1. Compute the inverse for each of the following functions.

   a) $f(x) = \frac{2}{3}x - 6$

   b) $f(x) = \sqrt[3]{2x+1}$

   c) $f(x) = (2x-3)^3 + 8$

   d) $f(x) = \frac{3x-1}{7x-5}$

   e) $f(x) = \frac{7x+10}{3x-7}$

   f) $f(x) = e^{4x-1} - 3$

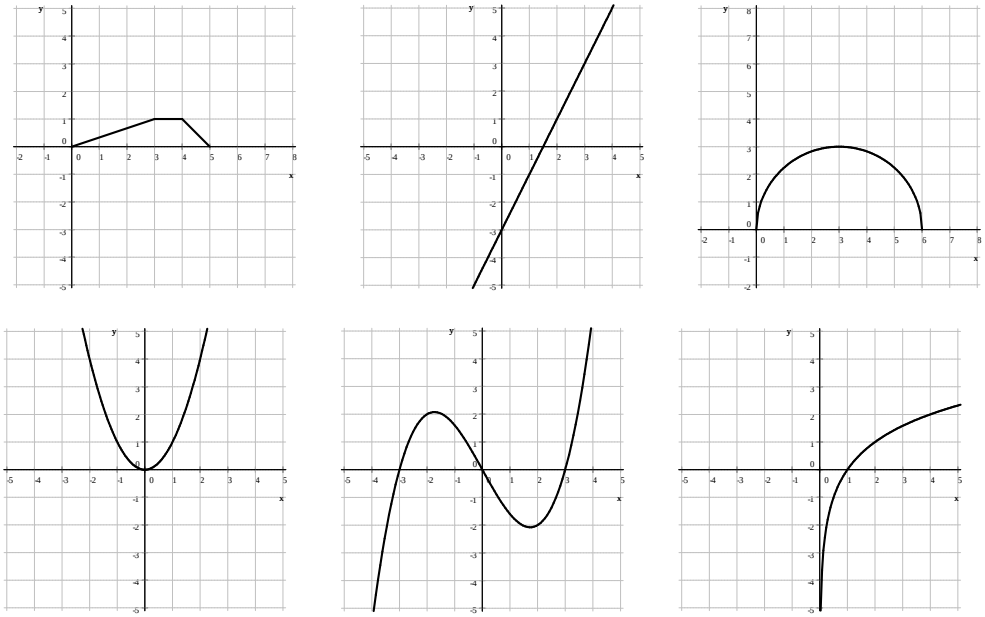
   g) $f(x) = \log_3(5x-4)$

2. Solve each of the following triangles.

   a) $b = 16\,\text{ft}$, $\alpha = 38°$, and $\beta = 83°$

   b) $\beta = 78°$ $\gamma = 49°$ and $c = 15\,\text{ft}$

3. Given the graph of a function f, graph the inverse relation f^{-1} in the same coordinate system.



4. Solve each of the following equations.

   a) $\sin 2x = \cos x$

   b) $\cos 2x = \sin x$

   c) $\sin 2x = \sin x$

   d) $\cos 2x = \cos x$

   e) $\ln(2x-8) = -2$

   f) $2^{3x-4} = 5$

   g) $\log_3(x+7) - \log_3(2x-20) = 2$

   h) $\log_2(3x+1) + \log_2(x-1) = 6$

   i) $\sin x - \sqrt{3}\cos x = -1$

   j) $\sin 5x - \sqrt{3}\cos 5x = -1$

   k*) $\sqrt{\log_3 x} = \log_3 \sqrt{x}$

5. Find the exact value of $\log_2\left(\log_2 \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}}}\right)$.

6. Re-write $0.3\overline{90} = 0.390909090....$ as a ratio of two integers. You do not need to reduce the fraction.

7. Simplify each of the following.

   a) $2^{\log_2 A} - 2^{\log_2 B} + 2^{\frac{1}{3}\log_2 C}$

   b) $2^{\log_2 A - \log_2 B + \frac{1}{3}\log_2 C}$

   c) $\tan\left(\frac{5\pi}{12}\right)$

   d) $\log_{\tan 60°}\left(2^3+1\right)$

   e) $\log_a\left(\left(3 - \frac{3a-2}{a+1}\right)\cdot\frac{a^2+a}{5}\right)$

   f) $\log 33 - \frac{1}{2}\log 44 - \log 15 - \log\sqrt{1100}$

8. Sketch the graph of each of the given functions.

   a) $f(x) = x^3 - 4x$

   b) $f(x) = 4x^2 - x^4$

   c) $f(x) = -2(x+3)^2 x(x-3)^2$

9. Find $\tan x$ if we know that $\tan 2x = \dfrac{48}{55}$.

10. Suppose that $x = \log_2 3$ and $y = \log_3 10$. Write $\log_6 20$ in terms of x and y.

11. Seattle, WA and San Francisco, CA are located approximately on the same longitude. The latitude of these cities are $47.5°$ N and $37.4°$ N. Find the distance between the two cities assuming that the Earth is a sphere with radius 3960 miles. Round your answer to the nearest mile.

