1. Solve each of the given triangles

   a) $b = 16\,\text{ft}$, $\alpha = 38°$, $\beta = 83°$ b) $a = 4\,\text{cm}$, $b = 7\,\text{cm}$, $\alpha = 58°$ c) $a = 6\,\text{in}$, $b = 4\sqrt{3}\,\text{in}$, $\alpha = 60°$

2. Suppose that $\underline{v} = 2\underline{i} - 3\underline{j}$ and $\underline{w} = -\underline{i} + 2\underline{j}$. Compute each of the following.

   a) $\|\underline{v}\|$ b) $\|\underline{w}\|$ c) $\underline{v} + \underline{w}$ d) $\|\underline{v} + \underline{w}\|$ e) $5\underline{w} + 3\underline{v}$ f) $\underline{v} + 2\underline{w}$ g) $\|\underline{v} + 2\underline{w}\|$

   h) Can you find real numbers A and B such that $A\underline{v} + B\underline{w} = 5\underline{i} - \underline{j}$?

3. Compute each of the given sums.

   a) $\displaystyle\sum_{k=1}^{60} \left(k^2 + 6k + 9\right)$ b) $\displaystyle\sum_{k=4}^{63} k^2$

   c) Can you explain the connection between your results in parts a) and b)?

4. Find $\tan x$ if we know that $\tan 2x = \dfrac{48}{55}$.

5. Solve each of the following inequalities.

   a) $\dfrac{2x-5}{3x+1} < -2$ b) $2x + \dfrac{1}{3}x^2 \leq -3$ c) $\dfrac{3}{x-1} \leq \dfrac{x}{2}$ d) $\dfrac{2}{x+1} \leq \dfrac{3}{4}$ e) $\dfrac{1}{x} \geq 2$

6. Solve each of the following equations.

   a) $\sin 2x = \cos x$
   b) $\cos 2x = \sin x$
   c) $\sin 2x = \sin x$
   d) $\cos 2x = \cos x$
   e) $5 \cdot 2^{3x-2} = 7 \cdot 3^{2x-1}$
   f) $2^{x+2} - 5 = 2^{x+3} + 1$
   g) $3^{x+1} + 13 = 3^{x+2} - 17$
   h) $4^x - 2^{x+3} = -7$

7. a) Find all solutions of the equation $2 - 3\sin 4x = \cos 8x$. Present your answer in degrees.
   b) Present the final answer in radians.
   c) List all solutions that fall between $0°$ and $360°$. Use degrees.

8. Find the domain for each of the following functions.

   a) $f(x) = \dfrac{1}{\log_2 x + \log_2 (x-4)}$ b) $g(x) = \sec x$ c) $f(x) = \dfrac{2x-3}{x^2+1}$ d) $g(x) = \sqrt{\dfrac{x}{x+1}}$

