1. Suppose that $\underset{\sim}{v} = 3\underset{\sim}{i} - 8\underset{\sim}{j}$ and $\underset{\sim}{w} = \underset{\sim}{i} + 2\underset{\sim}{j}$. Compute each of the following.

   a) $\|\underset{\sim}{v}\|$ b) $\|\underset{\sim}{w}\|$ c) $\underset{\sim}{v} + \underset{\sim}{w}$ d) $\|\underset{\sim}{v} + \underset{\sim}{w}\|$ e) $4\underset{\sim}{w} - \underset{\sim}{v}$ f) $\underset{\sim}{v} + 4\underset{\sim}{w}$

2. Find the approximate value of the angle that is formed between $\underset{\sim}{v} = 3\underset{\sim}{i} - 8\underset{\sim}{j}$ and the positive part of the $x-$axis.

3. The number of cells in a sample at time t (measured in hours) is $N(t) = 20\,000\left(1.2^{0.5t}\right)$.

   a) How many cells are in the sample at $t = 0$?

   b) How long will it take for the sample to double from the amount that it had at $t = 0$?

   c) How many cells are in the sample at $t = 6$?

   d) How long will it take for the sample to double from the amount that it had at $t = 6$?

   e) What do you observe? Can we make (and perhaps prove) a general statement?

4. Perform each of the following operations on the complex numbers given.

   a) $(3-i)^2$ b) $(3-i)^3$ c) $\dfrac{9+37i}{1-7i}$ d) $(1-i)^8$ e) i^{143} f) $\dfrac{1+i}{1-i}$ g) $\dfrac{3-2i}{1+i} \cdot \dfrac{3+2i}{1-i}$ h) $\dfrac{3-2i}{1+i} + \dfrac{3+2i}{1-i}$ i) $(1-3i)^4(1+3i)^4$

5. Simplify each of the following.

   a) $\left(\dfrac{8}{\sqrt{7}+\sqrt{3}} + \dfrac{12}{\sqrt{7}-\sqrt{3}}\right)\left(5\sqrt{7}-\sqrt{3}\right)$

   b) $\sqrt[3]{a\sqrt[3]{a\sqrt[3]{a}}}$

   c) $\dfrac{1}{\log_2 120} + \dfrac{1}{\log_3 120} + \dfrac{1}{\log_4 120} + \dfrac{1}{\log_5 120}$

   d) $1 - 2 + 3 - 4 + 5 - 6 + \ldots - 142 + 143$

   e) $\dfrac{\cos 35°}{\sin 20° \cos 35° + \cos 20° \sin 35°}$

   f) $\log_{\sin 45°} 2$

   g) $\log_{\sqrt{3}}\left(18x^2\right) - \log_{\sqrt{3}}\left(20x^3\right) + \dfrac{1}{2}\log_{\sqrt{3}}\left(900x^2\right)$

   h) $\sin\left(\dfrac{2015\pi}{6}\right)$

   i) $\dfrac{1 - \tan 75°}{1 + \tan 75°}$

   j) $\dfrac{\log_3 54 - \frac{1}{2}\log_3 108}{\log_2 12 - \frac{1}{2}\log_2 72}$

   l) $\cos 75° \sin 75°$

   m) $\left(\sin\left(\dfrac{11\pi}{4}\right) - \cos\left(\dfrac{11\pi}{4}\right)\right)^{10}$

6. Solve each of the following equations.

   a) $2 \cdot 3^{2x-1} = 5 \cdot 4^{x+1}$

   b) $\sqrt{x-4} + \sqrt{2x-1} = 4$

   c) $2^{2x-3} = 5^{3-x}$

   d) $2^{x+1} - 5 \cdot 2^{x-1} = -3$

   e) $2 \cdot 4^{x+1} - 65 \cdot 2^x + 8 = 0$

   f) $1 - \cos x = 2\sin^2 x$

   g) $\cos x = \cos x \sin x$

   h) $\tan^3 x = 3\tan x$

   i) $\log_2(x+1) + \log_2(x+5) = 5$

   j) $\log_x(-x+6) = 2$

   k) $\sin 5x = \cos 10x$

   l) $\sin 3x - \cos 3x = \sqrt{2}$

7. Prove that $\dfrac{\sqrt{2}+\sqrt{6}}{2} = \sqrt{\sqrt{3}+2}$ by squaring both sides.

8. Find an equation for all tangent lines drawn to the graph of $y = \dfrac{1}{2}x^2 - 5x + 8$ from the point $P(2, -8)$.