12. a) A sattelite is orbiting around Earth at a height of 500 miles just above a the equator. From a town located at the equator, the sattelite appears stationary. Find the speed of the sattelite in miles per hours. Assume that the Earth is a sphere with radius 3960 miles.

    b*) The latitude of Seattle, WA is $47.5°$ N. A satellite above Seattle appears stationary in the sky. Find the speed of the satellite if it is at a height of 1200 miles above the surface of Earth. Assume that the Earth is a sphere with radius 3960 miles.

13. Solve each of the following inequalities.

    a) $\dfrac{2x-5}{3x+1} < -2$ b) $2x + \dfrac{1}{3}x^2 \le -3$ c) $\dfrac{3}{x-1} \le \dfrac{x}{2}$ d) $\dfrac{2}{x+1} \le \dfrac{3}{4}$ e) $\dfrac{1}{x} \ge 2$

14. Find the domain for each of the following functions.

    a) $f(x) = \dfrac{1}{\log_2 x + \log_2 (x-4)}$ b) $g(x) = \sec x$ c) $f(x) = \dfrac{2x-3}{x^2+1}$ d) $g(x) = \sqrt{\dfrac{x}{x+1}}$

15. Find an equation for the tangent line drawn to $4y - 6x + x^2 + y^2 = 37$ at the point $(2, -9)$.

16. Find an equation for the tangent line drawn to $y = \dfrac{3}{2}x^2 - x + 2$ from the point $(3, 11)$.

17. Suppose that $f(x) = x^2 - 3x + 1$ and $g(x) = -2x + 1$. Compute each of the following.

    a) $f(g(-1))$ b) $g(f(-1))$ c) $f(2a)$ d) $g(2a)$ e) $f(g(x))$ f) $g(f(x))$

    g) Is there any value of x for which $f(g(x)) = g(f(x))$?

18. Graph each of the following functions.

    a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$ b) $g(x) = \log_{1/3} x$ c) $h(x) = \sqrt{9 - x^2}$ on its natural domain.

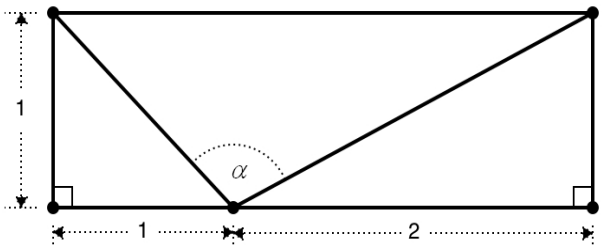
19. Perform the indicated operations on the complex numbers.

    a) $(3 - 5i)(5 + 2i)$ c) $(1 - i)^2$ d) $\dfrac{17 - 7i}{3 - 2i}$ g) $(8 - 5i)^2$

    h) $\left(\sqrt{3} - \sqrt{2}i\right)\left(\sqrt{3} + \sqrt{2}i\right)$

    b) i^{115} e) $(1 + 3i)^4 (1 - 3i)^4$ f) $|7i - 1|$

20. Compute the exact value of $\tan \alpha$ if α is the angle shown on the picture.



21. Point D is on side AB of triangle ABC, with $\angle ACD = \angle BCD = 60°$, $AC = 5$, and $BC = 15$. Find the length of line segment CD.