9. Find an equation for all tangent lines drawn to the given functions.

   a) to $f(x) = -\dfrac{1}{2}x^2 - 3x + 7$ from the point $(-1, 14)$

   b) to $f(x) = \dfrac{1}{2}x^2 + x - 1$ from the point $(-1, -6)$

10. Suppose that $f(x) = x^2 - 3x + 1$ and $g(x) = -2x + 1$. Compute each of the following.

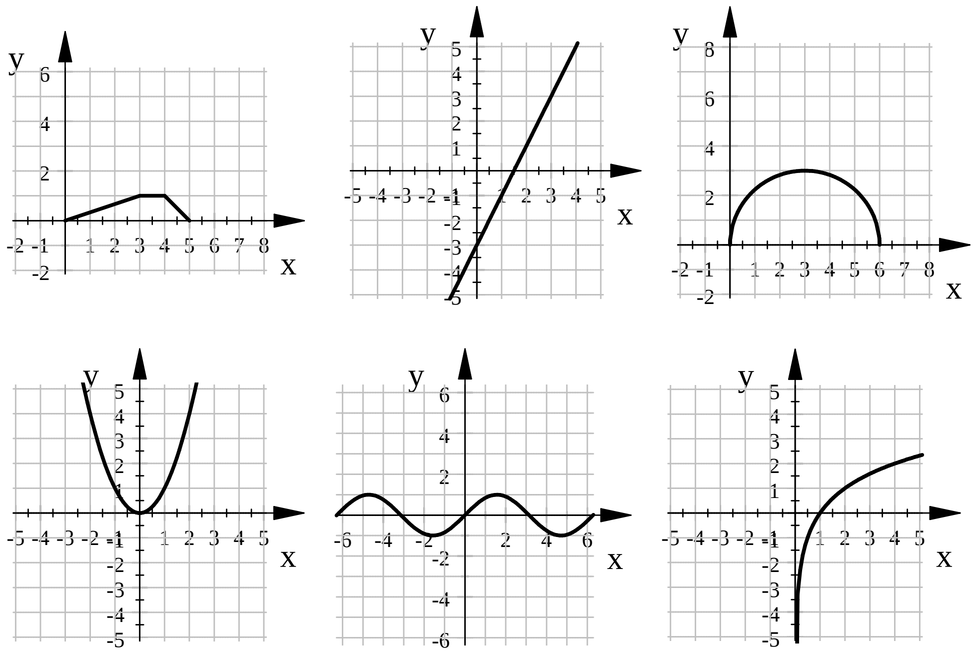
    a) $f(g(-1))$ b) $g(f(-1))$ c) $f(2a)$ d) $g(2a)$ e) $f(g(x))$ f) $g(f(x))$

    g) Is there any value of x for which $f(g(x)) = g(f(x))$?

11. Compute the inverse for each of the following functions.

    a) $f(x) = \dfrac{2}{3}x - 6$
    b) $f(x) = \sqrt[3]{2x+1}$
    c) $f(x) = (2x-3)^3 + 8$
    d) $f(x) = \dfrac{3x-1}{7x-5}$
    e) $f(x) = \dfrac{7x+10}{3x-7}$
    f) $f(x) = e^{4x-1} - 3$
    g) $f(x) = \log_3 (5x-4)$

12. Given the graph of a function f, graph the inverse relation f^{-1} in the same coordinate system.



13. If we take Q amount of a certain medication, the amount of it in our system, t hours after intake is

$$A(t) = Q\left(\frac{3}{4}\right)^{0.8t}$$

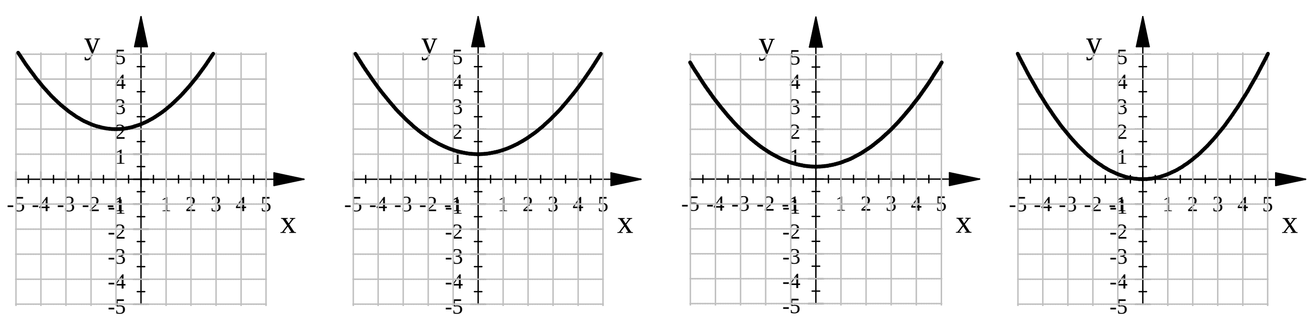
a) Approximately what percent of the medication is in our system 2 hours after taking it?
b) How long until we have only 20% left in our system?
c) How long until we have only 1% left in our system?

