1. Simplify each of the following.

   a) $\dfrac{3a^2b^{-3}\left(2a^3b^7\right)^0}{-2b^0a^5\left(3b^{-1}\right)}$

   b) $\dfrac{a^{-1}b^{-2}}{a^{-2}b^{-1}}$

   c) $\dfrac{a^{-1}+b^{-2}}{a^{-2}-b^{-1}}$

   d) $\left(3^{-1}+1\right)^{-1}$

   e) $(-1)^{-1}$

   f) $\left(\dfrac{3ab^5}{-2a^3b^{-1}}\right)^{-2}$

   g) $\left(\dfrac{2x^{1/2}y^{-1/2}}{-x^{3/2}y^{-3/2}}\right)^4$

2. Simplify each of the following.

   a) $4^{1/2}$ b) $5^{1/2}$ c) $8^{1/3}$ d) $100^{1/2}$ e) $100^{1/4}$ f) $25^{1/2}$ g) $25^{3/2}$ h) $4^{-3/2}$ i) $8^{2/3}$ j) $8^{-2/3}$ k) $-8^{-2/3}$ l) $(-8)^{-2/3}$ m) $-16^{5/4}$ n) $1^{-3/7}$ o) $-1^{-3/7}$ p) $(-8)^{1/3}$ q) $(-8)^{2/3}$

3. Simplify each of the following.

   a) $\sin 210^\circ - 2\cos 390^\circ$

   b) $\tan 60^\circ \tan 30^\circ$

   c) $\sin 120^\circ + \sin 150^\circ + \sin 210^\circ + \sin 240^\circ$

   d) $\dfrac{\tan 210^\circ - \tan 315^\circ + \tan 240^\circ}{\tan 210^\circ}$

4. Suppose that θ is an acute angle with $\tan\theta = A$. Express $\sin\theta$ in terms of A.

5. Prove each of the following identities.

   a) $\tan^2 x + 1 = \sec^2 x$

   b) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

   c) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

   d) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

   e) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1-\tan x}{1+\tan x}$

6. Simplify each of the following.

   a) -3^{-2} b) $(-3)^{-2}$ c) $81^{-3/4}$ d) $16^{7/4}$ e) $\log_{81} 3$ f) $\log_5 1$ g) $\log_5 0$ h) $\log_m\left(m^4\right)$ i) $\log_{1/2}\left(\dfrac{1}{8}\right)$ j) $\log_{49} 7$ k) $\log_4\left(\dfrac{1}{32}\right)$ l) $\log_8 16$ m) $5^{\log_5 125}$ n) $2^{\log_2 16}$ o) $\log_8\left(\dfrac{1}{16}\right)$ p) $\log_{27}\left(\dfrac{1}{9}\right)$ q) $3^{\log_3(1/9)}$ r) $\log_{100} 1000$ s) $\log_2\left(\dfrac{1}{32}\right)$ t) $2^{-\log_2 4}$

7. Place each of the following logarithms between two consecutive integers.

   a) $\log_2 20$ b) $\log_5 100$ c) $\log_3 10$ d) $\log_{10} 2012$ e) $\log_{10} 314$ f) $\log_2 100$ g) $\log_2 1000$ h) $\log_3 100$

8. Re-write each of the following logarithmic statements as an exponential statement.

   a) $\log_2 x = 10$ b) $\log_3 P = -2$ c) $\log_7 21 = y$ d) $\log_x y = 4$ e) $\log_A B = C$ f) $\log_2(x+1) = -2$ g) $\log_x 5 = 2$ h) $\log_{1.3}(x-5) = 1$

9. Solve each of the following basic logarithmic equations.

a) $\dfrac{2\log_2(3x-1)-4}{3}=2$

b) $\dfrac{2}{3}\log_2(3x-1)-4=2$

c) $4\ln(x-2)+1=-2$

d) $4(\ln(x-2)+1)=-2$

e) $\log_5(3x-1)=-2$

f) $\log_2\left(x^2-4\right)=5$

10. Re-write each of the following exponential statements as a logarithmic statement.

a) $A^B=C$ b) $4^x=7$ c) $3^{A-1}=12$ d) $e^x=24$

11. Solve each of the following basic exponential equations.

a) $3^x=243$
b) $3^x=38$
c) $2^{4x-1}=128$
d) $2^{4x-1}=42$
e) $10^{3x-1}=10^{41}$
f) $10^{3x-1}=19$
g) $e^{2x-8}=e^{10}$
h) $e^{3x-7}=5$
i) $3^{2x-5}=-27$