22. Compute the approximate value of the radius of the circle if we know that a sector subtended by a central angle of $38°$ has an area of $190\,\text{cm}^2$.

23. a) Compute $\sin\alpha$ and $\cos\alpha$ in terms of M if we know that α is an acute angle and that $\tan\alpha = M$.

    b) Compute $\sin\beta$ and $\cos\beta$ in terms of T if we know that β is an angle in the fourth quadrant such that $\tan\beta = T$.

24. Suppose that x is an angle belonging to the second quadrant and y is an angle belonging to the fourth quadrant. We also know that $\sin x = \dfrac{2}{3}$ and $\cos y = \dfrac{3}{5}$. Compute each of the following.

    a) $\cos x$ c) $\sin 2x$ e) $\sin(x-y)$ g) $\tan 2x$

    b) $\sin y$ d) $\cos 2x$ f) $\cos(x+y)$

25. Prove each of the following.

    a) $1-\sin 2x = (\sin x - \cos x)^2$

    b) $\cos\left(\dfrac{\pi}{4}-\alpha\right) - \cos\left(\dfrac{\pi}{4}+\alpha\right) = \sqrt{2}\sin\alpha$

    c) $\sin x + \sin\left(x+\dfrac{2\pi}{3}\right) + \sin\left(x+\dfrac{4\pi}{3}\right) = 0$

    d) $\sin^2 x - \sin^2 y = \sin(x+y)\sin(x-y)$

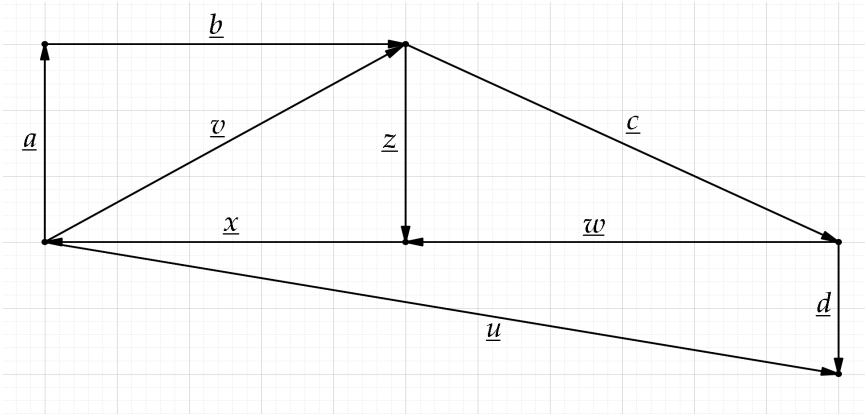
26. Consider a regular polygon of n sides that was written in a circle with radius R.

    a) Use right triangle trigonometry to express the area of the polygon.

    b) Use the formula $A = \dfrac{1}{2}ab\sin\gamma$ to express the area of the polygon.

    c) Compare the two results. Are they the same?

27. Consider the picture given. Find each of the following in terms of vector(s) shown.



    a) $\underline{a}+\underline{b}$ b) $\underline{z}-\underline{x}$ c) $\underline{d}-\underline{u}$ d) $\underline{a}+\underline{b}+\underline{c}+\underline{w}$

28. Suppose that $\underline{v} = 3\underline{i} - 8\underline{j}$ and $\underline{w} = \underline{i} + 2\underline{j}$. Compute each of the following.

    a) $\|\underline{v}\|$ b) $\|\underline{w}\|$ c) $\underline{v}+\underline{w}$ d) $\|\underline{v}+\underline{w}\|$ e) $4\underline{w}-\underline{v}$ f) $\underline{v}+4\underline{w}$