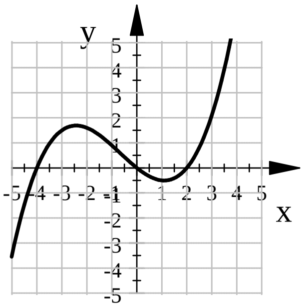
14. The number of cells in a sample at time t (measured in hours) is $N(t) = 20\,000\left(1.2^{0.5t}\right)$. How long will it take for the sample to double

a) from the amount at $t = 0$ b) from the amount at $t = 3$

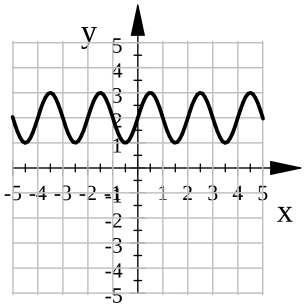
15. Graph $y = \dfrac{1}{f(x)}$ given the graph of $y = f(x)$.

a) b) c) d)

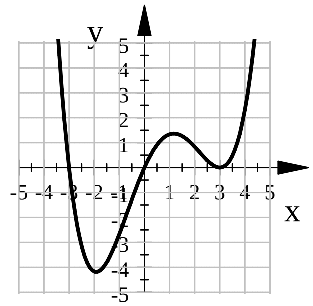




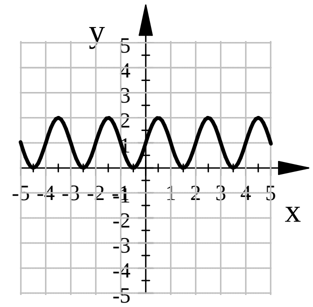
e)



f)



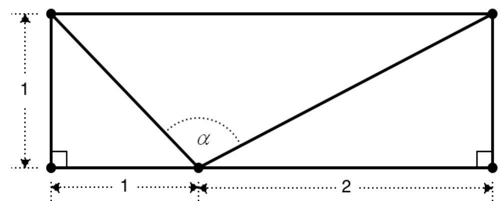
g)



h)

16. Graph each of the following functions on the intervals indicated.
    a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$
    b) $g(x) = \csc x$ on $[-2\pi, 2\pi]$
    c) $h(x) = \sqrt{9 - x^2}$ on its natural domain.
    d) $m(x) = \dfrac{1}{\sqrt{9 - x^2}}$ on its natural domain.

17. Compute the exact value of $\tan\alpha$ if α is the angle shown on the picture.



18. Which is greater, $143^{\log_3 2018}$ or $2018^{\log_3 143}$?

19. Suppose that $x = \log_2 3$ and $y = \log_3 10$. Write $\log_6 20$ in terms of x and y.

20. Suppose that x is an angle belonging to the second quadrant and y is an angle belonging to the fourth quadrant. We also know that $\sin x = \dfrac{2}{3}$ and $\cos y = \dfrac{3}{5}$. Compute each of the following.
    a) $\cos x$
    b) $\sin y$
    c) $\sin 2x$
    d) $\cos 2x$
    e) $\sin(x - y)$
    f) $\cos(x + y)$
    g) $\tan 2x$

21. Point D is on side AB of triangle ABC, with $\angle ACD = \angle BCD = 60^\circ$, $AC = 5$, and $BC = 15$. Find the length of line segment CD.

22. a) Compute $\sin\alpha$ and $\cos\alpha$ in terms of M if we know that α is an acute angle and that $\tan\alpha = M$.
    b) Compute $\sin 2\alpha$ and $\cos 2\alpha$ in terms of M.

23. Redo the previous problem if α is in the second quadrant and $\tan\alpha = M$.

24. Compute $\sin\beta$ and $\cos\beta$ in terms of T if we know that β is an angle in the fourth quadrant and that $\tan\beta = T$.

25. Sketch the graph of each of the following functions.
    a) $f(x) = -(x+3)^2(x+1)$
    b) $f(x) = (x+3)^2(x+1)(x-1)(x-3)^2$
    c) $f(x) = 8x^2 - 2x^4$

26. Sketch the graph of each of the following.
    a) $f(x) = x - 2$
    b) $g(x) = (x-2)^2$
    c) $h(x) = (x-2)^3$
    d) $k(x) = (x-2)^4$
    e) $f(x) = \dfrac{1}{x-2}$
    f) $g(x) = \dfrac{1}{(x-2)^2}$
    g) $h(x) = \dfrac{1}{(x-2)^3}$
    h) $k(x) = \dfrac{1}{(x-2)^4}$

27. *Find an equation for both tangent lines drawn to the graph of $(x-7)^2 + (y-4)^2 = 50$ from the external point $(-3, -16)$.

