1. Solve each of the following inequalities.

   a) $\dfrac{2}{x+3} < 1$ b) $\dfrac{3x+1}{x-1} \geq -2$ c) $\dfrac{-1}{x-5} \geq 2$ d) $\dfrac{x}{x+1} < \dfrac{3}{4}$ e) $\dfrac{3x-1}{x+4} \leq 3$

2. Solve each of the following equations.

   a) $\sqrt{2x-1} = 2 + \sqrt{x-4}$
   b) $\sqrt{x+1} + \sqrt{5x+1} = 6$
   c) $\sqrt{2x+1} = \sqrt{3x+4} - 1$
   d) $\sqrt{x+10} - \sqrt{x+3} = \sqrt{2x-11}$

3. Simplify each of the following.

   a) $2^{\log_2 4} + 2^{\log_2 8}$
   b) $2^{\log_2 4 + \log_2 8}$
   c) $\log_{25}\left(\dfrac{1}{\sqrt{125}}\right)$
   d) $\ln\left(e^{-7}\right)$
   e) $e^{-\ln 7}$
   f) $\log_{81} 3 \cdot \log_3 81$
   g) $5^{\log_5 M}$
   h) $25^{\log_5 M}$
   i) $5^{\log_{25} M}$
   j) $\log_{1/4}(8)$
   k) $3^{\log_3 9} + 3^{\log_3 27}$
   l) $3^{\log_3 9 + \log_3 27}$
   m) $\log_8(-64)$
   n) $e^{\ln A} + e^{\ln B}$
   o) $e^{\ln A + \ln B}$
   p) $\log_2(\sin 45°)$
   q) $\log_2\left(\sec^{10} 45°\right)$
   r) $\log_3(\tan 60°)$
   s) $\log_9\left(\dfrac{1}{\sqrt{27}}\right)$
   t) $\log_{\sqrt{2}} 32$
   u) $\log_A 1 - \log_A \sqrt{A} + \log_B \dfrac{1}{\sqrt[3]{B^2}}$
   v) $\ln\left(\dfrac{1}{\sqrt[3]{e}}\right) - 2\log_{10}(0.001) + \log_7 \dfrac{1}{49}$
   w) $2^{\log_2 x} + 2^{\log_2 y}$
   x) $2^{\log_2 x + \log_2 y}$

4. Simplify each of the following.

   a) $\log_3\left(\dfrac{1}{\sqrt{27}}\right)$ b) $2^{\log_2 8}$ c) $2^{\log_2 24}$ d) $\log_2(\cos 45°)$ e) $\log_{\sin 45°} 8$

5. Simplify each of the following.

   a) $\log_{10} 4 + \log_{10} 25$
   b) $\log_6 2 - \log_6 72$
   c) $\log_2 12 - \log_2 3$
   d) $\log_3 2 + \log_3 18 - 2\log_3 2$
   e) $\log_2 6 + \log_2 12 - 2\log_2 3$
   f) $\log_7 0.8 + \log_7 1.25$

6. Write each of the following as a single logarithm. Assume that A and B represent positive numbers.

   a) $\log_2 A + \log_2 B$
   b) $\log_2 A + 2\log_2 B$
   c) $\log_2 A - \log_2 B$
   d) $\dfrac{1}{2}\log_2 A + 2\log_2 B$
   e) $1 + \log_3 A$
   f) $2 - \log_{10} 5$
   g) $3 - \ln 8$
   h) $2 - \log_3 A$
   i) $2 - \dfrac{1}{4}\log_3 A$

7. a) Compute the area of the triangle determined by the points $A(5,2)$, $B(10,2)$, and $C(10,6)$.

   b) Compute the area of the triangle determined by the points $A(\log_2 x, \log_2 y)$, $B(\log_2 8x, \log_2 y)$, and $C(\log_2 x, \log_2 4y)$.

8. Graph each of the given functions.

   a) $f(x) = \dfrac{1}{x}$ b) $f(x) = 2^x$ c) $f(x) = \sqrt[3]{x}$

9. Suppose that x is an acute angle with $\tan x = \dfrac{3}{2}$. Compute the exact value of $\sin x \cos x$.

10. Simplify each of the following.

a) $\sin\left(\frac{3\pi}{2}\right)$ b) $\cos\left(\frac{7\pi}{4}\right)$ c) $\tan(-3\pi)$ d) $\tan\left(\frac{\pi}{2}\right)$ e) $\sec\left(-\frac{7\pi}{3}\right)$ f) $\csc\left(\frac{3\pi}{4}\right)$

11. Simplify each of the following. (Write it in terms of $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$.)

a) $\sin(\alpha+360°)$ c) $\tan(\alpha-180°)$ e) $\cos(-\alpha)$ g) $\sin(\alpha-180°)$

b) $\cos(\alpha+180°)$ d) $\sin(180°-\alpha)$ f) $\tan(-\alpha)$ h) $\tan(180°-\alpha)$

12. Consider the expression $\cos(180°-\alpha)$. All of the following expressions are equal to $\cos(180°-\alpha)$, except for one. Which one?