12. Solve each of the following equations.

a) $\sqrt{2x-1}=2+\sqrt{x-4}$ b) $\sqrt{x+1}+\sqrt{5x+1}=6$ c) $\sqrt{2x+1}=\sqrt{3x+4}-1$

13. Solve each of the following inequalities.

a) $12x-2x^2\le 20$
b) $3x^2-12x<3$
c) $10x+5x^2\le -25$
d) $10x+5x^2\ge 25$
e) $x^2+x>1$

14. Find the domain of each of the following functions.

a) $f(x)=\ln\left(x^2-3\right)$
b) $g(x)=\dfrac{1}{\ln(x^2-3)}$
c) $f(x)=\sqrt{5x-4}$
d) $g(x)=\dfrac{1}{6-\sqrt{5x-4}}$
e) $f(x)=\sqrt{x^2-4}+\sqrt{4-x^2}$

15. We are driving toward a tower. The angle of elevation is $27°$. Then we drive 100 ft toward the tower. Now the angle of elevation is $37°$. How tall is the tower? Present your answer as an approximation, accurate up to three or more decimal places.

16. A number is 2 less than its own reciprocal. Find this number.

17. Find the sides of a right triangle with hypotenuse 58 units and perimeter 140 units.

18. We are planning to build a fence around a rectangular a garden that is next to a river. Because of that, we only need to build a fence around three of its sides. What is the maximum area possible to fence in if we have 400 meters of fencing? What dimensions of a garden should we plan to achieve this maximum area?

19. Find an equation for the line connecting the points of intersections of the circles $(x+1)^2+(y-2)^2=10$ and $(x-1)^2+(y-6)^2=50$.

20. Find an equation of the tangent line drawn to the given circle at the given point.

a) $2y-6x+x^2+y^2=40$ at $P(10,-2)$ b) to $2y+x^2+y^2=3(2x+5)$ at $P(7,-4)$

21. Suppose that triangle ABC is determined by $A(-3,8)$, $B(-7,2)$, and $C(11,-2)$

a) Find an equation for the perpendicular bisector of line segment AB.

b) Find an equation for the altitude belonging to side AB.

22. True or false?

    a) $\sin 90^\circ = \sin 30^\circ + \sin 60^\circ$

    b) For all angles α, $\sin(\alpha + \beta) = \sin\alpha + \sin\beta$

    c) $\sin 60^\circ = 2\sin 30^\circ$

    d) For all angles α, $\sin 2\alpha = 2\sin\alpha$.

23. Find the exact value of each of the following expressions.

    a) $\log_2\left(\sqrt{\sqrt{\sqrt{2}}}\right)$ c) $\log_3(\tan 45^\circ)$

    b) $\log_2\left(\sec^{100} 45^\circ\right)$ d) $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos 180^\circ$

24. The sum of two numbers is 20. Find the minimal value of the sum of their squares.

25. Side AB of a rectangle $ABCD$ is located on the $x-$ axis, within the interval $[-4, 4]$. Another horizontal side is defined by points C and D, lying on the straight lines $y = 3x+12$ and $y = -3x+12$, respectively. Find the maximal possible area of the rectangle.

26. Consider the triangle with sides 16 ft, 17 ft, and 17 ft long.

    a) Compute the exact value of the area of the triangle.

    b) Compute the approximate value of the angles in the triangle. Present your answer in degrees, accurate to four or more decimal places.

27. Find the area of a regular $9-$sided polygon written into a circle of radius $10\,\mathrm{cm}$.

28. a) Find the exact and approximate value of the smaller angle that is formed between the line $y = 2x-3$ and the positive part of the $x-$axis.

    b) Find the exact and approximate value of the smaller angle that is formed between the line $y = 4x-3$ and the positive part of the $x-$axis.

    c) Based on the previous computations, determine whether doubling the slope of a line means doubling the angle or not.