9. Compute each of the following sums.

   a) $\dfrac{1}{143} + \dfrac{2}{143} + \dfrac{3}{143} + \ldots + \dfrac{142}{143}$ b) $\displaystyle\sum_{k=0}^{100}\left(k^2 - 2k + 1\right)$

10. Find the domain for each of the following functions.

a) $f(x) = \sin^{-1} x$

b) $f(x) = \sqrt{9 - x^2}$

c) $f(x) = \dfrac{x^2 - 4}{3 - \log_2(x+1)}$

d) $f(x) = \tan x$

e) $f(x) = \dfrac{\ln(x+4)}{\ln(x-5)}$

f) $g(x) = \ln\left(\dfrac{x+4}{x-5}\right)$

g) $h(x) = \dfrac{1}{\sin x - \cos x}$

h) $h(x) = \dfrac{\cos^4 x - \sin^4 x}{\sin 2x}$

11. Graph each of the pair of functions in the same coordinate system.

a) $f(x) = \sin x$ and $g(x) = \csc x$ b) $f(x) = \sin x$ and $g(x) = 3\sin x$ c) $f(x) = \cos x$ and $g(x) = \cos x + 1$

12. Graph $f(x) = \sin^{-1} x$. State its domain and range.

13. Solve each of the following triangles.

a) $a = 3\,\text{ft}$, $b = 7\,\text{ft}$, and $\alpha = 29°$

b) $a = 6\,\text{ft}$, $b = 7\,\text{ft}$, and $\alpha = 29°$

c) $a = 10\,\text{ft}$, $b = 7\,\text{ft}$, and $\alpha = 29°$

d) $a = 5\,\text{ft}$, $b = 12\,\text{ft}$, $c = 8\,\text{ft}$

e) $a = 17$, $b = 13$, and $\beta = 20°$

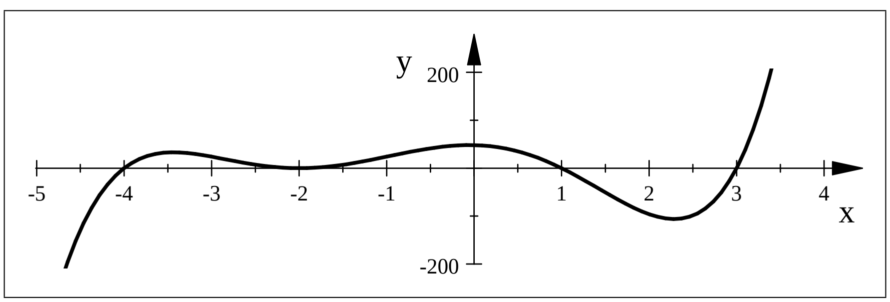
f) $a = 17$, $b = 13$, and $\alpha = 20°$

g) $a = 17$, $b = 13$, and $\gamma = 20°$

14. Sketch the graph of each of the following.

a) $f(x) = -2\left(x^5 - x^3\right)$ b) $g(x) = x\left(x^2 - 1\right)\left(4 - x^2\right)^2$

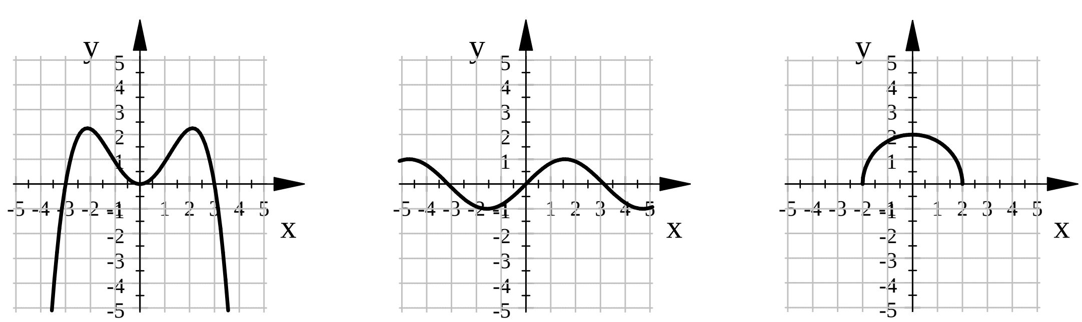
15. Find a possible equation for the function whose graph is shown. Assume the function is a polynomial.