A) $-\cos\alpha$ B) $-\sin(90°-\alpha)$ C) $\sin(\alpha-90°)$ D) $\cos\alpha$ E) $-\cos(-\alpha)$

13. Simplify each of the following.

a) $\log_3(\tan 60°)+\log_2(\sin 135°)-\ln(\tan 225°)$

b) $\sec 30°-\csc 45°+\cot 60°$

c) $\dfrac{\tan 120°-\tan 135°}{1+\tan 120°\tan 135°}$

d) $\tan^2 120°-\tan^2 150°$

14. a) Compute the exact value of $\tan 30°+\tan 30°$

b) Compute the exact value of $\tan 60°$

c) Based on your findings, determine whether the following statement is true or false:

$$\tan(\alpha+\beta)=\tan\alpha+\tan\beta$$

15. Convert each of the given angles to radians. Use exact values.

a) $120°$ b) $270°$ c) $30°$ d) $540°$ e) $135°$

16. Convert each of the given angles to degree. Use exact values.

a) $-\frac{5\pi}{3}$ b) $-\frac{\pi}{5}$ c) $\frac{7\pi}{4}$ d) $-\frac{3\pi}{4}$ e) $-\frac{11\pi}{3}$

17. Simplify each of the following. Present exact values. Rationalize the denominator.

a) $\sin\left(\frac{5\pi}{3}\right)-2\tan\left(-\frac{\pi}{6}\right)+\sin\left(\frac{7\pi}{6}\right)$

b) $\cos 1°+\cos 2°+\cos 3°+\ldots..+\cos 179°+\cos 180°$

18. Solve each of the following equations. You may present your answer in degrees.

a) $\sin x=-\frac{5}{2}$

b) $\cos x=-\frac{1}{\sqrt{2}}$

c) $\tan x=1$

d) $\sin x=0$

e) $\cos x=1$

f) $\tan x=-\frac{1}{\sqrt{3}}$

g) $\cos\alpha=\frac{\sqrt{3}}{2}$

h) $\sin\beta=-\frac{1}{2}$

19. Solve each of the following equations. Present exact values of all answers.

a) $\log_3(5x+1)=4$ b) $3^{2x-1}=18$ c) $\log_2(3x-2)=4$ d) $5^{0.5x-3}=10$

20. Solve each of the following equations. Make sure to check your solutions.

a) $\log_2(x-1)+\log_2(x-5)=5$

b) $\log_6(3-x)+\log_6(-x-2)=2$

c) $\log_2(x+1)+\log_2(x-1)=3$

d) $\log_5(x-16)-\log_5(4x+20)=-2$

e) $\log_2(3x+6)-\log_2(x+7)=3$

21. Consider the parabola $y=4x^2-3x-8$.

a) Find both coordinates of the vertex. b) Find all $x-$intercepts. Present exact values.

22. Find the domain for each of the following functions.

a) $f(x) = \dfrac{1}{x+2}$

b) $g(x) = \sqrt{x+2}$

c) $h(x) = \log_3(x+2)$

d) $f(x) = \dfrac{1}{x-x^2}$

e) $g(x) = \sqrt{x-x^2}$

f) $h(x) = \ln\left(x-x^2\right)$

g) $f(x) = \dfrac{2x-1}{x^2+9}$

h) $f(x) = \dfrac{3x-5}{8-\sqrt{x-1}}$

i) $f(x) = \ln\left(x^2-9\right) + \dfrac{1}{x^2-16}$

j) $f(x) = \tan x$

23. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

a) $y = x + x^2 - 24$ and $y = 3x - 16$

b) $(x+3)^2 + (y+2)^2 = 10$ and $(x-1)^2 + (y-2)^2 = 10$

c) $(x+3)^2 + (y+2)^2 = 8$ and $x^2 + (y-1)^2 = 2$

24. Consider the pyramid $ABCDE$ if its base is a square $ABCD$ with sides 6 m and side $AE = BE = CE = DE = 10$ m. Compute an approximate value for the angle that is formed between a triangular face and the base.

