Please note that Quiz 10 may also cover topics from Reviews for Quizzes 1-9 or from the Exam 1 Review.

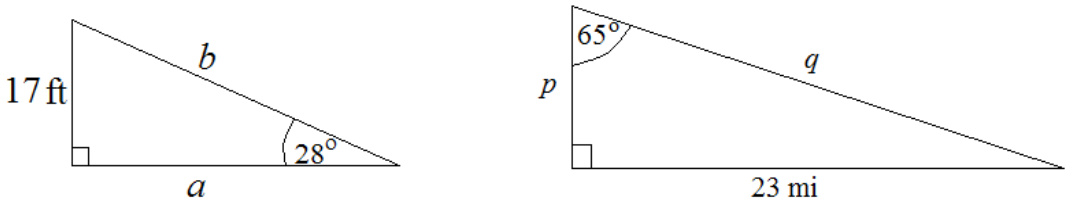
1. Compute each of the following sums.
   a) $2+4+6+...+200$ b) $11+21+31+...+651$ c) $78+83+88+...+1273$
2. Suppose that f is a function defined by $f(x)=-2x+5$. Compute or simplify each of the following.
   a) $f(3)+f(4)$ c) $f(1)-f(2)$ e) $3\cdot f(4)$ g) $f(f(3))$ i) $f(2x)$ k) $f\left(x^2\right)$
   b) $f(3+4)$ d) $f(1-2)$ f) $f(3\cdot 4)$ h) $f(f(f(3)))$ j) $2f(x)$ l) $(f(x))^2$
3. Suppose that g is a function defined by $g(x)=2x^2-10$. Simplify each of the following.
   a) $g(2)+g(3)$ b) $g(2+3)$ c) $g(a+1)$ d) $g(a)+g(1)$ e) $g(2a)$ f) $2g(a)$
4. List all subsets of $A=\{1,2,3,4\}$
5. List all two-element subsets of $U=\{1,2,3,4,5,6,7,8,9,10\}$.
6. How many diagonals are there in a regular polygon of n sides?
7. How many 3−digit numbers can be formed using the digits $1,3,5,7,8$, and 9, if
   a) repetition of digits is allowed b) repetition of digits is not allowed?
8. How many 4−digit numbers can be formed using the digits $1,3,5,7,8$, and 9, if
   a) repetition of digits is allowed b) repetition of digits is not allowed?
9. Compute the exact value of each of the following. Simplify your answer.
   a) $\sin 60^\circ-\tan 30^\circ$ b) $\sec 45^\circ-\sin 45^\circ$ c) $\tan 30^\circ\cdot\tan 45^\circ\cdot\tan 60^\circ$ d) $(\sin 30^\circ)^2+(\cos 30^\circ)^2$
10. Rationalize the denominator in $\dfrac{\sqrt{x}-\sqrt{2}}{\sqrt{x}+\sqrt{2}}$.
11. Simplify each of the following. Present all answers using only positive exponents.
    a) $\dfrac{2^{-1}-3^{-2}}{-2^{-2}+1}$ b) $\dfrac{2b^{-2}\left(-a^3\right)^{-2}b^0}{\left(-b^2\right)^{-3}a^{-5}}$ c) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ d) $\left(\dfrac{1-\sqrt{5}}{2}\right)^2-1$ e) $\dfrac{\sqrt{500}-\sqrt{20}}{\sqrt{45}-\sqrt{5}}$
12. Simplify each of the following.
    a) $2^{-3}+5^{-2}$
    b) $\left(x^{-1}y^{-1}\right)^{-1}$
    c) $\left(x^{-1}+y^{-1}\right)^{-1}$
    d) $\left(\dfrac{2a^3b^{-2}\left(-ab^{-2}\right)^{-3}ba^0}{b^{-1}\left(-2b^2a^{-2}\right)^3b}\right)$
    e) $\dfrac{2a^{-2}b^3}{a^5b^{-1}}$
    f) $\dfrac{2a^{-2}+b^3}{a^5-b^{-1}}$
    g) $\left(\sqrt{3}\right)^6$
    h) $\left(\dfrac{1}{\sqrt{2}}\right)^{10}$
13. Solve the equation $5(x-2)^2-3x+2=x-2$. Check your solution(s) using exact values.
14. a) Solve $3x^2+x=3x+2$.
    b) Check your solutions using exact values.
15. State the center and radius for each of the following circles.
    a) $x^2+y^2=25$ b) $(x-3)^2+(y+4)^2=20$ c) $6y+x^2+y^2=6+2x$
16. Consider the circle $(x+2)^2+(y-1)^2=25$. Find all points on the circle with
    a) x−coordinate -6 b) y−coordinate 6 c) x−coordinate 5

17. Find an equation for the tangent line drawn to the graph of $(x-3)^2+(y+6)^2=50$ at the point $P(2,-13)$.

18. A person is standing 3 ft away from a street light that is 15.6 ft tall. How long is his shadow if he is 5.2 ft tall?