29. Which is greater, $2^{\log_3 2017}$ or $2017^{\log_3 2}$?

30. Sketch the graph of each of the following functions.

    a) $f(x) = x-2$

    b) $f(x) = (x-2)^2$

    c) $f(x) = (x-2)^3$

    d) $f(x) = \dfrac{1}{x-2}$

    e) $f(x) = \dfrac{1}{(x-2)^2}$

    f) $f(x) = \dfrac{1}{(x-2)^3}$

    g) $f(x) = \dfrac{(x-2)^5}{(x-2)^7}$

    h) $f(x) = \dfrac{(x-2)^2}{(x-2)^5}$

    i) $f(x) = \dfrac{(x-2)^6}{(x-2)^6}$

    j) $f(x) = \dfrac{(x-2)^5}{(x-2)^4}$

    k) $f(x) = \dfrac{(x-2)^7}{(x-2)^5}$

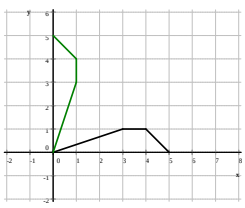
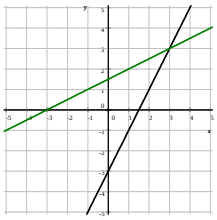
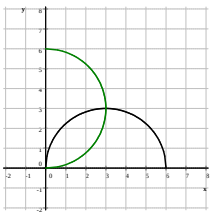
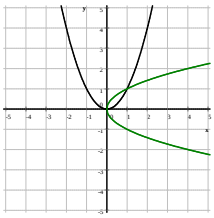
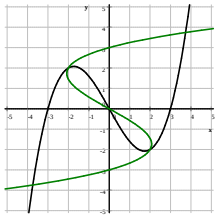
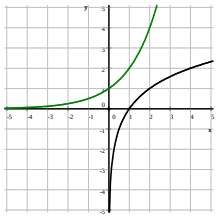
    l) $f(x) = \dfrac{(x-2)^5}{(x-2)^2}$

Answers

1. a) $f^{-1}(x) = \frac{3}{2}x + 9$ b) $f^{-1}(x) = \frac{1}{2}x^3 - \frac{1}{2}$
   c) $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-8} + \frac{3}{2}$ d) $f^{-1}(x) = \frac{5x-1}{7x-3}$
   e) $f^{-1}(x) = \frac{7x+10}{3x-7}$
   f) $f^{-1}(x) = \frac{1}{4}(\ln(x+3)+1)$
   g) $f^{-1}(x) = \frac{1}{5}(3^x + 4)$

2. a) $\gamma = 59°$, $a \approx 9.924\,56\,\text{ft}$, $c \approx 13.817\,671\,64\,\text{ft}$
   b) $\alpha = 53°$ $b \approx 19.44087\,\text{ft}$ $a \approx 15.8730364\,\text{ft}$

3. The green graph is the inverse.

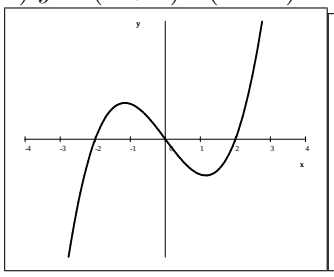
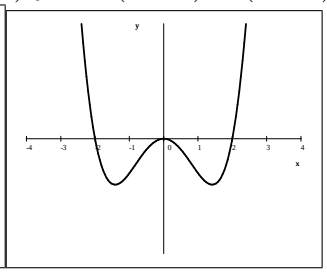







4. a) $\frac{\pi}{2} + k\pi$, $\frac{\pi}{6} + 2k\pi$, $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$
   b) $-\frac{\pi}{2} + 2k\pi$, $\frac{\pi}{6} + 2k\pi$, $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$
   c) $k\pi$, $\pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$
   d) $2k\pi$, $\pm\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$
   e) $\frac{1}{2e^2} + 4$ f) $\frac{1}{3}(4 + \log_2 5)$ g) 11 h) 5
   i) $-\frac{\pi}{2} + 2k\pi$ or $\frac{\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$
   j) $-\frac{\pi}{10} + \frac{2k\pi}{5}$ or $\frac{\pi}{30} + \frac{2k\pi}{5}$ where $k \in \mathbb{Z}$ k) 1, 81

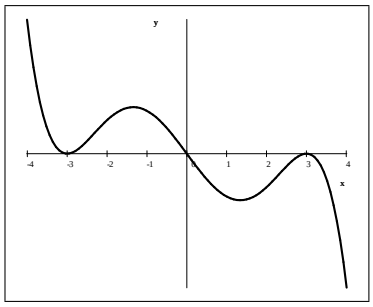
5. -7 6. $\frac{387}{990}$

7. a) $A - B + \sqrt[3]{C}$ b) $\frac{A\sqrt[3]{C}}{B}$ c) $2 + \sqrt{3}$
   d) 4 e) 1 f) -2

