1. a) Suppose that α is NOT an acute angle. Find the exact value of $\cos\alpha$ and $\tan\alpha$ if $\sin\alpha = \frac{1}{3}$. Rationalize the denominator in your answers.

   b) Suppose that β is an angle with $90° < \beta < 270°$. Find the value of $\sin\beta$ and $\cos\beta$ if $\tan\beta = 4$. Rationalize the denominator in your answer.

   c) Suppose that γ is an angle not in the third quadrant . Find the value of $\sin\gamma$, $\cos\gamma$, and $\tan\gamma$ if $\sec\gamma = -\frac{5}{2}$. Rationalize the denominator in your answer.

   d) Suppose that $\tan\theta = -\frac{3}{2}$. Compute $\sin 2\theta$.

2. Express each of the following in terms of x if $x = 2^{2018}$.

   a) $2^{2018} + 2^{2019} + 2^{2020}$ b) $2^{2018} + 4^{2018}$

3. We deposited \$2000 in a bank account with an annual compound interest rate of 7%. If we do not withdraw or add money to this account, how long do we need to wait until there is \$4000 in the account?

4. Compute the exact value of each of the following. Rationalize denominators and simplify your answer.

   a) $\cos 15°$ b) $\sin 15°$ c) $\tan 75°$

5. Prove each of the following identities.

   a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$ b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$ c) $(\sin x - \cos x)^2 = 1 - \sin 2x$

6. Prove each of the following co-function identities. For parts a) and b) , use an appropriate compund angle formula. Why is that not an option for part c)?

   a) $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$ b) $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$ c) $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$

7. Suppose that α and β are acute angles with $\sin\alpha = \frac{1}{3}$ and $\cos\beta = \frac{5}{13}$. Compute the exact value of each of the following.

   a) $\cos\alpha$ d) $\tan\beta$ g) $\sin(\alpha+\beta)$ j) $\cos(\alpha-\beta)$
   b) $\tan\alpha$ e) $\sin 2\alpha$ h) $\sin(\alpha-\beta)$ k) $\tan 2\alpha$
   c) $\sin\beta$ f) $\cos 2\beta$ i) $\cos(\alpha+\beta)$ l) $\tan 2\beta$

8. Suppose that $\sin A = -\frac{3}{5}$. Compute the exact value of each of the following.

   a) $\cos A$ b) $\tan A$ c) $\sin 2A$ d) $\cos 2A$

9. Suppose that $\sin B = \frac{1}{3}$ and B is not in the first quadrant. Compute the exact value of each of the following.

   a) $\sec B$ b) $\tan 2B$ c) $\cos 2B$ d) $\tan\left(B - \frac{\pi}{4}\right)$

10. The expression $\dfrac{2\sin x}{\cos x - \sin x \tan x}$ is equivalent to which of the following?

    A) $\tan 2x$ B) $\cot 2x$ C) $\tan x$ D) $\cot x$ E) $\sec x$

11. Compute the exact value of $\dfrac{\tan\left(\frac{4\pi}{3}\right) - \tan\left(\frac{\pi}{12}\right)}{1 + \tan\left(\frac{4\pi}{3}\right)\tan\left(\frac{\pi}{12}\right)}$. (Hint: there is an easy way and also a difficult way to do this.)

12. Graph each of the following functions.

a) $f(x) = \log_2 x$
b) $f(x) = \log_{1/2} x$
c) $f(x) = x^2 + 6x + 5$
d) $f(x) = \dfrac{1}{x}$
e) $f(x) = \sqrt[3]{x}$
f) $f(x) = \sqrt{4 - x^2}$
g) $f(x) = -|x|$
h) $f(x) = \left(\dfrac{1}{2}\right)^x$
i) $f(x) = \cos x$
j) $f(x) = \tan x$

13. Find the value of $\tan\alpha$ if we know that $\tan\beta = \dfrac{2}{3}$ and $\tan(\alpha + \beta) = \dfrac{11}{10}$.