19. A right triangle has sides 5 ft, 12 ft, and 13 ft long.
    a) State the value of all six trigonometric functions of α if α is the angle opposite the 5 ft long side.
    b) State the value of all six trigonometric functions of β if β is the angle opposite the 12 ft long side.
    c) Compute the approximate value (up to four or more decimal places) of the measure of the smallest angle in the triangle.

20. Compute the exact value and the approximate value for each of a, b, p, and q, based on the picture below.



21. The hypotenuse of a right triangle is 31 cm long. One angle in the triangle measures 42°.
    a) Find the exact value of the length of the missing sides in the triangle.
    b) Find an approximate value of the length of the missing sides in the triangle. Round your answer to four or more decimal places.

22. Draw a square with sides 1 unit long and draw in one of the diagonals. Use the isosceles right triangle you obtained to find all trigonometric function values of 45°.

23. The hypotenuse of a right triangle is 74 units. The difference between the lengths of the other two sides is 46 units.
    a) How long are the sides of this triangle?
    b) Use your calculator to find an approximate value of the smallest angle in the triangle. Present your answer in degrees, accurate up to four or more decimals.

24. Consider a circle of radius 12 units and with a center C. P is a point located 30 units away from C. We draw a tangent line from P to the circle. The point of tangency is Q.
    a) Compute the exact value of $d(P,Q)$ (that is the distance between P and Q).
    b) Let α denote the angle CPQ. Compute the exact value of all six trigonometric functions of α.

25. Find the exact value and an approximate value for the angle that is formed between the line $y=\frac{1}{2}x$ and the positive part of the $x-$axis.

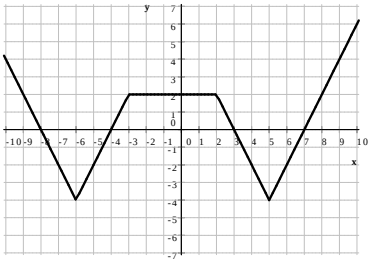
26. Compute the perimeter and area of the 15-sided regular polygon that is written into a circle with radius 6 cm. Present both exact and approximate values for the answers.

27. Find an equation for the line that
    a) is parallel to $5x+3y=-15$ and passes through the point $(-6,2)$.
    b) is perpendicular to $5x+3y=-15$ and passes through the point $(-6,2)$.
    c) passes through the points $(3,8)$ and $(-1,2)$.

28. Find an equation of the perpendicular bisector of the line segment AB where $A\,(8,-3)$ and $B\,(12,13)$. (Recall that the perpendicular bisector of a line segment AB is a line perpendicular to the line segment and passes through the midpoint of the line segment.)

29. Suppose that triangle ABC is defined by $A\,(-3,2)$, $B\,(1,7)$, and $C\,(6,-1)$. Find an equation of the altitude drawn to side AC.

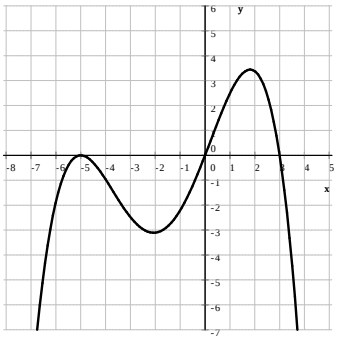
30. Find all values of x for which $P\,(x,y)$ is on the graph given and



a) $y \le 0$ c) $y \ge 2$ e) $y < -2$

b) $y > 2$ d) $y \le 2$ f) $y < -5$

31. Find all values of x for which $P\,(x,y)$ is on the graph given and



a) $y > 0$

b) $y < 0$

c) $y \ge 0$

d) $y \le 0$

32. Solve each of the following inequalities.

a) $6x + x^2 \le 16$ c) $x^2 > 4x$ e) $16x^2 \le 8x - 1$ g) $x^2 - 5x + 14 \ge x + 3$

b) $6x + x^2 \le 15$ d) $x^2 > 4x + 1$ f) $(2x-1)^2 \le 3\,(x-1)^2$

33. A company finds that if it prices its product at \$30, then it can sell 1000 items. For every dollar increase in the price, the company will sell 5 less items.
    a) What is the maximum revenue possible, and what price guarantees that maximal revenue?
    b) What price range will guarantee a revenue greater than \$56 000?

