1. Simplify each of the following.

a) $\dfrac{3x^3y^{-3}\left(-2x^3y^5\right)^0}{-2y^0x^5\left(3y^{-2}\right)}$

b) $\dfrac{a^{-5}b^{-10}}{a^{-10}b^{-5}}$

c) $\dfrac{a^{-5}-b^{-10}}{a^{-10}+b^{-5}}$

d) $\left(x^{-1}+1\right)^{-1}$

e) $\dfrac{a^{-2}-b^{-2}}{b^{-1}+a^{-1}}$

f) $\left(\dfrac{3a^0b^{-5}}{-2a^3b^{-3}}\right)^{-2}$

g) $\left(\dfrac{2a^{1/3}b^{-2/3}}{-b^{3/2}a^{-2/2}}\right)^6$

h) $(\sec 45°)^{10}$

i) $\sqrt[12]{x^9}$

j) $\sqrt[9]{x^{12}}$

2. Simplify each of the following.

a) $\sin 300°$

b) $\csc(-315°)$

c) $\sin 420°$

d) $\cot 960°$

e) $\cos(-240°)$

f) $\tan 630°$

g) $\cos(-540°)$

h) $\cot 450°$

i) $\cos 2070°$

j) $\sin(-150°)$

k) $\tan(-390°)$

l) $\cos(-180°)$

m) $\csc(-45°)$

n) $\cos 2880°$

o) $\tan 0°$

3. Simplify each of the following.

a) $\sin^2 3° + \cos^2 3°$

b) $\sin^2 150° - \cos^2 150°$

c) $\dfrac{\tan^3 60°}{\tan^3 30°}$

d) $2^{\sin 30°}$

e) $2^{\log_8(5x)}$

f) $e^{-\ln 3}$

g) $\log_{\sqrt{2}}(\sec 60°)$

h) $3^{\log_3 5}$

i) $3^{\log_9 10}$

j) $3^{\log_3 5+\log_9 10}$

k) $3^{\log_3 5}+3^{\log_9 10}$

l) $\log_{1/4} 8$

m) $49^{\log_7(-3)}$

n) $\log_4\left(\dfrac{1}{\sqrt{32}}\right)$

o) $2^{5\log_8 3}$

p) $5^{-2\log_5 2}$

q) $8^{\log_2 3}$

r) $2^{\log_8 2}$

s) $\log_{27}\left((\tan 60°)^4\right)$

t) $\dfrac{\sin^2(600°)\cos^2(300°)}{\sin^2(315°)}$

u) $\dfrac{\sin 210°\cos 210° - \sin 135°\cos 135°}{\tan 45° - \tan 30°}$

4. Place each of the following between two consecutive integers.

a) $\log_3 50$ b) $\sqrt[3]{80}$ c) $\log_2\left(\dfrac{1}{5}\right)$ d) $\log 2017$

5. Compute each of the following sums.

a) $73+76+79+\ldots+208$ b) $143+136+129+\ldots+(-53)$

6. Solve each of the following basic exponential equations.

a) $3^x = 243$

b) $3^x = 38$

c) $2^{4x-1} = 128$

d) $2^{4x-1} = 42$

e) $10^{3x-1} = 10^{41}$

f) $10^{3x-1} = 19$

g) $e^{2x-8} = e^{10}$

h) $e^{3x-7} = 5$

i) $3^{2x-5} = -27$

j) $5^{2x-7} = \dfrac{1}{125}$

k) $20 - 5^{3x+1} = 19.8$

l) $4\cdot 2^{5-x} = 64$

m) $3\cdot e^{-2x+5} = 24$

n) $3 = 3^{x^2+x} - 6$

o) $e^{5x-1} = \dfrac{1}{\sqrt[3]{e}}$

p*) $2^x + 2^{x+1} + 2^{x+2} = 56$

7. Re-write each of the logarithmic statements as an exponential statement.

a) $\log_A B = C$ b) $\log_2(3x-1) = 5$ c) $\ln\left(\dfrac{1}{3}x+1\right) = -2$

8. Solve each of the following basic logarithmic equations.

a) $\log_2(3x-1) = 5$ b) $\dfrac{2}{3}\ln(x+4) - 1 = 5$ c) $\dfrac{2\ln(x+4)-1}{3} = 5$

9. Let $f(x) = \dfrac{x-2}{3x+6}$. Simplify each of the following.

   a) $f(0)$
   b) $f(1)$
   c) $f(-1)$
   d) $f\left(\dfrac{2}{3}\right)$
   e) $f(-2)$
   f) $f(2) + f(3)$
   g) $f(2+3)$
   h) $f(a)$
   i) $2f(a)$
   j) $f(2a)$
   k) $f(f(2))$
   l) $f\left(f\left(\dfrac{1}{2}\right)\right)$

   m) Compute the value of x for which $f(x) = \dfrac{1}{7}$.

