

1. Simplify each of the following. Use exact values.

a) $\sqrt[10]{x^{16}}$	e) $\ln(e^{100})$	h) $\log_2\left(\frac{1}{\sqrt{8}}\right)$	k) $\log_4(\sqrt[3]{2})$
b) $(\sqrt[3]{2})^{12}$	f) $\ln\left(\frac{1}{e}\right)$	i) $\log_9(\sqrt{3^4})$	l) $\ln(-e)$
c) $(\sqrt[4]{2})^{12}$	g) $\log_2(8^{10})$	j) $\log_{25}(5^{10})$	m) $\log_8\left(\frac{1}{(\sqrt{2})^x}\right)$
d) $-32^{-3/5}$			

2. Approximate each of the following by placing them between two consecutive integers.

a) $\sqrt{70}$ b) $\log_3 70$ c) $\log_2 200$ d) $\log 144$

3. Re-write each of the following as an exponential statement.

a) $2 = \log_x(2x - 1)$ b) $3 = \ln(x + 1)$ c) $A = \log_B C$ d) $-1 = \log_3(x^2 - 1)$

4. Solve each of the following basic logarithmic equations.

a) $\log_3(2x - 7) = 2$	d) $\ln(2x + 1) = -3$	g) $\frac{2}{3}\log_2(x + 2) - 1 = 7$
b) $\log_3(x^2 - 19) = 4$	e) $\log_2(3x + 4) = -1$	h) $\frac{2\log_2(x + 2) - 1}{3} = 7$
c) $\ln(1 - 3x) = 28$	f) $3\log_2(7x - 1) - 2 = 10$	

5. Re-write each of the following as a logarithmic statement. Assume that all variables represent positive numbers.

a) $5^{A+1} = 20$ b) $2^{5x+1} = 35$ c) $e^{3x-x^2} = 12$ d) $P^Q = R$

6. Solve each of the following basic exponential equations.

a) $2^x = \frac{1}{16}$	c) $2^x = \frac{1}{\sqrt{2}}$	f) $3^{5x-1} = \sqrt{27}$	i) $3^{5x-1} = 40$	l) $e^{2x+8} = 1$
	d) $2^x = 21$	g) $3^{5x-1} = -9$	j) $e^{2x+8} = 0$	
b) $2^x = -8$	e) $2^x = -4$	h) $3^{5x-1} = \frac{1}{9}$	k) $e^{2x+8} = \frac{1}{e^5}$	m) $e^{2x+8} = 10$

7. a) Factor the given expression over \mathbb{R} by completing the square $12x - 18x^2 + 12$.

b) What is the greatest value of the expression $12x - 18x^2 + 12$?

8. Sketch the graph of each of the following.

a) $y = 2^x$ b) $y = \left(\frac{1}{2}\right)^x$ c) $y = \log_2 x$ d) $y = \log_{1/2}(x)$

9. Solve each of the following.

a) $3 + 2 \cdot 3^{2x-1} = 21$ b) $5 - 2e^{3x+8} = -7$ c) $\frac{3}{4} \cdot 2^{4x+1} = -12$ d) $\frac{1}{e^{x-3}} = \sqrt{e^7}$

10. Solve each of the following inequalities.

a) $x^2 > 9x$ c) $24x - 3x^2 - 48 \geq 0$ e) $x^2 - 2x + 3 < 0$ g) $x^2 - 4x \leq 3$
 b) $8x - 2x^2 + 2 \leq 0$ d) $x^2 < 9$ f) $10x - 5x^2 - 20 \leq 0$

11. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

a) $(x - 2)^2 + (y + 1)^2 = 20$ and $x - 2y = 19$ c) $(x + 1)^2 + (y - 4)^2 = 17$ and $x + 4y = 15$
 b) $(x + 1)^2 + (y - 3)^2 = 50$ and $x - 7y + 72 = 0$ d) $(x - 2)^2 + (y - 1)^2 = 25$ and $3y = 4x - 30$

$$e) (x - 1)^2 + (y + 7)^2 = 25 \text{ and } y = -\frac{1}{2}x - \frac{3}{2}$$

$$f) (x + 2)^2 + (y - 1)^2 = 10 \text{ and } (x - 1)^2 + (y + 2)^2 = 4$$

$$g) (x + 2)^2 + (y - 1)^2 = 10 \text{ and } (x - 4)^2 + (y + 2)^2 = 25$$

12. Find an equation of the tangent line drawn to $8x + x^2 + y^2 = 6y$ to the point $(-7, -1)$.

13. Two numbers a and b are such that $3a + b = 20$. Find the exact value of each of the following.

a) smallest value of $a^2 + b^2$. b) greatest value of ab . c) greatest value of $a^2 - b^2$.