29. Suppose that circle C_1 has radius 4 and circle C_2 has radius 7. The two centers are 9 units apart. Find an approximate value of the angle formed by the two common tangent lines drawn to the circles.

30. Suppose that $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b, c\}$.

    a) How many relations are possible with domain A and range any subset of B?

    b) How many functions are possible with domain A and range any subset of B?

    c) How many one-to-one functions are possible with domain A and range any subset of B?

31. *Solve the equation $\log_x(x + 20) = 2$

Answers

1. a) $-\dfrac{1}{2a^3b^2}$ b) $\dfrac{a}{b}$ c) $\dfrac{a^2+ab^2}{b^2-a^2b}$ d) $\dfrac{3}{4}$ e) -1 f) $\dfrac{4a^4}{9b^{12}}$ g) $\dfrac{16y^4}{x^4}$

2. a) 2 b) $\sqrt{5}$ c) 2 d) 10 e) $\sqrt{10}$ f) 5 g) 125 h) $\dfrac{1}{8}$ i) 4 j) $\dfrac{1}{4}$ k) $-\dfrac{1}{4}$ l) undefined m) -32 n) 1 o) -1 p) -2 q) undefined

3. a) $-\sqrt{3}-\dfrac{1}{2}$ b) 1 c) 0 d) $\sqrt{3}+4$

4. $\dfrac{A}{\sqrt{A^2+1}}$

5. a) $\tan^2 x+1=\sec^2 x$

$$\text{LHS}=\tan^2 x+1=\left(\frac{\sin x}{\cos x}\right)^2+1=\frac{\sin^2 x}{\cos^2 x}+1=\frac{\sin^2 x}{\cos^2 x}+\frac{\cos^2 x}{\cos^2 x}=\frac{\sin^2 x+\cos^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\text{RHS}$$

b) $\tan x+\dfrac{\cos x}{1+\sin x}=\sec x$

$$\begin{aligned}\text{RHS} &= \tan x+\frac{\cos x}{1+\sin x}=\frac{\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=\frac{\sin x\,(1+\sin x)+\cos^2 x}{\cos x\,(1+\sin x)}\\ &= \frac{\sin x+\sin^2 x+\cos^2 x}{\cos x\,(1+\sin x)}=\frac{\sin x+1}{\cos x\,(1+\sin x)}=\frac{1}{\cos x}=\sec x=\text{LHS}\end{aligned}$$

c) $\dfrac{\cos x}{1-\sin x}=\sec x+\tan x$

$$\begin{aligned}\text{LHS} &= \frac{\cos x}{1-\sin x}=\frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}=\frac{\cos x\,(1+\sin x)}{1-\sin^2 x}=\frac{\cos x\,(1+\sin x)}{\cos^2 x}=\frac{1+\sin x}{\cos x}\\ &= \frac{1}{\cos x}+\frac{\sin x}{\cos x}=\sec x+\tan x=\text{RHS}\end{aligned}$$

d) $\dfrac{\csc^2 x-1}{\csc^2 x}=\cos^2 x$

$$\text{LHS}=\frac{\csc^2 x-1}{\csc^2 x}=\frac{\dfrac{1}{\sin^2 x}-1}{\dfrac{1}{\sin^2 x}}=\frac{\dfrac{1}{\sin^2 x}-\dfrac{\sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}}=\frac{\dfrac{1-\sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}}$$

To divide is to multiply by the reciprocal:

$$\frac{\dfrac{1-\sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}}=\frac{1-\sin^2 x}{\sin^2 x}\cdot\frac{\sin^2 x}{1}=1-\sin^2 x=\cos^2 x=\text{RHS}$$

e) $\dfrac{\cot x-1}{\cot x+1}=\dfrac{1-\tan x}{1+\tan x}$

$$\text{LHS}=\frac{\cot x-1}{\cot x+1}=\frac{\dfrac{\cos x}{\sin x}-1}{\dfrac{\cos x}{\sin x}+1}$$

Multiply numerator and denominator by $\sin x$

$$\frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1} = \frac{\cos x - \sin x}{\cos x + \sin x}$$

