

1. Re-write the repeating decimal $0.7\overline{16} = 0.7161616\dots$ as a quotient of two integers.

2. Simplify each of the following.

a) $(-2a^3b^2)^{-2}(-2a^5b^6)$	e) $2^{-3} - (1 - 3^{-2})$	h) $\left(\frac{a^{-2}}{b^{-1}}\right)^0$	m) $(-32)^{-2/5}$
b) $\frac{a^3b^{-2}}{a^{-1}b^{-1}}$	f) $\left((2^{-2})^{-2}\right)^{-2}$	i) -2^2	n) $\left(\frac{a^2b^0}{a^{-2}b^{12}}\right)^{1/2}$
c) $\frac{a^3 - b^{-2}}{a^{-1} + b^{-1}}$	g) $\left(\frac{a^{-2}}{b^{-1}}\right)^{-3}$	j) $(-2)^{-2}$	o) $\frac{a^{1/2}b^{2/3}}{a^{-1/2}b^{-1/3}}$
d) $2^{-3}(1 - 3^{-2})$		k) $32^{-2/5}$	
		l) $-32^{-2/5}$	

3. Let k denote 2^{1000} . Re-write each of the following in terms of k .

a) 2^{1001} b) 2^{999} c) 2^{1002} d) 2^{3000} e) 4^{1000} f) 2^{500} g) $2^{1000} - 2^{1001} + 2^{1002}$

4. Let $f(x) = \frac{x-2}{x+5}$ and $g(x) = \frac{5x+2}{-x+1}$. Compute each of the following.

a) $f(0)$	d) $g(\sqrt{2})$	f) $f(g(0))$	i) $f(1) + f(3)$
b) $g(0)$	e) $f(\sqrt{20})$	g) $g(f(3))$	j) $2f(5)$
c) $g(1)$		h) $f(1+3)$	k) $f(2 \cdot 5)$

5. Simplify each of the following. Assume that x is positive.

a) $(-16)^{-3/4}$	d) $\frac{2}{(\sqrt{5}-1)^2}$	f) $(\sqrt{3+\sqrt{5}} - \sqrt{3-\sqrt{5}})^2$
b) $-16^{-3/4}$		
c) $(x^{2/3})^{3/4} \left(\frac{1}{\sqrt{x}}\right)$	e) $(1+\sqrt{3})^3(1-\sqrt{3})^3$	g) $\frac{3+\sqrt{3}}{3-\sqrt{3}}$

6. Simplify each of the following. Assume that a is positive.

a) $\log_4\left(\frac{1}{2}\right)$	e) $\log_2\left(\frac{1}{8}\right)$	j) $\log_8\left(\frac{1}{4}\right)$	o) $\log_a(a^{17})$	s) $\log_{\sqrt{3}}(81)$
b) $\log_4 8$	f) $\log_8 2$	k) $\log_{(1/5)} 125$	p) $\log_{(-5)} 25$	t) $\log_2(16^2)$
c) $\log_{10} 1000$	g) $\log_{\sqrt{2}} 8$	l) $\log_{0.1} 100\,000$	q) $\log_9\left(\frac{1}{\sqrt{27}}\right)$	u) $\log_2(8^x)$
d) $\log_{100} 1000$	h) $\log_4 8$	m) $\log_5 1$	r) $\log_a(\sqrt[7]{a^2})$	v) $\log_8(2^x)$
	i) $\log_4(-8)$	n) $\log_2(\sec 315^\circ)$		

7. Simplify each of the following. Assume that x represents a positive number.

a) $(\sqrt{3})^6$	d) $(\sqrt[12]{x})^4$	g) $\log_3(\tan 240^\circ)$	i) $\frac{\sqrt{x}}{\sqrt[3]{\sqrt{x}} \cdot \sqrt{\sqrt{x}}}$	j) $\log_3\left(\frac{\sqrt[3]{9}}{\sqrt{3}}\right)$
b) $(\sqrt[7]{x})^{21}$	e) $\sqrt[2]{\sqrt[3]{x}}$	h) $\left(\frac{1}{\sqrt[3]{x}}\right)^{-12}$		k) $\ln(\tan 225^\circ)$
c) $\sqrt{\sqrt{\sqrt{x}}}$	f) $(\sec 45^\circ)^{10}$			
l) $\log_2(\log_2(\sin 135^\circ + \cos 315^\circ))$	m) $\log_3(\tan 210^\circ)$			