14. Prove that $\log_{3/4}\left(\dfrac{15}{8}\right) = \dfrac{\ln 3 + \ln 5 - 3\ln 2}{\ln 3 - 2\ln 2}$.

15. Find the exact value of each of the following expressions.

a) $\sin\left(-\dfrac{5\pi}{2}\right) - \cos\left(\dfrac{7\pi}{3}\right) + \tan\left(\dfrac{3\pi}{4}\right) - \cos(7\pi)$
b) $\log_{10}(0.01) + \log_8\left(\dfrac{1}{2}\right) - \ln\left(\dfrac{1}{e^5}\right)$
c) $\cos 420^\circ - \tan 210^\circ + \sec 240^\circ + \cot 135^\circ$
d) $\cos 100^\circ \cos 40^\circ + \sin 100^\circ \sin 40^\circ$

16. Simplify each of the following.

a) $\ln\left(e^{-5}\right)$
b) $e^{-\ln 3}$
c) $3^{\log_9 A}$
d) $\log_m\left(m^4\right)$
e) $\log_{27} 9$
f) $\ln\left(-e^3\right)$
g) $\ln\left(e^{-3}\right)$
h) $9^{\log_3 7}$
i) $e^{3\ln 2}$
j) $\ln 1$
k) $25^{\log_5 10}$
l) $5^{\log_{25} 10}$
m) $2^{\log_8 x}$
n) $\log_5\left(5^b\right)$
o) $\log_5\left(125^m\right)$
p) $\log_2\left[\left(\dfrac{1}{8}\right)^p\right]$
q) $\log_{16}\left(2^x\right)$

17. Simplify each of the following.

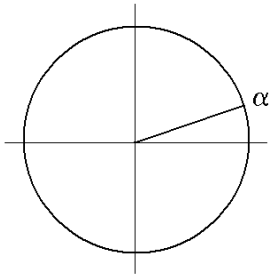
a) $\log_{10} 2 + \log_{10} 5$
b) $\log_6 180 - \log_6 5$
c) $\log_2 24 - \log_2 3$
d) $\log_{10} 40 - 2\log_{10} 2$
e) $\log_3 18 + \log_3 24 - 4\log_3 2$
f) $\log_5 0.4 + \log_5 2.5$

18. Write each of the following as a single logarithm. Assume that all variables represent positive numbers.

a) $\log_{10} a + 2\log_{10} b$
b) $\dfrac{1}{3}\log_2 a - 3\log_2 b$
c) $1 + \log_3 2$
d) $2 + \log_5 3$
e) $\dfrac{1}{2} - \log_7 x$
f) $2 + \ln 3 + \ln x - 2\ln y$

19. a) Compute the area of the triangle determined by the points $A(5, 2)$, $B(10, 2)$, and $C(10, 6)$.
b) Compute the area of the triangle determined by the points $A(\log_2 x, \log_2 y)$, $B(\log_2 8x, \log_2 y)$, and $C(\log_2 x, \log_2 4y)$.

20. The picture shows an angle α on the unit circle. Draw each of the following angles in the same circle.
a) $-\alpha$ b) $\alpha - 180^\circ$ c) $\alpha + 90^\circ$



21. Simplify $\log_3 6 - \log_9 12$

22. Find the radius of the circle in which a sector subtended by a central angle of 50° has area $14\,\text{cm}^2$.

23. Simplify each of the following. (i.e. write it in terms of trigonometric functions of α.)

a) $\sin(90^\circ - \alpha)$ d) $\sin(\alpha + 180^\circ)$ g) $\cos(-\alpha)$ j) $\tan(180^\circ - \alpha)$

b) $\sin(180^\circ - \alpha)$ e) $\cos(90^\circ - \alpha)$ h) $\cos(\alpha + 180^\circ)$ k) $\tan(-\alpha)$

c) $\sin(-\alpha)$ f) $\cos(180^\circ - \alpha)$ i) $\tan(90^\circ - \alpha)$ l) $\tan(\alpha + 180^\circ)$

24. Find all angles β so that twice β is coterminal with 120°. Express your answer

a) in degrees b) in radians

c) Find all coterminal angles β such that $-500^\circ < \beta < 500^\circ$.