34. Solve each of the following systems over the real numbers.

a) $\begin{cases} x + y = 8 \\ 2xy = 30 \end{cases}$ b) $\begin{cases} x + y = -1 \\ \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{6} \end{cases}$ c) $\begin{cases} x^2 + y^2 = 10 \\ xy = 3 \end{cases}$ d) $\begin{cases} x^2 + 4x + 4 = -2 \\ x + y = 3 \end{cases}$

35. Suppose that m and n are real number such that m is 10 less than three times n. Find each of the following.
    a) the smallest value of $n^2 + m^2$
    b) the smallest value of nm
    c) the greatest value of $n^2 - m^2$

36. Suppose that m and n are real number such that m is 10 less than three times n. For what values of n will the value of $m^2 + n^2$ be greater than 260?

37. Find both coordinates of all point(s) where the graphs of the given equations intersect each other.

a) $\begin{cases} (x-1)^2+(y+3)^2=10 \\ 3y=-x-8 \end{cases}$

b) $\begin{cases} (x+8)^2+(y-2)^2=50 \\ y+7x=-4 \end{cases}$

c) $\begin{cases} (x-4)^2+(y+6)^2=20 \\ x+2y=7 \end{cases}$

d) $\begin{cases} (x+8)^2+(y-2)^2=50 \\ 7y=x+22 \end{cases}$

e) $\begin{cases} (x-2)^2+(y-2)^2=20 \\ x^2+(y-8)^2=100 \end{cases}$

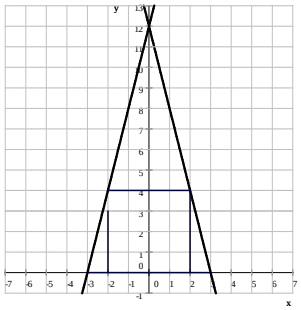
f) $\begin{cases} (x-1)^2+(y-3)^2=5 \\ (x-3)^2+(y-4)^2=20 \end{cases}$

38. An arch is in the shape of a semicircle. At a point along the base 4 feet from an end of the arch, the height of the arch is 10 feet. Find the maximum height of the arch.

39. The hypotenuse of a right triangle is 50 feet long. Find the other two sides, given that the perimeter of the triangle is 112 feet.

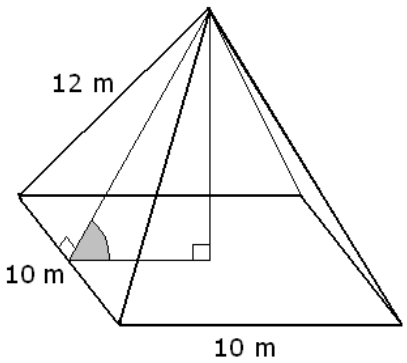
40. Prove that there are no real values for x and y that would satisfy the equation $x^2-4x+5=-6y-y^2-11$.

41. Let l_1 and l_2 denote the lines $y=4x+12$ and $y=-4x+12$, respectively. Let R be a rectangle with vertical and horizontal sides, where one horizontal side is on the $x-$axis and the vertices connecting the other horizontal side lie on the lines l_1 and l_2, above the $x-$axis. What is the maximal value of the area of such a rectangle?



42. The first row in a theater has 35 seats in it. The second row has three more seats than the first row. The third row has three more seats than the second row. And so on, each row has three more seats than the row before. If the last row has 164 seats in it, how many seats are there in the entire theater?

43. Consider a square based straight pyramid as shown on the picture. The base is a square with sides 10 m long, and all other edges are 12 m long. Find the exact and approximate value of the angle that is formed between a triangular face and the square base. The angle is marked on the picture.