8. Which of the following angles are co-terminal? Group co-terminal angles together.

$$\begin{array}{cccccccc} -585^\circ & 630^\circ & 270^\circ & -225^\circ & -180^\circ & 660^\circ & 180^\circ & -540^\circ \\ 135^\circ & 300^\circ & 540^\circ & -90^\circ & 495^\circ & -780^\circ & -60^\circ & \end{array}$$

9. Find the exact value of each of the following.

a) $\sin 210^\circ$	f) $\tan 315^\circ$	k) $\cos 180^\circ$	p) $\sin 600^\circ$	u) $\tan(-90^\circ)$
b) $\cos 210^\circ$	g) $\sin(-60^\circ)$	l) $\tan 180^\circ$	q) $\cos 600^\circ$	v) $\sin(-225^\circ)$
c) $\tan 210^\circ$	h) $\cos(-60^\circ)$	m) $\sin(-150^\circ)$	r) $\tan 600^\circ$	w) $\cos(-225^\circ)$
d) $\sin 315^\circ$	i) $\tan(-60^\circ)$	n) $\cos(-150^\circ)$	s) $\sin(-90^\circ)$	x) $\tan(-225^\circ)$
e) $\cos 315^\circ$	j) $\sin 180^\circ$	o) $\tan(-150^\circ)$	t) $\cos(-90^\circ)$	

10. Suppose that a and b are real numbers so that a is eight less than three times b . Find the greatest possible value of $5b^2 - a^2$.

11. Suppose that α is the smallest angle in a right triangle with sides 8 cm, 15 cm, and 17 cm long. Compute the exact value of each of the following.

a) $\sin \alpha$ b) $\cos \alpha$ c) $\tan \alpha$ d) $\sin^2 \alpha + \cos^2 \alpha$ e) $\sec^2 x - \tan^2 x$

12. Find the exact value of each of the following expressions and simplify.

a) $\sin 150^\circ - \cos 225^\circ \tan 120^\circ$	e) $(\sin 45^\circ + \cos 45^\circ)^2$	i) $\log_3(\tan 60^\circ)$
b) $\sin^2 45^\circ \cos 45^\circ - \cos^2 30^\circ$	f) $\sin^2 45^\circ + \cos^2 45^\circ$	j) $\log_2(\sin 45^\circ)$
c) $\frac{\sin 10^\circ}{\cos 80^\circ}$	g) $(\sin 45^\circ + \cos 45^\circ)^{10}$	k) $\frac{\tan 120^\circ - \tan 135^\circ}{1 + \tan 120^\circ \tan 135^\circ}$
d) $\sin 60^\circ \sin 30^\circ + \cos 60^\circ \cos 30^\circ$	h) $\sin^{10} 45^\circ + \cos^{10} 45^\circ$	l) $\frac{\tan^2 120^\circ - \tan^2 150^\circ}{\tan^2 120^\circ \tan^2 150^\circ}$

13. Approximate each of the following by placing them between two consecutive integers.

a) $\log_2 100$ b) $\log_2 200$ c) $\log_{10} 2014$ d) $\log_5 2014$ e) $\log_2 \left(\frac{1}{3}\right)$

14. Re-write each of the following as an exponential statement.

a) $x = \log_7(2y - 1)$ c) $p = \log_q T$ e) $2 = \log_x 20$
 b) $3 = \log_B(A - 1)$ d) $x + 2 = \log_a(x^2 + 1)$

15. Re-write each of the following as a logarithmic statement. Assume that all variables represent positive numbers.

a) $3^{x-2} = 60$ b) $10^{2x-5} = 2012$ c) $\left(\frac{1}{3}\right)^{a+1} = b - 2$ d) $A^B = C$

16. Solve each of the following equations. Make sure to check your solutions.

a) $\frac{2x+1}{5} - \frac{x-1}{3} = 2x+16$	d) $\sqrt{3x+10} - \sqrt{x+4} = 2$	g) $2x^5 = 32x^4$
b) $(2x-1)^2 - (x-2)^2 = x+1$	e) $\sqrt{x-3} + 1 = \sqrt{x+2}$	h) $2x^5 = 32x^3$
c) $\sqrt{x+6} + \sqrt{11-x} = 5$	f) $\sqrt{x-1} - 1 = \sqrt{2x-9}$	i) $2x^5 = 32x$
		j) $x^2 + x = 1$