25. Solve each of the following.

a) $\dfrac{3x+1}{x-10} \le 2$ b) $\sqrt{2x+5} - \sqrt{x+6} = 1$

26. Solve each of the following equations.

a) $\tan x = -\dfrac{1}{\sqrt{3}}$ c) $\sin x = 2\sin x \cos x$ e) $\cos x - 2\sin^2 x = 1$

b) $\sin^2 x = \dfrac{1}{2}$ d) $\sin x + 2\cos^2 x = 1$ f*) $\tan x = \tan 2x$

27. Solve each of the following equations for x. Present the exact value of each solution.

a) $2^{3x-1} = 4$ b) $3^{2x-1} = 5$ c) $e^{t-2} = 5$ d) $2 + 5e^{x-1} = 2012$

28. Solve each of the following equations.

a) $\log_2(x-1) + \log_2(x-5) = 5$

b) $\log_6(3-x) + \log_6(-x-2) = 2$

c) $\log_2(x+1) + \log_2(x-1) = 3$

d) $\log_5(x-16) - \log_5(4x+20) = -2$

e) $\log_2(3x+6) - \log_2(x+7) = 3$

f) $\log_2(1-x) + \log_2(9-x) = 7$

g) $\log_6(x-3) + \log_6 3 + \log_6(x+1) = 2$

h) $\log_2(x-5) - \log_2(2x-14) = -2$

i*) $\log_{x-3}(x+20) \cdot \log_x(x-3) = 2$

29. Compute the exact value of $\dfrac{1-\tan 15^\circ}{1+\tan 15^\circ}$. (Hint: there is a really smart way to do this that involves very little computation.)

30. Compare the domains of $f(x) = \log_3(x^2-4)$ and $g(x) = \log_3(x+2) + \log_3(x-2)$.

31. Graph each of the following pairs of functions together, in the same coordinate system.

a) $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$ b) $f(x) = 2^x$ and $g(x) = \left(\dfrac{1}{2}\right)^x$ c) $f(x) = 2^x$ and $g(x) = \log_2 x$

32. Find the inverse of each of the given functions.

a) $f(x) = 2x + 5$ b) $f(x) = 3\sqrt[5]{x-1} + 2$ c) $f(x) = \dfrac{8x+1}{3x-2}$ d) $f(x) = 5 - \log_3(2x-1)$

33. *Which is greater: $2^{\log_{10} 3}$ or $3^{\log_{10} 2}$?

34. *Solve each of the following.

a) $\sqrt{x^2-3x+8} + 3x = x^2 - 22$ b) $\sqrt{x^2-9} - x = 3 - \sqrt{9-x^2}$ c) $5^{\log_2(x-1)} + (x-1)^{\log_2 5} = 50$

Answers

1. a) $\cos\alpha = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$ $\tan\alpha = -\frac{\sqrt{2}}{4}$ b) $\sin\beta = -\frac{4\sqrt{17}}{17}$ $\cos\beta = -\frac{\sqrt{17}}{17}$
   c) $\sin\gamma = \frac{\sqrt{21}}{5}$ $\cos\gamma = -\frac{2}{5}$ $\tan\gamma = -\frac{\sqrt{21}}{2}$ d) $-\frac{12}{13}$

2. a) $7x$ b) $x + x^2$ 3. It will happen sometime during the 11th year. $\log_{1.07} 2 = \frac{\ln 2}{\ln 1.07} \approx 10.\,244\,77$

4. a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ b) $\frac{\sqrt{6}-\sqrt{2}}{4}$ c) $\sqrt{3}+2$