8. a) $y = (x+2)x(x-2)$ b) $y = -(x+2)x^2(x-2)$

   c) $f(x) = -2(x+3)^2 x(x-3)^2$



9. $\frac{3}{8}$ or $-\frac{8}{3}$ 10. $\frac{xy+1}{x+1}$ 11.698 mi

12. a) $1167.625\,27\frac{\text{mi}}{\text{h}}$ b) $912.644\,57\frac{\text{mi}}{\text{h}}$

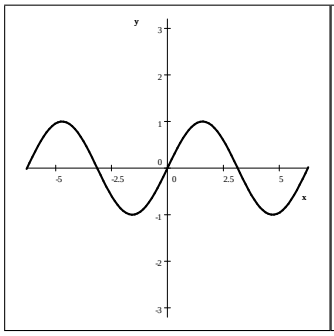
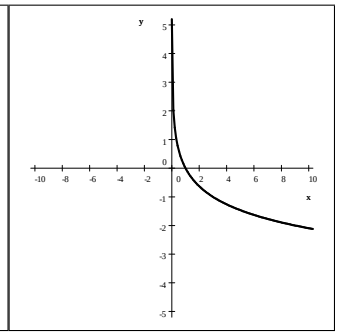
13. a) $\left(-\frac{1}{3}, \frac{3}{8}\right)$ b) $\{-3\}$ c) $[-2,1) \cup [3,\infty)$
    d) $(-\infty,-1) \cup \left[\frac{5}{3},\infty\right)$ e) $\left(0, \frac{1}{2}\right]$

14. a) $\left\{x : x > 4 \text{ and } x \neq 2 + \sqrt{5}\right\}$
    b) $\left\{x : x \neq \frac{\pi}{2} + 2k\pi \text{ where } k \in \mathbb{Z}\right\}$ c) $\mathbb{R}$
    d) $(-\infty,-1) \cup [0,\infty)$ 15. $-\frac{1}{7}(x-2) = (y+9)$

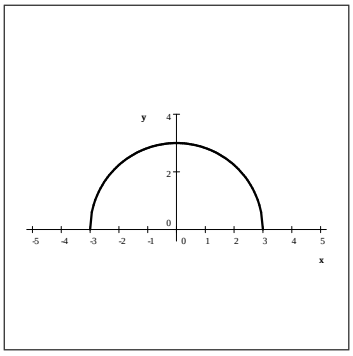
16. $y = 5x - 4$ and $y = 11x - 22$

17. a) 1 b) -9 c) $4a^2 - 6a + 1$ d) $-4a + 1$
    e) $4x^2 + 2x - 1$ f) $-2x^2 + 6x - 1$ g) 0 and $\frac{2}{3}$

18. a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$ b) $g(x) = \log_{1/3} x$

c) $h(x) = \sqrt{9 - x^2}$ on $[-3, 3]$



19. a) $25 - 19i$ b) $-i$ c) $-2i$ e) $10\,000$
d) $5 + i$ f) $5\sqrt{2}$ g) $39 - 80i$ h) 5

20. -3 21. $\dfrac{15}{4}$ 22. $23.936536\,\text{cm}$

23. a) $\sin\alpha = \dfrac{M}{\sqrt{M^2+1}}$ and $\cos\alpha = \dfrac{1}{\sqrt{M^2+1}}$
b) T is negative.
$\sin\beta = \dfrac{T}{\sqrt{T^2+1}}$ and $\cos\beta = \dfrac{1}{\sqrt{T^2+1}}$

24. a) $-\dfrac{\sqrt{5}}{3}$ b) $-\dfrac{4}{5}$ c) $-\dfrac{4\sqrt{5}}{9}$ d) $\dfrac{1}{9}$
e) $\dfrac{6-4\sqrt{5}}{15}$ f) $-\dfrac{3\sqrt{5}-8}{15}$ g) $4\sqrt{5}$