17. Solve each of the following basic logarithmic equations.

a) $\log_3(2x - 7) = 2$

d) $\ln(2x + 1) = -3$

g) $\frac{2}{3} \log_2(x + 2) - 1 = 7$

b) $\log_3(x^2 - 19) = 4$

e) $\log_2(3x + 4) = -1$

h) $\frac{2 \log_2(x + 2) - 1}{3} = 7$

c) $\ln(1 - 3x) = 28$

f) $3 \log_2(7x - 1) - 2 = 10$

18. Solve each of the following basic exponential equations.

a) $2^x = \frac{1}{16}$

c) $2^x = \frac{1}{\sqrt{2}}$

f) $3^{5x-1} = \sqrt{27}$

i) $3^{5x-1} = 40$

l) $e^{2x+8} = 1$

d) $2^x = 21$

g) $3^{5x-1} = -9$

j) $e^{2x+8} = 0$

b) $2^x = -8$

e) $2^x = -4$

h) $3^{5x-1} = \frac{1}{9}$

k) $e^{2x+8} = \frac{1}{e^5}$

m) $e^{2x+8} = 10$

19. Solve each of the following inequalities.

a) $3x^2 \geq 15x$

b) $28x + 88 \leq -2x^2$

c) $\frac{1}{3}x^2 < 4x - 12$

d) $\frac{1}{3}x^2 \leq 4x - 12$

20. Find the domain of each of the following functions.

a) $f(x) = \frac{1}{5x + 3}$

d) $f(x) = \sqrt{x^2 - 10x + 24}$

g) $f(x) = \frac{3x - 1}{x^2 + 4}$

b) $f(x) = \sqrt{5x + 3}$

e) $f(x) = \sqrt{x - 3} - 2 + \frac{1}{x - 10}$

h) $f(x) = \sqrt{9 - x^2} - \sqrt{x + 1}$

c) $f(x) = \frac{1}{x^2 - 10x + 24}$

f) $f(x) = \frac{1}{\sqrt{x - 3} - 2}$

21. The sum of a and three times b is 100. What is the smallest possible value of $a^2 - (2b)^2$?

22. Find the height of a building if the angle of elevation from the ground to its top changes from 15° to 25° as the observer advances 120 ft toward it.

23. A straight pyramid has a square base with sides 18 units long. All other edges are 20 units long. Compute the angle formed by the base and a triangular face.

24. Compute the area and perimeter of a regular 12-sided polygon written into a circle with radius 9 cm. Present the exact value and approximation of your answer.

25. Find all points of intersections of the circles given.

a) $(x + 4)^2 + (y - 4)^2 = 25$ and $(x - 10)^2 + (y - 2)^2 = 125$

b) $x^2 + (y - 4)^2 = 25$ and $(x - 6)^2 + (y - 1)^2 = 10$

c) $(x + 2)^2 + (y + 2)^2 = 5$ and $(x - 2)^2 + y^2 = 5$

d) $(x + 5)^2 + (y - 3)^2 = 13$ and $(x - 4)^2 + (y + 3)^2 = 52$

26. Consider the circle $x^2 - 20x + y^2 = 21 + 4y$. Find the equation of the tangent line drawn to the circle at $(0, 7)$.

27. Compute the area of a triangle with sides 15 m, 15 m, and 22 m.

28. Suppose that α is an acute angle. (An acute angle is one between 0 and 90°)

a) Find the exact value of $\tan \alpha$ if $\cos \alpha = \frac{1}{3}$.

b) Find the exact value of $\cos \alpha$ if $\tan \alpha = 2$.

c) Find the value of $\sin \alpha$ and $\cos \alpha$ in terms of M if we know that $\tan \alpha = M$.