Divide numerator and denominator by $\cos x$

$$\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}} = \frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}} = \frac{1 - \tan x}{1 + \tan x} = \text{RHS}$$

6. a) $-\frac{1}{9}$ b) $\frac{1}{9}$ c) $\frac{1}{27}$ d) 128 e) $\frac{1}{4}$ f) 0 g) undefined h) 4 i) 3 j) $\frac{1}{2}$ k) $-\frac{5}{2}$ l) $\frac{4}{3}$ m) 125 n) 16 o) $-\frac{4}{3}$ p) $-\frac{2}{3}$ q) $\frac{1}{9}$ r) $\frac{3}{2}$ s) -5 t) $\frac{1}{4}$

7. a) $4 < \log_2 20 < 5$ b) $2 < \log_5 100 < 3$ c) $2 < \log_3 10 < 3$ d) $3 < \log_{10} 2012 < 4$ e) $2 < \log_{10} 314 < 3$ f) $6 < \log_2 100 < 7$ g) $9 < \log_2 1000 < 10$ h) $4 < \log_3 100 < 5$

8. a) $2^{10} = x$ b) $3^{-2} = P$ c) $7^y = 21$ d) $x^4 = y$ e) $A^C = B$ f) $\frac{1}{4} = x + 1$ g) $x^2 = 5$ h) $1.3 = x - 5$

9. a) 11 b) 171 c) $e^{-3/4} + 2 = \frac{1}{(\sqrt[4]{e})^3} + 2$ d) $e^{-3/2} + 2 = \frac{1}{(\sqrt{e})^3} + 2$ e) $\frac{26}{75}$ f) $-6, 6$

10. a) $\log_A C = B$ b) $x = \log_4 7$ c) $A - 1 = \log_3 12$ d) $x = \ln 24$

11. a) 5 b) $\log_3 38$ c) 2 d) $\frac{1}{4}(1 + \log_2 42)$ e) 14 f) $\frac{1}{3}(1 + \log_{10} 19)$ g) 9 h) $\frac{1}{3}(7 + \ln 5)$ i) no solution

12. a) $13, 5$ b) 3 (24 doesn't work) c) $0, 4$

13. a) all real numbers b) $\left(2 - \sqrt{5}, 2 + \sqrt{5}\right)$ c) no solution d) $\left(-\infty, -1 - \sqrt{6}\right] \cup \left[-1 + \sqrt{6}, \infty\right)$ e) $\left(-\infty, -\frac{1 + \sqrt{5}}{2}\right) \cup \left(\frac{-1 + \sqrt{5}}{2}, \infty\right)$

14. a) $\left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$ b) $(-\infty, -2) \cup \left(-2, \sqrt{3}\right) \cup \left(\sqrt{3}, 2\right) \cup (2, \infty)$ c) $\left[\frac{4}{5}, \infty\right)$ d) $\left[\frac{4}{5}, 8\right) \cup [8, \infty)$ e) $\{-2, 2\}$ (this set has only two elements)

15. 157.340 15 ft

16. $\sqrt{2} - 1$ and $-\sqrt{2} - 1$

17. 40, 42, and 58 units

18. 100 m by 200 m will give us an area of 20 000 m^2

19. $y = -\frac{1}{2}x - 1$

20. a) $y + 2 = 7(x - 10)$ or $y = 7x - 72$ b) $\frac{4}{3}(x - 7) = y + 4$ or $y = \frac{4}{3}x - \frac{40}{3}$

21. a) $-\frac{2}{3}(x+5) = y-5$ b) $-\frac{2}{3}(x-11) = y+2$

22. a) false b) false c) false d) false

23. a) $\frac{1}{8}$ b) 50 c) 0 d) -1

24. 200

25. 24

26. a) $120\,\text{ft}^2$ b) $61.927\,513°$, $61.927\,513°$, and $56.144\,974°$

27. $900\cos 20°\sin 20°\,\text{cm}^2 \approx 289.254\,424\,36\,\text{cm}^2$

28. a) $\tan^{-1} 2 \approx 63.434948823°$ b) $\tan^{-1}(4) \approx 75.963756\,532°$ c) Nope!

29. $38.942\,44°$

30. a) 7^5 b) 3^5 c) 0

31. 5

Last revised: October 23, 2018