5. a) $\tan x + \frac{\cos x}{1+\sin x} = \sec x$

$$\begin{aligned} \text{RHS} &= \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x\,(1+\sin x) + \cos^2 x}{\cos x\,(1+\sin x)} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x + 1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \sec x = \text{LHS} \end{aligned}$$

   b) $\frac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} \\ &= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = \text{RHS} \end{aligned}$$

   c) $(\sin x - \cos x)^2 = 1 - \sin 2x$

$$\text{LHS} = (\sin x - \cos x)^2 = \sin^2 x + \cos^2 x - 2\sin x\cos x = 1 - \sin 2x = \text{RHS}$$

6. a) $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \sin\frac{\pi}{2}\cos\alpha - \cos\frac{\pi}{2}\sin\alpha = 1\cdot\cos\alpha - 0\cdot\sin\alpha = \cos\alpha$$

   b) $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0\cdot\cos\alpha + 1\cdot\sin\alpha = \sin\alpha$$

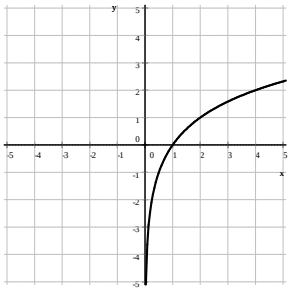
   c) $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$ We can not use the difference formula for tangent here, because $\tan\frac{\pi}{2}$ is undefined. Instead, we need to use a compound angle formula separately for sine and cosine (see parts a and b).

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\cos\left(\frac{\pi}{2} - \alpha\right)} = \frac{\cos\alpha}{\sin\alpha} = \cot\alpha$$

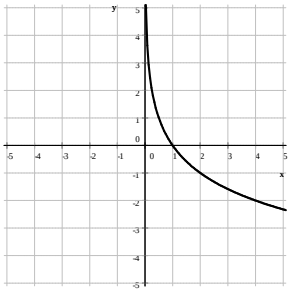
7. a) $\frac{2\sqrt{2}}{3}$ b) $\frac{\sqrt{2}}{4}$ c) $\frac{12}{13}$ d) $\frac{12}{5}$ e) $\frac{4\sqrt{2}}{9}$ f) $-\frac{119}{169}$ g) $\frac{5+24\sqrt{2}}{39}$ h) $\frac{5+24\sqrt{2}}{39}$
   i) $\frac{10\sqrt{2}-12}{39}$ j) $\frac{10\sqrt{2}+12}{39}$ k) $\frac{4\sqrt{2}}{7}$ l) $-\frac{120}{119}$

8. a) $\pm\frac{4}{5}$ b) $\pm\frac{3}{4}$ c) $\pm\frac{24}{25}$ d) $\frac{7}{25}$ 9. a) $-\frac{3\sqrt{2}}{4}$ b) $-\frac{4\sqrt{2}}{7}$ c) $\frac{8}{9}$ d) $-\frac{1}{2}$ 10. A 11. 1