25. see at the end with solutions

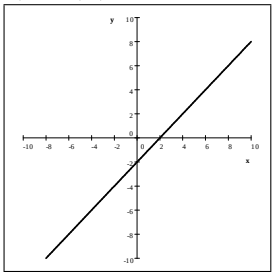
26. a) $A = nR^2 \cos\left(\dfrac{180^\circ}{n}\right) \sin\left(\dfrac{180^\circ}{n}\right)$
b) $\dfrac{1}{2}R^2 \sin\left(\dfrac{360^\circ}{n}\right)$
c) see solutions at the end

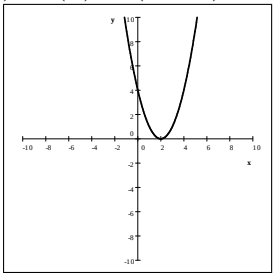
27. a) $\underline{v}$ b) $\underline{c}$ c) $\underline{w} + \underline{x}$ d) $-\underline{x}$

28. a) $\sqrt{73}$ b) $\sqrt{5}$ c) $4\underline{i} - 6\underline{j}$
d) $\sqrt{52}$ e) $\underline{i} + 16\underline{j}$ f) $7\underline{i}$

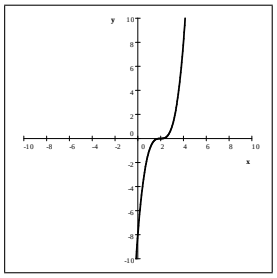
29. They are equal. (Hint: take $\log_3$ of both numbers.)

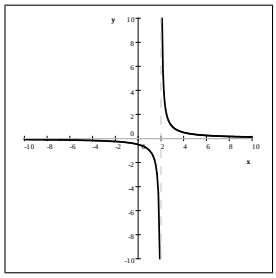
30. a) $f(x) = x - 2$ b) $f(x) = (x-2)^2$



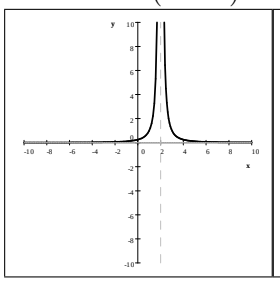


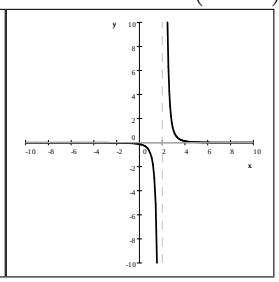
c) $f(x) = (x-2)^3$ d) $f(x) = \dfrac{1}{x-2}$



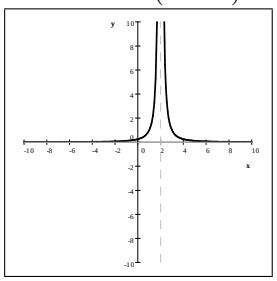


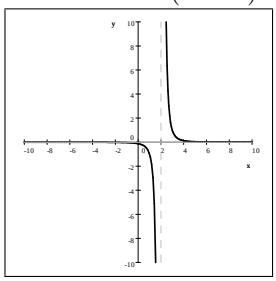
e) $f(x) = \dfrac{1}{(x-2)^2}$ f) $f(x) = \dfrac{1}{(x-2)^3}$



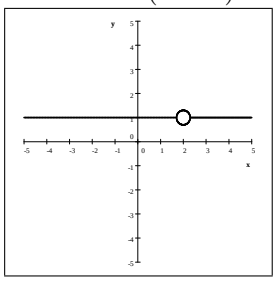


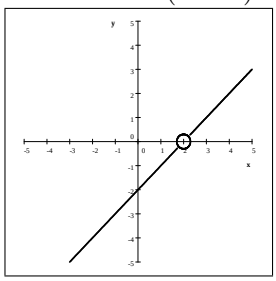
g) $f(x) = \dfrac{(x-2)^5}{(x-2)^7}$ h) $f(x) = \dfrac{(x-2)^2}{(x-2)^5}$