14. Find the domain for each of the given functions.

$$a) f(x) = \frac{1}{3x - 8}$$

$$e) g(x) = \sqrt{x^2 - 6x}$$

$$i) h(x) = \ln(9 - x^2)$$

$$b) g(x) = \sqrt{3x - 8}$$

$$f) h(x) = \log_2(x^2 - 6x)$$

$$j) f(x) = \frac{1}{x - 2} + \sqrt{x + 5}$$

$$c) h(x) = \log_3(3x - 8)$$

$$g) f(x) = \frac{1}{9 - x^2}$$

$$k) f(x) = \sqrt{x - 6} + \sqrt{6 - x}$$

$$d) f(x) = \frac{1}{x^2 - 6x}$$

$$h) g(x) = \sqrt{9 - x^2}$$

$$k) f(x) = \frac{\sqrt{x - 2}}{\sqrt{5 - x}}$$

15. Consider the triangle with sides 6 m, 6 m, and 8 m long.

- Compute the exact value of the area of the triangle.
- Compute the exact value of the smallest angle in the triangle.
- Compute an approximate value of the smallest angle in the triangle.

16. Let C be the circle $2y + x^2 + y^2 = 6x + 26$ and $P(9, -9)$. Compute the angle that is formed by the two tangent lines drawn to the circle from P . Present your answer as a) an exact value b) an approximation

17. Consider a pyramid with a square base of sides 1 unit long. If the triangular faces form an angle of 52.5° with the base, compute

- the height of the pyramid
- the height of a triangular face
- the edges connecting the base with the highest point of the pyramid.

18. We are driving toward a tower. The angle of elevation from the car to the top of the tower is 15° . Then we drive 120 ft toward the tower. Now the angle of elevation is 25° . How tall is the tower? Present your answer as an approximation.

19. Simplify each of the following.

$$a) \sin 30^\circ \sin 45^\circ \sin 60^\circ + \cos 30^\circ \cos 45^\circ \cos 60^\circ$$

$$c) \cos^2 60^\circ + \sin^2 60^\circ$$

$$b) \tan 30^\circ - \tan 45^\circ + \tan 60^\circ$$

$$d) \cos^2 30^\circ - \sin^2 30^\circ$$

20. a) Let α be an acute angle. Compute the exact value of $\cos \alpha$ if $\tan \alpha = 3$.

b) Let β be an acute angle. Compute the exact value of $\sin \beta$ if $\tan \beta = x$.

21. Suppose that $\{a_n\}$ is an arithmetic sequence determined by $a = -125$ and $d = 3$. Compute each of the following.

a) a_{30} b) s_{30} c) a_{50} d) s_{50} e) a_{100} f) s_{100}

22. Find the first term and common difference in the arithmetic sequence $\{a_n\}$ if we know that $a_{10} = 174$ and $a_{20} = 334$.

23. Find the first term and common difference in the arithmetic sequence $\{a_n\}$ if we know that $a_{15} = 22$ and $s_{15} = 1065$.

24. Find the first term and common difference in the arithmetic sequence $\{a_n\}$ if we know that $a_{30} = 700$ and $s_{50} = 30\,500$.

25. Find n if $s_n = -360$ in the arithmetic sequence determined by $a = -60$ and $d = 5$.

26. Which of the following functions are one-to-one?

a) $f(x) = \log_3 x$ b) $f(x) = -|x|$ c) $h(x) = x^2 - x - 2$ d) $g(x) = \left(\frac{1}{2}\right)^x$ e) $g(x) = x^3$

27. Suppose that $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$.

- a) How many different functions are possible whose domain is A and its range is a subset of B ?
 b) How many different relations are possible whose domain is A and its range is a subset of B ?

Answers

1. a) $\sqrt[5]{x^8}$ b) 16 c) 8 d) $-\frac{1}{8}$ e) 100 f) -1 g) 30 h) $-\frac{3}{2}$ i) $\frac{A}{4}$ j) 20 k) $\frac{1}{6}$ l) undefined m) $-\frac{x}{6}$

2. a) $8 < \sqrt{70} < 9$ b) $3 < \log_3 70 < 4$ c) $7 < \log_2 200 < 8$ d) $2 < \log 144 < 3$

3. a) $x^2 = 2x - 1$ b) $e^3 = x + 1$ c) $B^A = C$ d) $3^{-1} = x^2 - 1$

4. a) 8 b) ± 10 c) $\frac{1}{3} - \frac{e^{28}}{3}$ d) $\frac{1}{2e^3} - \frac{1}{2}$ e) $-\frac{7}{6}$ f) $\frac{17}{7}$ g) 4094 h) 2046