10. Which of the following relations are functions?

    a) f assigns y to x where $x^2 = y^2$ b) g assigns y to x where $y = x^2$ c) h assigns y to x where $y^2 = x$

11. The sum of a number a and four times a number b is 30.

    a) Find the greatest value of ab. b) Find the greatest value of $6b^2 - a^2$

12. We are driving toward a tower. The angle of elevation is 35°. Then we drive 50 ft toward the tower. Now the angle of elevation is 40°. How tall is the tower? Present your answer as an approximation, accurate up to three decimal places.

13. A number is 3 greater than its own reciprocal. Find this number.

14. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

    a) $y = x^2 + 7x - 6$ and $y = 3x - 1$
    b) $y = -\dfrac{1}{2}x^2 + 5x - 3$ and $y = 3x - 1$
    c) $y = 3x^2 - x + 7$ and $y = x - 4$
    d) $x^2 + (y+4)^2 = 20$ and $x - 3y = 2$
    e) $(x-2)^2 + (y+5)^2 = 10$ and $3y - x = -7$
    f) $x^2 + (y+1)^2 = 16$ and $y + 2x = 15$

15. Find an equation for the line connecting the points of intersections of the circles $(x+1)^2 + (y-2)^2 = 10$ and $(x-1)^2 + (y-6)^2 = 50$.

16. Find an equation of the tangent line drawn
    a) to $2y - 6x + x^2 + y^2 = 40$ at the point $(10, -2)$. b) to $2y + x^2 + y^2 = 3(2x+5)$ at the point $(7, -4)$.

17. We are planning to build a fence around a rectangular a garden that is next to a river. Because of that, we only need to build a fence around three of its sides. What is the maximum area possible to fence in if we have 400 meters of fencing? What dimensions of a garden should we plan to achieve this maximum area?

18. Consider the right triangle ABC. Find all sides if $\gamma = 90°$, $\beta = 24°$ and $a = 17$. Present your answer as an approximation, accurate up to at least four decimal places.

19. True or false? For all angles α, $\sin 2\alpha = 2\sin\alpha$. (Hint: is it true for 30° and 60°?)

20. Prove each of the following identities.

    a) $1 + \tan^2 x = \sec^2 x$
    b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$
    c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$
    d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$
    e) $\cos x\,(\sec x - \cos x) = \sin^2 x$
    f) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$
    g) $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 4\tan x \sec x$

21. Solve each of the following inequalities.

a) $x^2 \geq x$ b) $8x - 2x^2 > 58$ c) $x^2 + 7 \leq 6x$ d) $4x^2 + 1 \leq 4x$

22. Solve each of the following equations.

a) $\sqrt{x+6} + \sqrt{11-x} = 5$ b) $\sqrt{3x+10} - \sqrt{x+4} = 2$ c) $\sqrt{x-3} + 1 = \sqrt{x+2}$ d) $\sqrt{x-1} - 1 = \sqrt{2x-9}$

23. A square-based pyramid has all of its edges measuring the same length, 10 meters.

a) What is the height of the pyramid?

b) Find the exact and approximate value of the angle that is formed between the square base and a triangular face.

24. a) Find the exact and approximate value of the smaller angle that is formed between the line $y = 2x - 3$ and the positive part of the $x-$axis.

b) Find the exact and approximate value of the smaller angle that is formed between the line $y = 4x - 3$ and the positive part of the $x-$axis.

c) Based on the previous computations, determine whether doubling the slope of a line means doubling the angle or not.

25. Find the first term and common difference in the arithmetic sequence $\{a_n\}$ if we know that $a_{10} = 80$ and $s_{20} = 1530$.