25. Prove each of the following identities.

a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$ b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$ c) $1 + 2\sin x \cos x = (\sin x + \cos x)^2$

26. Compute an approximate value for each of the angles in a triangle with sides 7 cm, 7 cm, and 10 cm.

27. One number a is 10 greater than twice another number b. Find each of the following.

a) The minimal value of $a^2 + b^2$.

b) The minimal value of ab

c) The maximal value of $b^2 - a^2$

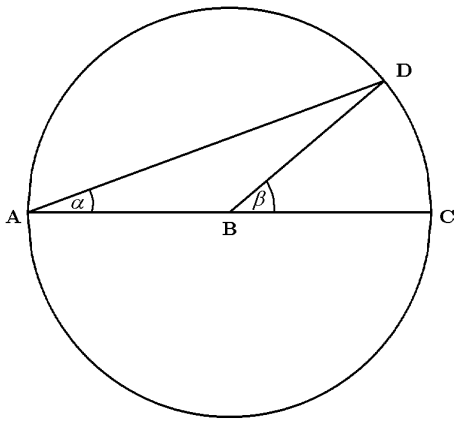
28. We drew an n-sided regular polygon into a circle with radius R. In terms of R and n, express

a) the perimeter of the polygon

b) the area of the polygon.

29. Find the angles in a right triangle if we know that its sides $a < b < c$, in this order, form the first, fourth, and twenty-eightth terms of an increasing arithmetic sequence.

30. Consider the picture shown. Given that B is the center of the circle, prove that $\beta = 2\alpha$.



Answers

1. a) $(-1,\infty)\cup(-\infty,-3)$ b) $\left(-\infty,\frac{1}{5}\right]\cup(1,\infty)$ c) $\left[\frac{9}{2},5\right)$ d) $(-1,3)$ e) $(-4,\infty)$

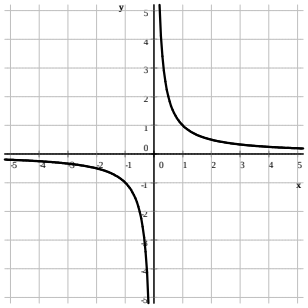
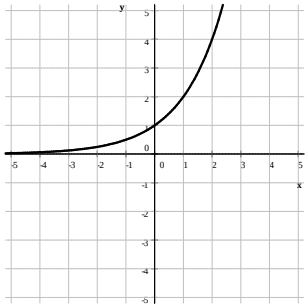
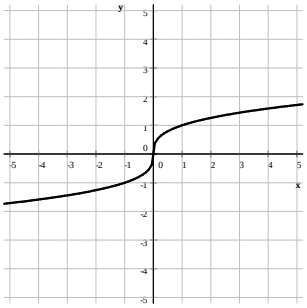
2. a) $13,5$ b) 3 (24 doesn't work) c) $0,4$ d) 6

3. a) 12 b) 32 c) $-\frac{3}{4}$ d) -7 e) $\frac{1}{7}$ f) 1 g) M h) M^2 i) $\sqrt{M}$ j) $-\frac{3}{2}$ k) 5
   l) 243 m) undefined n) $A+B$ o) AB p) $-\frac{1}{2}$ q) 5 r) $\frac{1}{2}$ s) $-\frac{3}{4}$ t) 10
   u) $-\frac{7}{6}$ v) $\frac{11}{3}$ w) $x+y$ x) xy 4. a) $-\frac{3}{2}$ b) 8 c) 24 d) $-\frac{1}{2}$ e) -6

5. a) 2 b) -2 c) 3 d) 2 e) 3 f) 0

6. a) $\log_2 AB$ b) $\log_2\left(AB^2\right)$ c) $\log_2\left(\frac{A}{B}\right)$ d) $\log_2\left(B^2\sqrt{A}\right)$ e) $\log_3(3A)$ f) $\log_{10}20$
   g) $\ln\left(\frac{e^3}{8}\right)$ h) $\log_3\left(\frac{9}{A}\right)$ i) $\log_3\left(\frac{9}{\sqrt[3]{A}}\right)$ 7. a) 10 unit^2 b) 3 unit^2

8. a) $f(x)=\frac{1}{x}$ b) $f(x)=2^x$ c) $f(x)=\sqrt[3]{x}$

9. $\frac{6}{13}$ 10. a) -1 b) $\frac{\sqrt{2}}{2}$ c) 0 d) undefined e) 2 f) $\sqrt{2}$

11. a) $\sin\alpha$ b) $-\cos\alpha$ c) $\tan\alpha$ d) $\sin\alpha$ e) $\cos\alpha$ f) $-\tan\alpha$ g) $-\sin\alpha$ h) $-\tan\alpha$

12. D 13. a) 0 b) $\sqrt{3}-\sqrt{2}$ c) $2-\sqrt{3}$ d) $\frac{8}{3}$ 14. a) $\frac{2}{3}\sqrt{3}$ b) $\sqrt{3}$ c) false

15. a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{2}$ c) $\frac{\pi}{6}$ d) 3π e) $\frac{3\pi}{4}$