29. Find all real numbers that are exactly four greater than their own reciprocal.
30. A circle has a radius of 10 units. A point P is 17 units away from the center of the circle. Find the measure of the angle formed by the tangent lines drawn from P to the circle.
31. A person is standing 35 feet away from a street light that is 28 ft tall. How long is his shadow if he is 5.6 feet tall?
32. Compute each of the following sums.
- a) $210 + 223 + 236 + \dots + 1302$ b) $100 + 103 + 106 + \dots + 847$ c) $1010 + 1000 + 990 + \dots + 120$
33. Paul is starting at his new job today. His starting salary is \$2000 a month (after all deductions and taxes). If this amount is expected to increase by \$100 after each year, how much money in total would he make at this company during the next 10 years?
34. Suppose that $A = \{2, 3, 5, 8, 9, 10\}$.
- a) How many subsets does the set A have?
b) List all two-element subsets of A .
c) How many different ways can we list the elements of A ? (Don't list them.)
35. Find an equation for the line that
- a) is parallel to $5x + 3y = -15$ and passes through the point $(-6, 2)$.
b) is perpendicular to $5x + 3y = -15$ and passes through the point $(-6, 2)$.
c) passes through the points $(3, 8)$ and $(-1, 2)$.
36. An arch is in the shape of a semicircle. At a point along the base 2 feet from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch.
37. The shortest side of a right triangle is 12 cm. The difference between the other two sides is 2 cm. Find an approximate value for the smallest angle in the triangle.
38. Graph each of the following.
- a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$ c) $(x - 2)^2 + (y - 1)^2 = 25$ d) $f(x) = \left(\frac{1}{2}\right)^x$
39. a) Find the exact and approximate values of the smaller angle that is formed between the positive part of the x -axis and the line $y = 2x - 1$.
b) Find an equation for the straight line that passes through the point $P(4, -2)$ and forms a 60° angle with the positive part of the x -axis.
40. Suppose that C_1 is a circle with radius 12 units and C_2 is a circle with radius 15 units. The distance between the centers is 20 units.
- a) Find the exact value and approximate value for the angle formed by the common tangent lines drawn to the circles.
b) Compute the distance between the two points of tangency on a common tangent line.
41. * Suppose that $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$.
- a) How many relations are possible with domain A and range any non-empty subset of B ?
b) How many functions are possible with domain A and range any non-empty subset of B ?
c) How many one-to-one functions are possible with domain A and range any non-empty subset of B ?
42. * Find the equation of the circle that passes through $A(1, -3)$, $B(3, 3)$, and $C(7, -5)$.
43. * Solve the equation $\sqrt{x^2 + x + 3} + x^2 + x - 7 = 2$
44. * Simplify $\frac{\sqrt{1009 + \sqrt{2017}} - \sqrt{1009 - \sqrt{2017}}}{\sqrt{2}}$