26. Find the area of a regular 9−sided polygon written into a circle of radius 10 cm.

27. Consider the triangle with sides 6 m, 6 m, and 8 m long.
a) Compute the exact value of the area of the triangle.
b) Compute the exact value of the smallest angle in the triangle.
c) Compute an approximate value of the smallest angle in the triangle.

28. A person is standing 3 ft away from a street light that is 15. 6 ft tall. How long is his shadow if he is 5.2 ft tall?

29. Label each of the given statements are true or false.

a) If α is co-terminal to $60°$, then 2α is co-terminal to $120°$.

b) If 2α is co-terminal to $120°$, then α is co-terminal to $60°$.

30. * Simplify: $\dfrac{\sqrt{1009 + \sqrt{2017}} - \sqrt{1009 - \sqrt{2017}}}{\sqrt{2}}$

Answers

1. a) $-\dfrac{1}{2x^2y}$ b) $\dfrac{a^5}{b^5}$ c) $\dfrac{a^5b^{10}-a^{10}}{b^{10}+a^{10}b^5}=\dfrac{a^5\left(b^{10}-a^5\right)}{b^5\left(b^5+a^{10}\right)}$ d) $\dfrac{x}{x+1}$ e) $\dfrac{b-a}{ab}$ f) $\dfrac{4}{9}a^6b^4$ g) $64\dfrac{a^8}{b^{13}}$
   h) 32 i) $\sqrt[4]{x^3}$ j) $\sqrt[3]{x^4}$

2. a) $-\dfrac{\sqrt{3}}{2}$ b) $\sqrt{2}$ c) $\dfrac{\sqrt{3}}{2}$ d) $\dfrac{\sqrt{3}}{3}$ e) $-\dfrac{1}{2}$ f) undefined g) -1 h) 0 i) 0
   j) $-\dfrac{1}{2}$ k) $-\dfrac{\sqrt{3}}{3}$ l) -1 m) $-\sqrt{2}$ n) 1 o) 0

3. a) 1 b) $-\dfrac{1}{2}$ c) 27 d) $\sqrt{2}$ e) $\dfrac{5x}{3}$ f) $\dfrac{1}{3}$ g) 2 h) 5 i) $\sqrt{10}$ j) $5\sqrt{10}$
   k) $5+\sqrt{10}$ l) $-\dfrac{3}{2}$ m) undefined n) $-\dfrac{5}{4}$ o) $\left(\sqrt[3]{3}\right)^5=3\sqrt[3]{9}$ p) $\dfrac{1}{4}$ q) 27 r) $\sqrt[3]{2}$
   s) $\dfrac{2}{3}$ t) $\dfrac{3}{8}$ u) $\dfrac{5\sqrt{3}+9}{8}$

4. a) $3<\log_3 50<4$ b) $4<\sqrt[3]{80}<5$ c) $-3<\log_2\left(\dfrac{1}{5}\right)<-2$ d) $3<\log 2017<4$

5. a) 6463 b) 1305

6. a) 5 b) $\log_3 38$ c) 2 d) $\dfrac{1}{4}\left(1+\log_2 42\right)$ e) 14 f) $\dfrac{1}{3}\left(1+\log_{10} 19\right)$ g) 9 h) $\dfrac{1}{3}\left(7+\ln 5\right)$
   i) no solution j) 2 k) $-\dfrac{2}{3}$ l) 1 m) $\dfrac{5-\ln 8}{2}$ n) $-2, 1$ o) $\dfrac{2}{15}$ p) 3

7. a) $A^C=B$ b) $2^5=3x-1$ c) $e^{-2}=\dfrac{1}{3}x+1$ 8. a) 11 b) e^9-4 c) e^8-4

9. a) $-\dfrac{1}{3}$ b) $-\dfrac{1}{9}$ c) -1 d) $-\dfrac{1}{6}$ e) undefined f) $\dfrac{1}{15}$ g) $\dfrac{1}{7}$ h) $\dfrac{a-2}{3a+6}$ i) $\dfrac{2a-4}{3a+6}$ j) $\dfrac{a-1}{3a+3}$
   k) $-\dfrac{1}{3}$ l) $-\dfrac{11}{27}$ m) 5 10. only g 11. a) $\dfrac{225}{4}$ b) 540 12. 211.510 92 ft 13. $\dfrac{3-\sqrt{13}}{2}$, $\dfrac{3+\sqrt{13}}{2}$