16. a) $-300°$ b) $-36°$ c) $315°$ d) $-135°$ e) $-660°$ 17. a) $\frac{\sqrt{3}}{6}-\frac{1}{2}$ b) -1

18. a) no solution b) $\pm135°+k\cdot360°$ where $k\in\mathbb{Z}$ c) $45°+k\cdot180°$ where $k\in\mathbb{Z}$ d) $k\cdot180°$ where $k\in\mathbb{Z}$
   e) $k\cdot360°$ where $k\in\mathbb{Z}$ f) $-30°+k\cdot180°$ where $k\in\mathbb{Z}$ g) $\pm30°+k\cdot360°$ where $k\in\mathbb{Z}$
   h) $-30°+k\cdot360°$ and $210°+k\cdot360°$ where $k\in\mathbb{Z}$

19. a) 16 b) $\frac{1}{2}(1+\log_3 18)$ c) 6 d) $6+2\log_5 10$

20. a) 9 (-3 does not work) b) -6 (7 does not work) c) 3 (-3 does not work) d) 20 e) no solution

21. a) $\left(\frac{3}{8},-\frac{137}{16}\right)$ b) $\left(\frac{3-\sqrt{137}}{8},0\right)$ and $\left(\frac{3+\sqrt{137}}{8},0\right)$

22. a) $(-\infty,-2)\cup(-2,\infty)$ b) $[-2,\infty)$ c) $(-2,\infty)$ d) $(-\infty,0)\cup(0,1)\cup(1,\infty)$ e) $[0,1]$ f) $(0,1)$
g) $\mathbb{R}$ h) $[1,65)\cup(65,\infty)$ i) $(-\infty,-4)\cup(-4,-3)\cup(3,4)\cup(4,\infty)$ j) $x\neq\dfrac{\pi}{2}+k\pi$ where $k\in\mathbb{Z}$

23. a) $(-2,-22)$ and $(4,-4)$ b) $(-2,1)$ and $(0,-1)$ c) $(-1,0)$ 24. $71.670\,246\,2°$

25. a) $\tan x+\dfrac{\cos x}{1+\sin x}=\sec x$

$$\begin{aligned}\text{RHS} &= \tan x+\frac{\cos x}{1+\sin x}=\frac{\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=\frac{\sin x\,(1+\sin x)+\cos^2 x}{\cos x\,(1+\sin x)}\\ &= \frac{\sin x+\sin^2 x+\cos^2 x}{\cos x\,(1+\sin x)}=\frac{\sin x+1}{\cos x\,(1+\sin x)}=\frac{1}{\cos x}=\sec x=\text{LHS}\end{aligned}$$

b) $\dfrac{\cos x}{1-\sin x}=\sec x+\tan x$

$$\begin{aligned}\text{LHS} &= \frac{\cos x}{1-\sin x}=\frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}=\frac{\cos x\,(1+\sin x)}{1-\sin^2 x}=\frac{\cos x\,(1+\sin x)}{\cos^2 x}=\frac{1+\sin x}{\cos x}\\ &= \frac{1}{\cos x}+\frac{\sin x}{\cos x}=\sec x+\tan x=\text{RHS}\end{aligned}$$

c) $1+\sin 2x=(\sin x+\cos x)^2$

$$\text{RHS}=(\sin x+\cos x)^2=\sin^2 x+\cos^2 x+2\sin x\cos x=1+\sin 2x=\text{LHS}$$

26. $44.415\,3°$, $44.415\,3°$, and $91.169\,4°$ 27. a) 20 b) $-\dfrac{25}{2}$ c) $\dfrac{100}{3}$

28. a) $2nR\sin\left(\dfrac{180°}{n}\right)$ b) $nR\sin\left(\dfrac{180°}{n}\right)R\cos\left(\dfrac{180°}{n}\right)=nR^2\sin\left(\dfrac{180°}{n}\right)\cos\left(\dfrac{180°}{n}\right)$

29. $\alpha\approx 43.602\,819°$, $\beta\approx 46.397\,181°$, $\gamma=90°$

30. Line segments AB and BD are both radii in the circle, and so they are equal. So ABD triangle is isosceles and so the angles opposite AB and BD are also equal to each other. Thus $\measuredangle ADB=\alpha$. The third angle in triangle ADB is $180°-2\alpha$. Angles ABD and DBC are supplementary beause together they form a straight angle. Thus

$$\begin{aligned}\measuredangle ABD+\measuredangle DBC &= 180° &&\\ 180°-2\alpha+\beta &= 180° && \text{subtract } 180°\\ -2\alpha+\beta &= 0 && \text{add } 2\alpha\\ \beta &= 2\alpha &&\end{aligned}$$

Last revised: November 2, 2017