Answers

1. a) $\gamma = 59°$, $a \approx 9.924\,56\,\text{ft}$, $c \approx 13.817\,671\,64\,\text{ft}$ b) no solution c) $\beta = 90°$, $\gamma = 30°$, $c = 2\sqrt{3}\,\text{in}$

2. a) $\sqrt{13}$ b) $\sqrt{5}$ c) $\underline{i} - \underline{j}$ d) $\sqrt{2}$ e) $7\underline{i} - 9\underline{j}$ f) $\underline{j}$ g) 1 h) $A = 9$ and $B = 13$

3. a) 85 330 b) 85 330 c) $k^2 + 6k + 9 = (k+3)^2$ and so $\sum_{k=1}^{60} \left(k^2 + 6k + 9\right) = \sum_{k=1}^{60} (k+3)^2$

   Let $n = k + 3$ Then $\sum_{k=1}^{60} (k+3)^2 = \sum_{n=4}^{63} n^2$

4. $\frac{3}{8}$ or $-\frac{8}{3}$

5. a) $\left(-\frac{1}{3}, \frac{3}{8}\right)$ b) $\{-3\}$ c) $[-2, 1) \cup [3, \infty)$ d) $(-\infty, -1) \cup \left[\frac{5}{3}, \infty\right)$ e) $\left(0, \frac{1}{2}\right]$

6. a) $\frac{\pi}{2} + k\pi$, $\frac{\pi}{6} + 2k\pi$, $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ b) $-\frac{\pi}{2} + 2k\pi$, $\frac{\pi}{6} + 2k\pi$, $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

   c) $k\pi$, $\pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ d) $2k\pi$, $\pm\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ e) $\log_{9/8}\left(\frac{15}{28}\right)$

   f) no solution g) $\log_3 5$ h) $0, \log_2 7$

7. a) $x = 22.5° + k \cdot 90°$ or $x = 7.5° + k \cdot 90°$ or $x = 37.5° + k \cdot 90°$ where $k \in \mathbb{Z}$

   b) $x = \frac{\pi}{8} + \frac{k\pi}{4}$ or $x = \frac{\pi}{24} + \frac{k\pi}{4}$ or $x = \frac{5\pi}{24} + \frac{k\pi}{4}$ where $k \in \mathbb{Z}$

   c) 7.5°, 22.5°, 37.5°, 97.5°, 112.5°, 127.5°, 187.5°, 202.5°, 217.5°, 277.5°, 292.5°, 307.5°

8. a) $\left\{x : x > 4 \text{ and } x \neq 2 + \sqrt{5}\right\}$ b) $\left\{x : x \neq \frac{\pi}{2} + 2k\pi \text{ where } k \in \mathbb{Z}\right\}$ c) $\mathbb{R}$ d) $(-\infty, -1) \cup [0, \infty)$

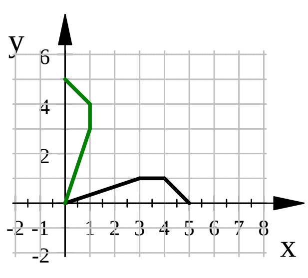
9. a) $y = -5x + 9$ and $y = x + 15$ b) $y = -3x - 9$ and $y = 3x - 3$

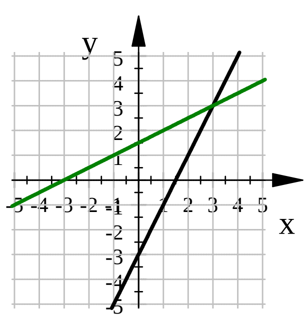
10. a) 1 b) -9 c) $4a^2 - 6a + 1$ d) $-4a + 1$ e) $4x^2 + 2x - 1$ f) $-2x^2 + 6x - 1$ g) 0 and $\frac{2}{3}$