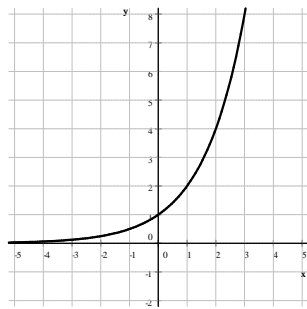
5. a) $A + 1 = \log_5 20$ b) $5x + 1 = \log_2 35$ c) $3x - x^2 = \ln 12$ d) $Q = \log_P R$

6. a) -4 b) no solution c) $-\frac{1}{2}$ d) $\log_2 21$ e) no solution f) $\frac{1}{2}$ g) no solution

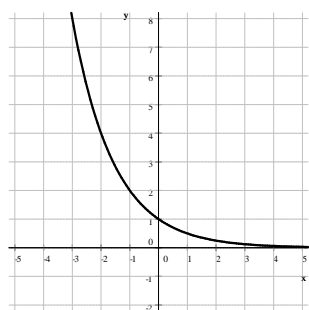
h) $-\frac{1}{5}$ i) $\frac{1}{5}(1 + \log_3 40)$ j) no solution k) $-\frac{13}{2}$ l) -4 m) $\frac{1}{2}(-8 + \ln 10)$

7. a) $-18 \left(x - \frac{1 - \sqrt{7}}{3}\right) \left(x - \frac{1 + \sqrt{7}}{3}\right)$ b) 14, (when $x = \frac{1}{3}$)

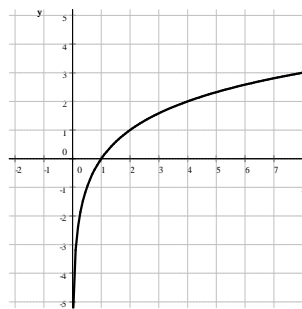
8. a) $y = 2^x$



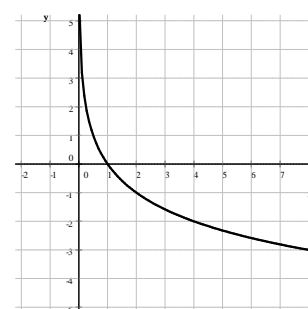
b) $y = \left(\frac{1}{2}\right)^x$



c) $y = \log_2 x$



d) $y = \log_{1/2}(x)$



9. a) $\frac{3}{2}$ b) $\frac{-8 + \ln 6}{3}$ c) no solution d) $-\frac{1}{2}$ 10. a) $(-\infty, 0) \cup (9, \infty)$ b) $(-\infty, 2 - \sqrt{5}] \cup [2 + \sqrt{5}, \infty)$

c) 4 d) $(-3, 3)$ e) no solution f) \mathbb{R} g) $[-\sqrt{7} + 2, \sqrt{7} + 2]$

11. a) no intersection point b) $(-2, 10)$ c) $(3, 3)$ and $(-5, 5)$ d) $(6, -2)$ e) $(1, -2)$ and $(5, -4)$
f) $(1, 0)$ and $(-1, -2)$ g) $(-1, -2)$ and $(1, 2)$ 12. $-\frac{3}{4}(x+7) = y+1$ or $y = -\frac{3}{4}x - \frac{25}{4}$
13. 8. a) 40 b) $\frac{100}{3}$ c) 50 14. a) $x \neq \frac{8}{3}$ b) $x \geq \frac{8}{3}$ c) $x > \frac{8}{3}$ d) $x \neq 0, 6$ e) $x \leq 0$ or $x \geq 6$
f) $x < 0$ or $x > 6$ g) $x \neq \pm 3$ h) $-3 \leq x \leq 3$ i) $-3 < x < 3$ i) $x \geq -5$ but $x \neq 2$ j) $\{6\}$ k) $2 \leq x < 5$
15. a) $8\sqrt{5} \text{ m}^2$ b) $\cos^{-1}\left(\frac{4}{6}\right)$ c) 48.1896851° 16. a) $2 \sin^{-1}\left(\frac{3}{5}\right)$ b) 73.73980°
17. a) $\frac{1}{2} \tan 52.5^\circ \approx 0.6516127$ unit b) $\frac{1}{2 \cos 52.2^\circ} \approx 0.81578438$ unit c) 0.95682 unit
18. 75.58846151 ft 19. a) $\frac{\sqrt{6}}{4}$ b) $\frac{4\sqrt{3}}{3} - 1$ c) 1 d) $\frac{1}{2}$ 20. a) $\frac{1}{\sqrt{10}}$ b) $\frac{x}{\sqrt{x^2+1}}$
21. a) -38 b) -2445 c) 22 d) -2575 e) 172 f) 2350 22. $a = 30, d = 16$
23. $a = 120, d = -7$ 24. $a = 120, d = 20$ 25. 9, 16 26. a,d,e 27. a) $3^4 = 81$ b) $7^4 = 2401$

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