin\gamma$ b) $12\sqrt{2}\,\text{cm}$ 30. $8.75\,\text{ft}$

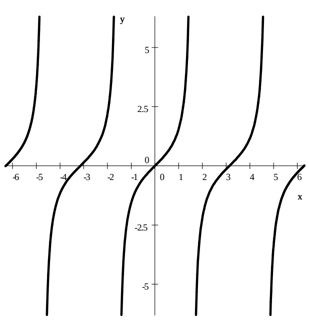
31. a) $f(x) = \sin x$



b) $f(x) = \cos x$



c) $f(x) = \tan x$



32. a) $\dfrac{\ln 10 - \ln 3}{\ln 1.04} = 30.6974$ during the 31st year

b) $\dfrac{\ln 20 - \ln 3}{\ln 1.04} \approx 48.37036$ during the 49th year

c) $\dfrac{\ln 10}{\ln 1.04} \approx 58.7084$ during the 59th year

d) $\dfrac{\ln 40 - \ln 3}{\ln 1.04} \approx 66.043347$ during the 67th year

e) $\dfrac{\ln 50 - \ln 3}{\ln 1.04} \approx 71.7328$ during the 72nd year

f) $\dfrac{\ln 20}{\ln 1.04} \approx 76.3814$ during the 77th year

33. $60°$ 34. $\dfrac{1}{64}$

Solution for 8:

a) Given that $a = \log_5 4$ and $b = \log_2 6$. Express $\log_3 5$ in terms of a and b.
Solution: In the statement $a = \log_5 4$ we switch to base 2:

$a = \log_5 4 = \dfrac{\log_2 4}{\log_2 5} = \dfrac{2}{\log_2 5}$ Thus $a = \dfrac{2}{\log_2 5}$ we solve for $\log_2 5 = \dfrac{2}{a}$

$b = \log_2 6 = \log_2 (2 \cdot 3) = \log_2 3 + \log_2 2 = \log_2 3 + 1$ Thus $b = \log_2 3 + 1$ We solve for $\log_2 3 = b - 1$

Now we're ready: $\log_3 5 = \dfrac{\log_2 5}{\log_2 3} = \dfrac{\dfrac{2}{a}}{b-1} = \dfrac{2}{a(b-1)}$

b) Given that $\log_3 6 = x$ and $\log_5 9 = y$. Express $\log_{18} 100$ in terms of x and y.
Solution: We will switch to base 3.

$x=\log_3 6=\log_3(2\cdot 3)=\log_3 3+\log_3 2=1+\log_3 2$ Thus $x=1+\log_3 2$, we solve for $\log_3 2$ and get $\log_3 2=x-1$

$y=\log_5 9=\dfrac{\log_3 9}{\log_3 5}=\dfrac{2}{\log_3 5}$ Thus $y=\dfrac{2}{\log_3 5}$ We solve for $\log_3 5$ and get $\log_3 5=\dfrac{2}{y}$

We are now ready to solve the problem:

$$\begin{aligned}\log_{18}100 &= \frac{\log_3 100}{\log_3 18}=\frac{\log_3\left(2^2\cdot 5^2\right)}{\log_3\left(2\cdot 3^2\right)}=\frac{\log_3\left(2^2\right)+\log_3\left(5^2\right)}{\log_3(2)+\log_3\left(3^2\right)}=\frac{2\log_3 2+2\log_3 5}{\log_3 2+2}\\ &= \frac{2(x-1)+2\frac{2}{y}}{x-1+2}=\frac{2x-2+\frac{4}{y}}{x+1}\cdot\frac{y}{y}=\frac{2xy-2y+4}{xy+y}\end{aligned}$$

Answer: $\dfrac{2x-2+\frac{4}{y}}{x+1}=\dfrac{2xy-2y+4}{xy+y}=\dfrac{2(xy-y+2)}{y(x+1)}$ All three forms are acceptable as final answer.

Last revised: November 9, 2017