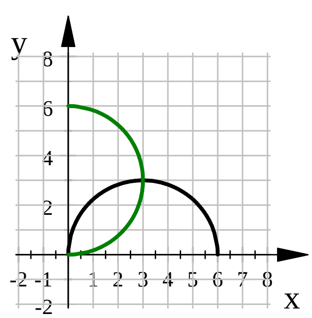
11. a) $f^{-1}(x) = \frac{3}{2}x + 9$ b) $f^{-1}(x) = \frac{1}{2}x^3 - \frac{1}{2}$ c) $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-8} + \frac{3}{2}$ d) $f^{-1}(x) = \frac{5x-1}{7x-3}$

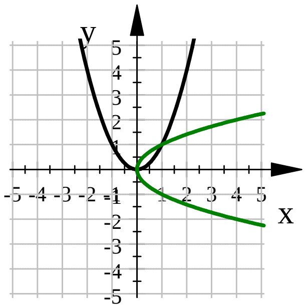
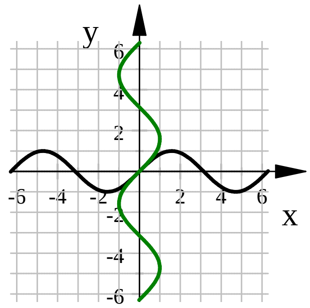
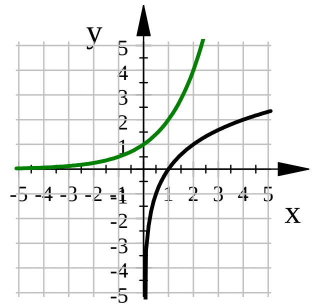
    e) $f^{-1}(x) = \frac{7x+10}{3x-7}$ f) $f^{-1}(x) = \frac{1}{4}(\ln(x+3) + 1)$ g) $f^{-1}(x) = \frac{1}{5}(3^x + 4)$

12. The green graph is the inverse.



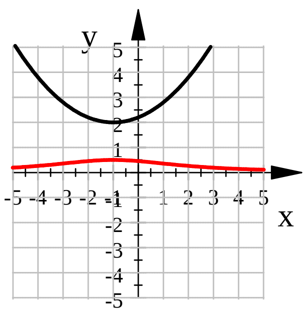
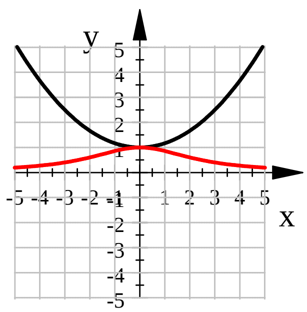
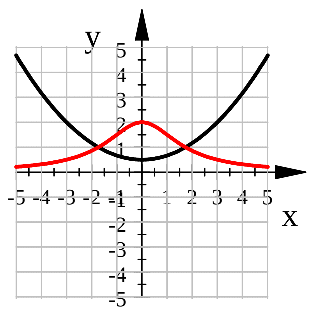
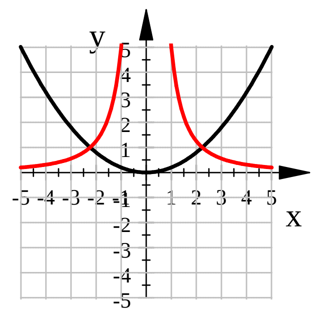


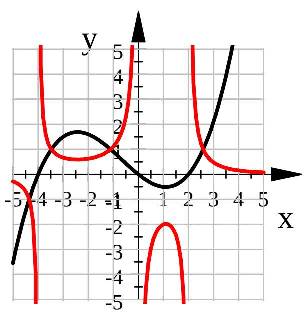
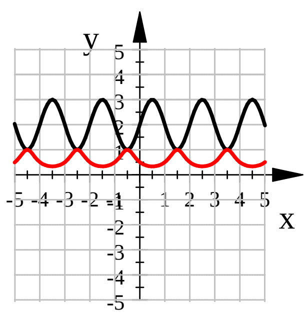
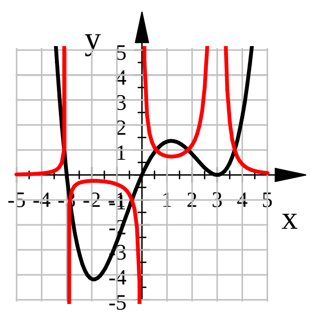
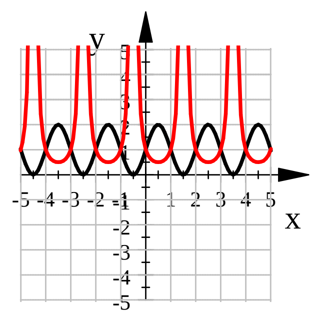