Answers

1. $\frac{709}{990}$
2. a) $-\frac{b^2}{2a}$ b) $\frac{a^4}{b}$ c) $\frac{a^4b^2 - a}{ab + b^2}$ d) $\frac{1}{9}$ e) $-\frac{55}{72}$
 f) $\frac{1}{256}$ g) $\frac{a^6}{b^3}$ h) 1 i) -4 j) $\frac{1}{4}$ k) $\frac{1}{4}$
 l) $-\frac{1}{4}$ m) undefined n) $\frac{a^2}{b^6}$ o) ab
3. a) $2k$ b) $\frac{k}{2}$ c) $4k$ d) k^3 e) k^2
 f) \sqrt{k} g) $3k$
4. a) $-\frac{2}{5}$ b) 2 c) undefined
 d) $-7\sqrt{2} - 12$ e) $\frac{14}{5}\sqrt{5} - 6$ f) 0
 g) 3 h) $\frac{2}{9}$ i) $-\frac{1}{24}$ j) $\frac{3}{5}$ k) $\frac{8}{15}$
5. a) undefined b) $-\frac{1}{8}$ c) 1 d) $\frac{\sqrt{5} + 3}{4}$
 e) -8 f) 2 g) $\sqrt{3} + 2$
6. a) $-\frac{1}{2}$ b) $\frac{3}{2}$ c) 3 d) $\frac{3}{2}$ e) -3 f) $\frac{1}{3}$ g) 6
 h) $\frac{3}{2}$ i) undefined j) $-\frac{2}{3}$ k) -3 l) -5 m) 0
 n) $\frac{1}{2}$ o) 17 p) undefined q) $-\frac{3}{4}$ r) $\frac{2}{7}$
 s) 8 t) 8 u) $3x$ v) $\frac{x}{3}$
7. a) 27 b) x^3 c) $\sqrt[8]{x}$ d) $\sqrt[3]{x}$ e) $\sqrt[6]{x}$ f) 32
 g) $\frac{1}{2}$ h) x^4 i) $\sqrt[12]{x}$ j) $\frac{1}{6}$ k) 0 l) -1 m) $-\frac{1}{2}$
8. Group 1: $135^\circ, 495^\circ, -225^\circ, -585^\circ$
 Group 2: $-60^\circ, 300^\circ, 660^\circ, -780^\circ$
 Group 3: $180^\circ, -180^\circ, 540^\circ, -540^\circ$
 Group 4: $-90^\circ, 270^\circ, 630^\circ$
9. a) $-\frac{1}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{3}$ d) $-\frac{\sqrt{2}}{2}$ e) $\frac{\sqrt{2}}{2}$
 f) -1 g) $-\frac{\sqrt{3}}{2}$ h) $\frac{1}{2}$ i) $-\sqrt{3}$ j) 0
 k) -1 l) 0 m) $-\frac{1}{2}$ n) $-\frac{\sqrt{3}}{2}$ o) $\frac{\sqrt{3}}{3}$
 p) $-\frac{\sqrt{3}}{2}$ q) $-\frac{1}{2}$ r) $\sqrt{3}$ s) -1 t) 0
 u) undefined v) $\frac{\sqrt{2}}{2}$ w) $-\frac{\sqrt{2}}{2}$ x) -1
10. 80
11. a) $\frac{8}{17}$ b) $\frac{15}{17}$ c) $\frac{8}{15}$ d) 1 e) 1
12. a) $\frac{1}{2}(1 - \sqrt{6})$ b) $\frac{1}{2} - \frac{1}{4}\sqrt{3}$ c) 1 d) $\frac{\sqrt{3}}{2}$
 e) 2 f) 1 g) 32 h) $\frac{1}{16}$ i) $\frac{1}{2}$ j) $-\frac{1}{2}$
 k) $\sqrt{3} - 2$ l) $\frac{8}{3}$
13. a) $6 < \log_2 100 < 7$ b) $7 < \log_2 200 < 8$
 c) $3 < \log_{10} 2014 < 4$ d) $4 < \log_5 2014 < 5$
 e) $-2 < \log_2 \left(\frac{1}{3}\right) < -1$
14. a) $7^x = 2y - 1$ b) $B^3 = A - 1$
 c) $q^p = T$ d) $a^{x+2} = x^2 + 1$ e) $x^2 = 20$
15. a) $x - 2 = \log_3 60$ b) $2x - 5 = \log_{10} 2012$
 c) $a + 1 = \log_{1/3}(b - 2)$ d) $B = \log_A C$
16. a) -8 b) $-1, \frac{4}{3}$ c) -5, 10 d) 5 (-3 extraneous)
 e) 7 f) 5 (17 extraneous) g) 0, 16 h) -4, 0, 4
 i) -2, 0, 2 j) $\frac{-1 \pm \sqrt{5}}{2}$
17. a) 8 b) ± 10 c) $\frac{1}{3} - \frac{e^{28}}{3}$ d) $\frac{1}{2e^3} - \frac{1}{2}$
 e) $-\frac{7}{6}$ f) $\frac{17}{7}$ g) 4094 h) 2046
18. a) -4 b) no solution c) $-\frac{1}{2}$ d) $\log_2 21$
 e) no solution f) $\frac{1}{2}$ g) no solution
 h) $-\frac{1}{5}$ i) $\frac{1}{5}(1 + \log_3 40)$ j) no solution k) $-\frac{13}{2}$
 l) -4 m) $\frac{1}{2}(-8 + \ln 10)$
19. a) $(-\infty, 0] \cup [5, \infty)$ b) $[-7 - \sqrt{5}, -7 + \sqrt{5}]$
 c) no solution d) 6
20. a) $x \neq -\frac{3}{5}$ b) $x \geq -\frac{3}{5}$ c) $x \neq 4, 6$
 d) $x \leq 4$ or $x \geq 6$ e) $x \geq 3$ and $x \neq 10$
 f) $x \geq 3$ and $x \neq 7$ g) \mathbb{R} h) $-1 \leq x \leq 3$
21. -8000