Answers

1. a) 10 100 b) 21 515 c) 162 120

2. a) -4 b) -9 c) 2 d) 7 e) -9
   f) -19 g) 7 h) -9 i) $-4x+5$
   j) $-4x+10$ k) $-2x^2+5$ l) $4x^2-20x+25$

3. a) 6 b) 40 c) $2a^2+4a-8$ d) $2a^2-18$
   e) $8a^2-10$ f) $4a^2-20$

4. All subsets of $A=\{1,2,3,4\}$

   0-element subsets: $\varnothing$
   1-element subsets: $\{1\},\{2\},\{3\},\{4\}$
   2-element subsets: $\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}$
   3-element subsets: $\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\}$
   4-element subsets: $\{1,2,3,4\}$

5. All two-element subsets of $U=\{1,2,3,4,5,6,7,8,9,10\}$

$\{1,2\}$								
$\{1,3\}$	$\{2,3\}$							
$\{1,4\}$	$\{2,4\}$	$\{3,4\}$						
$\{1,5\}$	$\{2,5\}$	$\{3,5\}$	$\{4,5\}$					
$\{1,6\}$	$\{2,6\}$	$\{3,6\}$	$\{4,6\}$	$\{5,6\}$				
$\{1,7\}$	$\{2,7\}$	$\{3,7\}$	$\{4,7\}$	$\{5,7\}$	$\{6,7\}$			
$\{1,8\}$	$\{2,8\}$	$\{3,8\}$	$\{4,8\}$	$\{5,8\}$	$\{6,8\}$	$\{7,8\}$		
$\{1,9\}$	$\{2,9\}$	$\{3,9\}$	$\{4,9\}$	$\{5,9\}$	$\{6,9\}$	$\{7,9\}$	$\{8,9\}$	
$\{1,10\}$	$\{2,10\}$	$\{3,10\}$	$\{4,10\}$	$\{5,10\}$	$\{6,10\}$	$\{7,10\}$	$\{8,10\}$	$\{9,10\}$

6. $\dfrac{n(n-3)}{2}$

7. a) $6^3=216$ b) $6\cdot5\cdot4=120$

8. a) $6^4=1296$ b) $6\cdot5\cdot4\cdot3=360$

9. a) $\dfrac{\sqrt{3}}{6}$ b) $\dfrac{\sqrt{2}}{2}$ c) 1 d) 1

10. $\dfrac{x+2-2\sqrt{2x}}{x-2}$

11. a) $\dfrac{14}{27}$ b) $-\dfrac{2b^4}{a}$ c) $2-\sqrt{3}$
    d) $\dfrac{1-\sqrt{5}}{2}$ e) 4

12. a) $\dfrac{1}{3}$ b) $\sqrt[3]{7}$ c) 4 d) $\dfrac{1}{4}$ e) -4
    f) undefined g) $-\dfrac{1}{4}$ h) $\sqrt{5}$ i) $\dfrac{1}{2}$
    j) 1 k) undefined l) $-\dfrac{1}{2}$ m) $\dfrac{1}{8}$
    n) 1 o) $\dfrac{1}{8}$ p) undefined q) $10\sqrt{10}$
    r) $\dfrac{1}{\sqrt{6}}=\dfrac{\sqrt{6}}{6}$

13. $x_{1,2}=\dfrac{12\pm2\sqrt{6}}{5}$

14. a) $\dfrac{1\pm\sqrt{7}}{3}$ b) If $x=\dfrac{1-\sqrt{7}}{3}$, then

$$\begin{aligned}\text{LHS} &= 3\left(\frac{1-\sqrt{7}}{3}\right)^2+\frac{1-\sqrt{7}}{3}\\ &= 3\cdot\frac{8-2\sqrt{7}}{9}+\frac{1-\sqrt{7}}{3}\\ &= \frac{8-2\sqrt{7}}{3}+\frac{1-\sqrt{7}}{3}=\frac{9-3\sqrt{7}}{3}\\ &= \frac{3(3-\sqrt{7})}{3}=3-\sqrt{7}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= 3\left(\frac{1-\sqrt{7}}{3}\right)+2\\ &= 1-\sqrt{7}+2=3-\sqrt{7}\end{aligned}$$

Checking the other solution goes similarly.

15. a) $C(0,0)$, $r=5$ b) $C(3,-4)$, $r=\sqrt{20}$
    c) $C(1,-3)$, $r=4$

16. a) $(-6,-2)$ and $(-6,4)$ b) $(-2,6)$
    c) There is no such point.