14. a) $(1,2)$ and $(-5,-16)$ b) $(2,5)$ c) no intersection point d) $(-4,-2)$ and $(2,0)$ e) $(1,-2)$
   f) no intersection point 15. $y=-\dfrac{1}{2}x-1$

16. a) $y+2=7\left(x-10\right)$ or $y=7x-72$ b) $\dfrac{4}{3}\left(x-7\right)=y+4$ or $y=\dfrac{4}{3}x-\dfrac{40}{3}$

17. 100 m by 200 m will give us an area of 20 000 m^2 18. $c\approx 18.608\,816\,7$ $b\approx 7.568\,888$

19. False. For example, consider $\alpha=30°$.
   Then $\sin 2\alpha=\sin 60°=\dfrac{\sqrt{3}}{2}$ and $2\sin\alpha=2\sin 30°=2\left(\dfrac{1}{2}\right)=1$.

20. a) $1+\tan^2 x=\sec^2 x$

$$\text{LHS}=1+\tan^2 x=1+\frac{\sin^2 x}{\cos^2 x}=\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\sec^2 x=\text{RHS}$$

   b) $\dfrac{\cot x-1}{\cot x+1}=\dfrac{1-\tan x}{1+\tan x}$

$$\text{LHS}=\frac{\cot x-1}{\cot x+1}=\frac{\dfrac{1}{\tan x}-1}{\dfrac{1}{\tan x}+1}\cdot\frac{\tan x}{\tan x}=\frac{1-\tan x}{1+\tan x}=\text{RHS}$$

c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right) \cos x$

$$\text{RHS} = \left(\sin^2 x - \sin^4 x\right) \cos x = \sin^2 x \left(1 - \sin^2 x\right) \cos x = \sin^2 x \cos^2 x \cos x = \text{LHS}$$

d) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$$\text{LHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x = \text{RHS}$$

e) $\cos x \left(\sec x - \cos x\right) = \sin^2 x$

$$\text{LHS} = \cos x \left(\sec x - \cos x\right) = \cos x \left(\frac{1}{\cos x} - \cos x\right) = 1 - \cos^2 x = \sin^2 x = \text{RHS}$$

f) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$

$$\text{LHS} = \frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\sin x \left(1 + \cos x\right)}{1 - \cos^2 x} = \frac{\sin x \left(1 + \cos x\right)}{\sin^2 x} = \frac{1 + \cos x}{\sin x} = \text{RHS}$$

g) $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

$$\begin{aligned} \text{LHS} &= \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{(1 + \sin x)^2 - (1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{\left(1 + \sin^2 x + 2 \sin x\right) - \left(1 + \sin^2 x - 2 \sin x\right)}{1 - \sin^2 x} \\ &= \frac{1 + \sin^2 x + 2 \sin x - 1 - \sin^2 x + 2 \sin x}{\cos^2 x} = \frac{4 \sin x}{\cos^2 x} = 4 \frac{\sin x}{\cos x} \frac{1}{\cos x} = 4 \tan x \sec x = \text{RHS} \end{aligned}$$

21. a) $(-\infty, 0] \cup [1, \infty)$ b) no solution c) $\left[-\sqrt{2} + 3, \sqrt{2} + 3\right]$ d) $x = \dfrac{1}{2}$

22. a) $-5, 10$ b) 5 (-3 does not work) c) 7 d) 5 (17 doesn't work)

23. a) $5\sqrt{2}\,\text{m}$ b) $\tan^{-1}\left(\sqrt{2}\right) \approx 54.735\,61°$ 24. a) $\tan^{-1} 2 \approx 63.43495°$ b) $\tan^{-1}(4) \approx 75.963757°$ c) no

25. $a = 143, d = -7$ 26. $900 \cos 20° \sin 20°\,\text{cm}^2 \approx 289.254\,424\,36\,\text{cm}^2$

27. a) $8\sqrt{5}\,\text{m}^2$ b) $\cos^{-1}\left(\dfrac{2}{3}\right)$ c) $48.1897°$ 28. $1.5\,\text{ft}$ 29. a) true b) false (try $240°$)

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