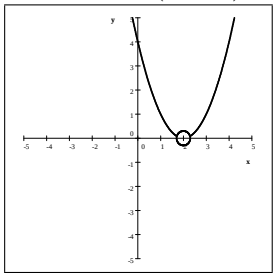


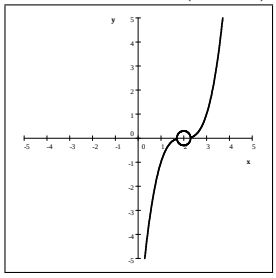
i) $f(x) = \dfrac{(x-2)^6}{(x-2)^6}$ j) $f(x) = \dfrac{(x-2)^5}{(x-2)^4}$





k) $f(x) = \dfrac{(x-2)^7}{(x-2)^5}$ l) $f(x) = \dfrac{(x-2)^5}{(x-2)^2}$





Solutions

10. $\log_6 20 = \dfrac{\log_3 20}{\log_3 6} = \dfrac{\log_3 2 + \log_3 10}{1 + \log_3 2} = \dfrac{\dfrac{1}{x} + y}{1 + \dfrac{1}{x}} = \dfrac{1 + xy}{x + 1} = \dfrac{xy + 1}{x + 1}$

25. a) $1 - \sin 2x = (\sin x - \cos x)^2$

$$\text{RHS} = (\sin x - \cos x)^2 = \sin^2 x + \cos^2 x - 2 \sin x \cos x = 1 - \sin 2x = \text{LHS}$$

b) $\cos\left(\dfrac{\pi}{4} - \alpha\right) - \cos\left(\dfrac{\pi}{4} + \alpha\right) = \sqrt{2} \sin \alpha$

$$\begin{aligned}
\text{LHS} &= \cos\left(\frac{\pi}{4} - \alpha\right) - \cos\left(\frac{\pi}{4} + \alpha\right) = \cos\frac{\pi}{4}\cos\alpha + \sin\frac{\pi}{4}\sin\alpha - \left(\cos\frac{\pi}{4}\cos\alpha - \sin\frac{\pi}{4}\sin\alpha\right) \\
&= \frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha - \left(\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha\right) \\
&= \frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha - \frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha = 2 \cdot \frac{\sqrt{2}}{2}\sin\alpha = \sqrt{2}\sin\alpha = \text{RHS}
\end{aligned}$$

c) $\sin x + \sin\left(x + \dfrac{2\pi}{3}\right) + \sin\left(x + \dfrac{4\pi}{3}\right) = 0$

$$\begin{aligned}
\text{LHS} &= \sin x + \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x + \frac{4\pi}{3}\right) = \\
&= \sin x + \sin x \cos\left(\frac{2\pi}{3}\right) + \cos x \sin\left(\frac{2\pi}{3}\right) + \sin x \cos\left(\frac{4\pi}{3}\right) + \cos x \sin\left(\frac{4\pi}{3}\right) \\
&= \sin x + \sin x\left(-\frac{1}{2}\right) + \cos x\left(\frac{\sqrt{3}}{2}\right) + \sin x\left(-\frac{1}{2}\right) + \cos x\left(-\frac{\sqrt{3}}{2}\right) \\
&= \sin x - \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x = 0 = \text{RHS}
\end{aligned}$$

d) $\sin^2 x - \sin^2 y = \sin(x + y)\sin(x - y)$

$$\begin{aligned}
\text{RHS} &= \sin(x + y)\sin(x - y) = (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\
&= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y = \sin^2 x\left(1 - \sin^2 y\right) - \left(1 - \sin^2 x\right)\sin^2 y \\
&= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y = \sin^2 x - \sin^2 y = \text{LHS}
\end{aligned}$$

26. c) They are the same because of the double angle formula for sine:

$$\begin{aligned}
A_1 &= nR^2 \cos\left(\frac{180^\circ}{n}\right)\sin\left(\frac{180^\circ}{n}\right) = nR^2 \cdot \frac{1}{2} \cdot 2\cos\left(\frac{180^\circ}{n}\right)\sin\left(\frac{180^\circ}{n}\right) \\
&= nR^2\frac{1}{2}\sin\left(2 \cdot \frac{180^\circ}{n}\right) = \frac{1}{2}nR^2 \sin\left(\frac{360^\circ}{n}\right) = A_2
\end{aligned}$$

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