22. 75.588 461 ft

23. $\cos^{-1}\left(\frac{9}{\sqrt{319}}\right) \approx 59.74142^\circ$

24. $P = 216 \sin 15^\circ \text{ cm} \approx 55.90491 \text{ cm}$
 $A = 972 \sin 15^\circ \cos 15^\circ \text{ cm}^2 \approx 243 \text{ cm}^2$

25. a) (0, 7) and (-1, 0) b) (5, 4) and (3, 0)
c) (0, -1) d) (-2, 1)

26. $y = 2x + 7$

27. $A = 22\sqrt{26} \text{ m}^2 \approx 112.17843 \text{ m}^2$

28. a) $2\sqrt{2}$ b) $\frac{1}{\sqrt{5}}$

c) $\sin \alpha = \frac{M}{\sqrt{M^2 + 1}}$ $\cos \alpha = \frac{1}{\sqrt{M^2 + 1}}$

29. $2 - \sqrt{5}, 2 + \sqrt{5}$

30. $2 \sin^{-1}\left(\frac{10}{17}\right) \approx 72.063758^\circ$

31. 8.75 ft

32. a) 64260 b) 118375 c) 50850

33. \$294 000

34. a) 64 b) see below c) 720

$\{1, 2\}$

$\{1, 3\}$ $\{2, 3\}$

$\{1, 4\}$ $\{2, 4\}$ $\{3, 4\}$

$\{1, 5\}$ $\{2, 5\}$ $\{3, 5\}$ $\{4, 5\}$

$\{1, 6\}$ $\{2, 6\}$ $\{3, 6\}$ $\{4, 6\}$ $\{5, 6\}$

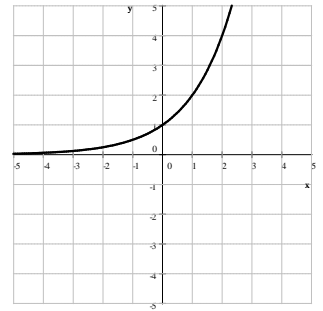
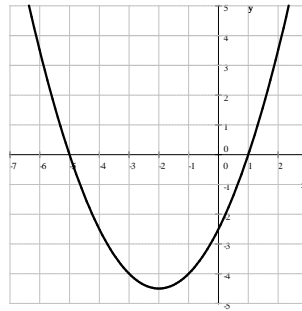
35. a) $y = -\frac{5}{3}x - 8$ b) $y = \frac{3}{5}x + \frac{28}{5}$

c) $y = \frac{3}{2}x + \frac{7}{2}$

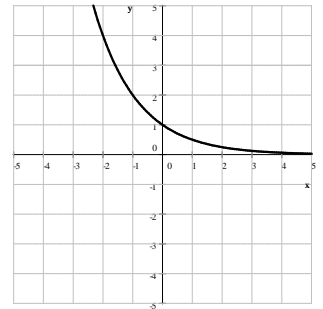
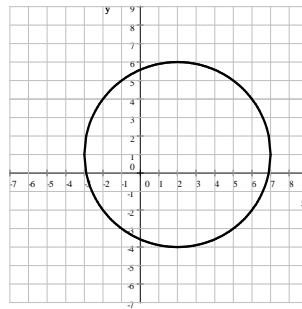
36. 17 feet

37. $\sin^{-1}\left(\frac{12}{37}\right) \approx 18.92464^\circ$

38. a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$



c) $(x - 2)^2 + (y - 1)^2 = 25$ d) $f(x) = \left(\frac{1}{2}\right)^x$



39. a) $\tan^{-1} 2 \approx 63.434949$ b) $y + 2 = \sqrt{3}(x - 4)$

40. a) $2 \sin^{-1}\left(\frac{3}{20}\right) \approx 17.253853^\circ$ b) $\sqrt{391}$ unit

41. a) 15^4 b) 4^4 c) 24

42. $(x - 5)^2 + (y + 1)^2 = 20$

43. Hint: try introducing a new variable!