17. $-\dfrac{1}{7}(x-2)=y+13$ or $y=-\dfrac{1}{7}x-\dfrac{89}{7}$

18. 1.5 ft

19. a) $\sin\alpha = \frac{5}{13}$ $\cos\alpha = \frac{12}{13}$ $\tan\alpha = \frac{5}{12}$
$\csc\alpha = \frac{13}{5}$ $\sec\alpha = \frac{13}{12}$ $\cot\alpha = \frac{12}{5}$
b) $\sin\beta = \frac{12}{13}$ $\cos\beta = \frac{5}{13}$ $\tan\beta = \frac{12}{5}$
$\csc\beta = \frac{13}{12}$ $\sec\beta = \frac{13}{5}$ $\cot\beta = \frac{5}{12}$
c) $22.619\,865°$

20. $a = \frac{17}{\tan 28°}\,\text{ft} \approx 31.9724\,\text{ft}$
$b = \frac{17}{\sin 28°}\,\text{ft} \approx 36.21093\,\text{ft}$
$p = \frac{23}{\tan 65°}\,\text{mi} \approx 10.72507614\,\text{mi}$
$q = \frac{23}{\sin 65°}\,\text{mi} \approx 25.377\,692\,\text{mi}$

21. a) $(31\sin 42°)$ cm and $(31\cos 42°)$ cm
b) 20.743 0488 cm and 23.037 49 cm

22. $\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 45° = 1$
$\csc 45° = \sqrt{2}$ $\sec 45° = \sqrt{2}$ $\cot 45° = 1$

23. a) 24, 70, and 74 units long b) $18.924\,644°$

24. a) $\sqrt{756} = 6\sqrt{21}$
b) $\sin\alpha = \frac{2}{5}$ $\cos\alpha = \frac{\sqrt{21}}{5}$ $\tan\alpha = \frac{2}{\sqrt{21}}$
$\csc\alpha = \frac{5}{2}$ $\sec\alpha = \frac{5}{\sqrt{21}}$ $\cot\alpha = \frac{\sqrt{21}}{2}$

25. exact value: $\tan^{-1}\left(\frac{1}{2}\right)$
approximation: $26.565\,051\,18°$

26. $P = 180\sin 12°\,\text{cm} \approx 37.424\,104\,347\,2\,\text{cm}$
$A = 540\cos 12°\sin 12°\,\text{cm}^2 \approx 109.818\,9\,\text{cm}^2$

27. a) $y = -\frac{5}{3}x - 8$ b) $y = \frac{3}{5}x + \frac{28}{5}$
c) $y = \frac{3}{2}x + \frac{7}{2}$

28. $-\frac{1}{4}(x - 10) = y - 5$ or $y = -\frac{1}{4}x + \frac{15}{2}$

29. $3(x - 1) = y - 7$ or $y = 3x + 4$

30. a) $[-8, -4] \cup [3, 7]$ b) $(-\infty, -9) \cup (8, \infty)$
c) $(-\infty, -9] \cup [-3, 2] \cup [8, \infty)$ d) $[-9, 8]$
e) $(-7, -5) \cup (4, 6)$ f) no solution

31. a) $(0, 3)$ b) $(-\infty, -5) \cup (-5, 0) \cup (3, \infty)$
c) $[0, 3] \cup \{-5\}$ d) $(-\infty, 0] \cup [3, \infty)$

32. a) $[-8, 2]$ b) $\left[-3 - 2\sqrt{6}, -3 + 2\sqrt{6}\right]$
c) $(-\infty, 0) \cup (4, \infty)$
d) $\left(-\infty, 2 - \sqrt{5}\right) \cup \left(2 + \sqrt{5}, \infty\right)$ e) $\left\{\frac{1}{4}\right\}$
f) $-1 - \sqrt{3} \le x \le -1 + \sqrt{3}$ g) $\mathbb{R}$

33. a) a price of \$115 for a revenue of \$66 125
b) a price between \$100 and \$130

34. a) $(3, 5)$ and $(5, 3)$ b) $(2, -3)$ and $(-3, 2)$
c) $(-1, -3)$, $(3, 1)$, $(1, 3)$, $(-3, -1)$
d) no real solution

35. a) 10 b) $-\frac{25}{3}$ c) $\frac{25}{2}$

36. interval notation: $(-\infty, -2) \cup (8, \infty)$
inequality notation: $n < -2$ or $n > 8$

37. a) $(-2, -2)$ and $(4, -4)$ b) $(-1, 3)$
c) no intersection point d) $(-1, 3)$ and $(-15, 1)$
e) $(0, -2)$, and $(6, 0)$ f) $(-1, 2)$

38. 14.5 feet

39. 14 and 48 feet

40. We complete the square on both sides.

$$(x - 2)^2 + 1 = -(y + 3)^2 - 2$$

Then we see that the value of the right-hand side is 1 or greater for all values of x. Meanwhile, the value of the other side is -2 or less. So the two sides can never be equal.

41. 18

42. 4378

43. exact value: $\cos^{-1}\left(\frac{5}{\sqrt{119}}\right)$
approximation: $62.719\,361\,28°$