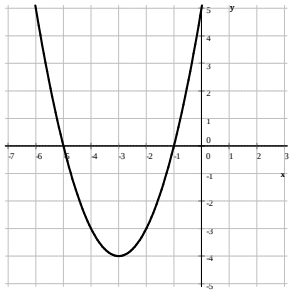
12. a) $f(x)=\log_2 x$



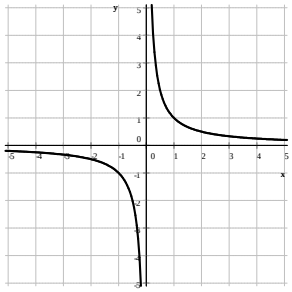
b) $f(x)=\log_{1/2} x$



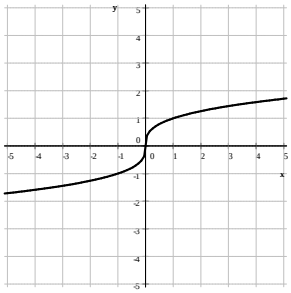
c) $f(x)=x^2+6x+5$



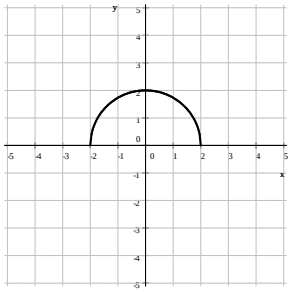
d) $f(x)=\frac{1}{x}$



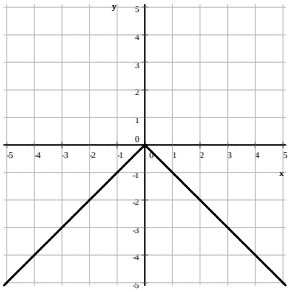
e) $f(x)=\sqrt[3]{x}$



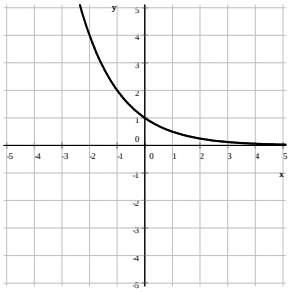
f) $f(x)=\sqrt{4-x^2}$



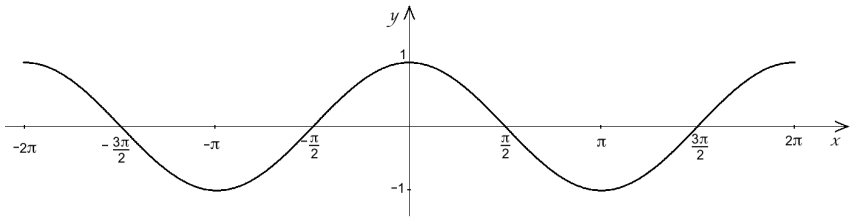
g) $f(x)=-|x|$



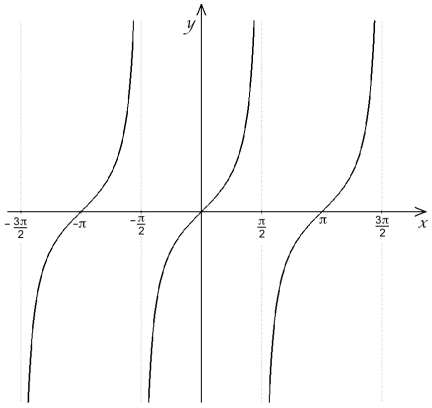
h) $f(x)=\left(\frac{1}{2}\right)^x$



i) $f(x)=\cos x$



j) $f(x)=\tan x$



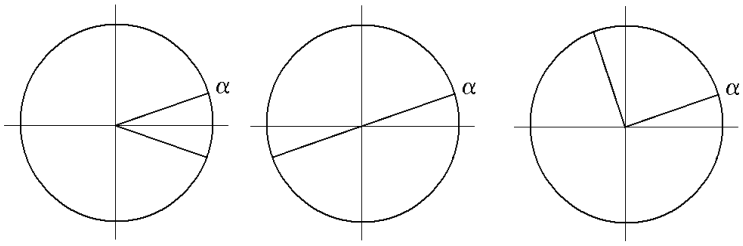
13. $\frac{1}{4}$ 14. $\log_{3/4}\left(\frac{15}{8}\right)=\dfrac{\ln\left(\frac{15}{8}\right)}{\ln\left(\frac{3}{4}\right)}=\dfrac{\ln 15-\ln 8}{\ln 3-\ln 4}=\dfrac{(\ln 3+\ln 5)-\ln\left(2^3\right)}{\ln 3-\ln\left(2^2\right)}=\dfrac{\ln 3+\ln 5-3\ln 2}{\ln 3-2\ln 2}$

15. a) $-\frac{3}{2}$ b) $\frac{8}{3}$ c) $-\frac{\sqrt{3}}{3}-\frac{5}{2}$ d) $\frac{1}{2}$