13. a) 63.11% b) 6.99 hours c) 20 hours 14. a) 7.604 hours b) 7.604 hours

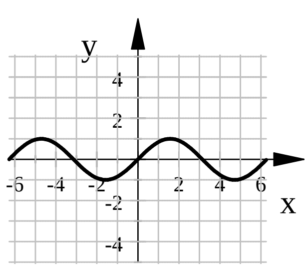
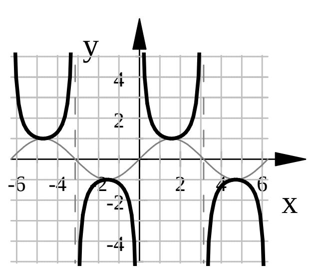
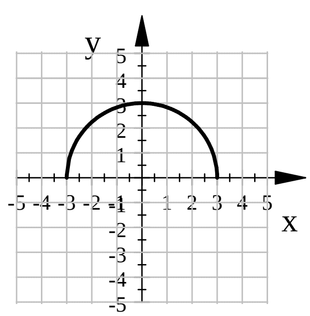
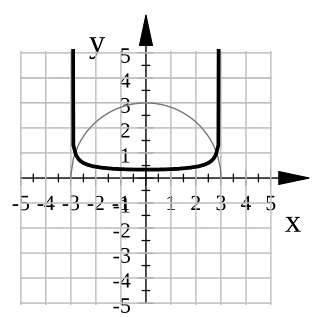
15. a) b) c) d)

e) f) g) h)

16. a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$ b) $g(x) = \csc x$ on $[-2\pi, 2\pi]$ c) $h(x) = \sqrt{9-x^2}$ on $[-3, 3]$ d) $m(x) = \dfrac{1}{\sqrt{9-x^2}}$ on $(-3, 3)$

17. -3 18. They are equal. Hint: take $\log_3$ of both expressions! 19. $\dfrac{xy+1}{x+1}$

20. a) $-\dfrac{\sqrt{5}}{3}$ b) $-\dfrac{4}{5}$ c) $-\dfrac{4\sqrt{5}}{9}$ d) $\dfrac{1}{9}$ e) $\dfrac{6-4\sqrt{5}}{15}$ f) $-\dfrac{3\sqrt{5}-8}{15}$ g) $4\sqrt{5}$

21. $\frac{15}{4}$ (Hint: Use the area formula $A = \frac{1}{2}ab\sin\gamma$ and write an equation expressing that $A_{ABC} = A_{ACD} + A_{BCD}$)

22. a) $\sin\alpha = \frac{M}{\sqrt{M^2+1}}$ and $\cos\alpha = \frac{1}{\sqrt{M^2+1}}$ b) $\sin 2\alpha = \frac{2M}{M^2+1}$ and $\cos 2\alpha = \frac{1-M^2}{M^2+1}$

23. If α is in the second quadrant, then $M < 0$. In the second quadrant, sine is positive and cosine is negative. Therefore,

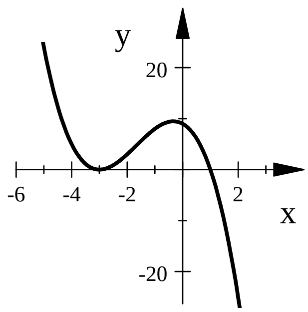
a) $\sin\alpha = \frac{|M|}{\sqrt{M^2+1}}$ and $\cos\alpha = \frac{-1}{\sqrt{M^2+1}}$ b) $\sin 2\alpha = \frac{-|M|}{M^2+1} = \frac{2M}{M^2+1}$ and $\cos 2\alpha = \frac{1-M^2}{M^2+1}$