16. a) -5 b) $\frac{1}{3}$ c) $\sqrt{A}$ d) 4 e) $\frac{2}{3}$ f) undefined g) -3 h) 49 i) 8 j) 0 k) 100 l) $\sqrt{10}$ m) $\sqrt[3]{x}$ n) b o) $3m$ p) $-3p$ q) $\frac{1}{4}x$ 17. a) 1 b) 2 c) 3 d) 1 e) 3 f) 0

18. a) $\log_{10} ab^2$ b) $\log_2\left(\frac{\sqrt[3]{a}}{b^3}\right)$ c) $\log_3 6$ d) $\log_5 75$ e) $\log_7\left(\frac{\sqrt{7}}{x}\right)$ f) $\ln\left(\frac{3e^2x}{y^2}\right)$

19. a) $10\ \text{unit}^2$ b) $3\ \text{unit}^2$ 20. a) b) c)



21. $\frac{1}{2}$ 22. $\sqrt{\frac{504}{5\pi}}\,\text{cm} \approx 5.664\,42\,\text{cm}$

23. a) $\cos\alpha$ b) $\sin\alpha$ c) $-\sin\alpha$ d) $-\sin\alpha$ e) $\sin\alpha$ f) $-\cos\alpha$ g) $\cos\alpha$ h) $-\cos\alpha$ i) $\cot\alpha$ j) $-\tan\alpha$ k) $-\tan\alpha$ l) $\tan\alpha$

24. a) $60° + k \cdot 180°$ where $k \in \mathbb{Z}$ b) $\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$ c) $-480°, -300°, -120°, 60°, 240°, 420°$

25. a) $[-21, 10)$ b) 10 (-2 does not work)

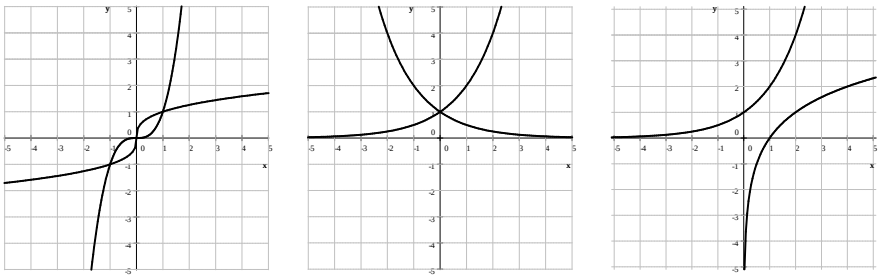
26. a) $-\frac{1}{6}\pi + k\pi$ where $k \in \mathbb{Z}$ b) $\frac{\pi}{4} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$ c) $k\pi$ or $\pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$
d) $x = \frac{\pi}{2} + 2k\pi$ or $x = -\frac{\pi}{6} + 2k\pi$ or $x = -\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$
e) $x = 2k\pi$ where $k \in \mathbb{Z}$ f) $x = k\pi$ where $k \in \mathbb{Z}$

27. a) 1 b) $\frac{1}{2}(1 + \log_3 5)$ or $\log_9 15$ c) $2 + \ln 5$ d) $1 + \ln 402$

28. a) 9 (-3 does not work) b) -6 (7 does not work) c) 3 (-3 does not work) d) 20 e) no solution
f) -7 (16 does not work) g) 5 (-3 does not work) h) no solution (3 does not work) i) 5

29. $\frac{\sqrt{3}}{3}$ 30. The domain of f is $(-\infty, -2) \cup (2, \infty)$ and the domain of g is $(2, \infty)$

31. a) b) c)



32. a) $f^{-1}(x) = \frac{1}{2}x - \frac{5}{2}$ b) $f^{-1}(x) = \left(\frac{x-2}{3}\right)^5 + 1$ c) $f^{-1}(x) = \frac{2x+1}{3x-8}$ d) $f^{-1}(x) = \frac{1}{2}\left(3^{5-x} + 1\right)$

33. Hint: take the common logarithm of both sides and compare.

34. a) $-4, 7$ b) -3 c) 5