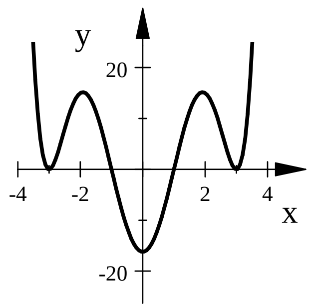
24. If β is in the fourth quadrant, then T is negative, $\sin\beta$ is negative, and $\cos\beta$ is positive.

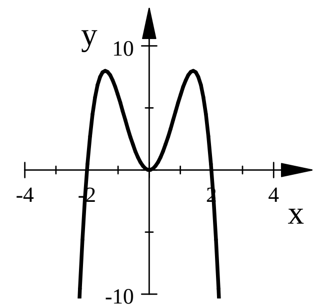
$\sin\beta = \frac{T}{\sqrt{T^2+1}}$ and $\cos\beta = \frac{1}{\sqrt{T^2+1}}$

25. a) $f(x) = -(x+3)^2(x+1)$ b) $f(x) = (x+3)^2(x+1)(x-1)(x-3)^2$ c) $f(x) = -2x^2(x+2)(x-2)$

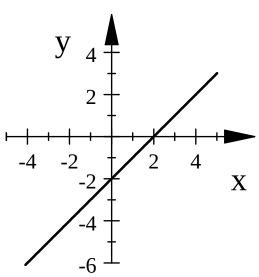
Note: Please ignore the numbers on the $y-$axis. We are not supposed to know how tall those humps are.

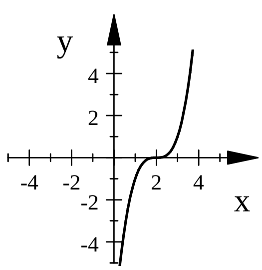


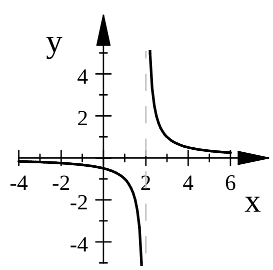


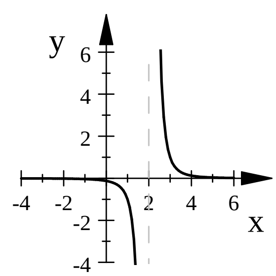


26. a) $f(x) = x - 2$ c) $h(x) = (x-2)^3$ e) $f(x) = \frac{1}{x-2}$ g) $h(x) = \frac{1}{(x-2)^3}$

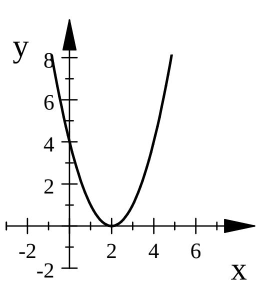


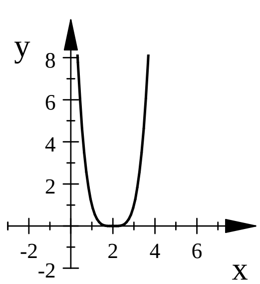


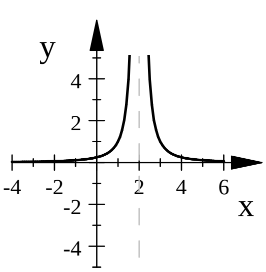


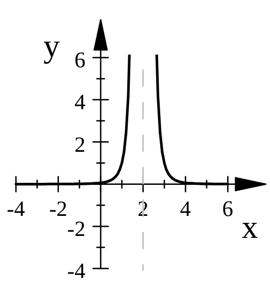


b) $g(x) = (x-2)^2$ d) $k(x) = (x-2)^4$ f) $g(x) = \frac{1}{(x-2)^2}$ h) $k(x) = \frac{1}{(x-2)^4}$









27. $y = 7x + 5$ and $y = x - 13$