16. Classify the discontinuities of each of the following functions as a hole or a vertical asymptote.

a) $f(x) = \dfrac{x^3 - x}{x^2(x-1)}$ b) $g(x) = \dfrac{(x+3)^2(x+1)^3 x^4 (x-2)^5 (x-5)^8}{(x+3)^3(x+1)^2 x^4 (x-2)^8 (x-5)^9}$

17. Given the graph of a function $y = f(x)$, graph $y = \dfrac{1}{f(x)}$ in the same coordinate system.



18. Given the same graphs as in the previous problem, graph the inverse relation in the same coordinate system.

19. Find the exact value of the cosine of the smallest angle in a triangle with sides 3, 5, and 6.

20. Find the **exact value** of the area of a triangle with sides 2, 3, and 4.

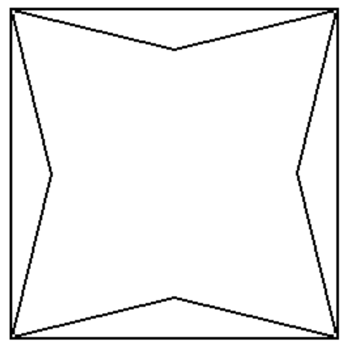
21. Two sides of a triangle are 8 ft and 15 ft long. Find the exact value of the third side if we know that the area of the triangle is $48\,\text{ft}^2$.

22. Triangle SML has sides of length 6, 7, and 8. Find the exact value of $\cos S + \cos M + \cos L$.

23. The population of a town is growing exponentially. From 2007, it took A years for the population to double. From 2007, it took B years for the population to triple. Express B in terms of A.

24. Find the inverse for each of the following functions.

a) $f(x) = \ln(2x-1)$ b) $f(x) = \dfrac{2}{3-x}$ c) $f(x) = 5(x-4)^3$



25. Consider a square with sides 1 unit long. To the inside of each side, we draw an isosceles triangle with its greatest angle, opposite the unit long base, measures 150°. Consider all vertices of these triangles that are not on the square. If we connect these vertices, we obtain a square. Compute the exact value of the area of this square.

26. a) How many 5−digit numbers are there?
b) How many 5−digit numbers are there with all of its digits being different?
c) How many 5−digit numbers are there with at least one digit repeating?

27. Graph each of the following.

a) $f(x) = x-2$

b) $f(x) = (x-2)^2$

c) $f(x) = (x-2)^3$

d) $f(x) = \dfrac{1}{x-2}$

e) $f(x) = \dfrac{1}{(x-2)^2}$

f) $f(x) = \dfrac{1}{(x-2)^3}$

g) $f(x) = \dfrac{(x-2)^5}{(x-2)^7}$

h) $f(x) = \dfrac{(x-2)^2}{(x-2)^5}$

i) $f(x) = \dfrac{(x-2)^6}{(x-2)^6}$

j) $f(x) = \dfrac{(x-2)^5}{(x-2)^4}$

k) $f(x) = \dfrac{(x-2)^7}{(x-2)^5}$

l) $f(x) = \dfrac{(x-2)^5}{(x-2)^2}$

Answers

1. a) $\sqrt{73}$ b) $\sqrt{5}$ c) $4\underline{i} - 6\underline{j}$ d) $\sqrt{52}$ e) $\underline{i} + 16\underline{j}$ f) $7\underline{i}$

2. $\tan^{-1}\left(-\frac{8}{3}\right) \approx 69.443955°$

3. a) 20 000 b) 7.603 568 hours c) 34 560 d) $\frac{\ln 2\left(1.2^3\right)}{0.5 \ln 1.2} - 6 \approx 7.603\,568$ hours

   e) For any time t, we will need to wait until $t + \frac{2 \ln 2}{\ln 1.2}$ for the sample to double. So, the doubling time is independent of t and is constant. Let t_1 be any time and t_2 is the time when the amount is doubled from $A(t_1)$. In other words, $A(t_2) = 2A(t_1)$.

$$\begin{aligned} A(t_2) &= 2A(t_1) \\ 20\,000\left(1.2^{0.5t_2}\right) &= 2 \cdot 20\,000\left(1.2^{0.5t_1}\right) \\ 1.2^{0.5t_2} &= 2\left(1.2^{0.5t_1}\right) \\ \frac{1.2^{0.5t_2}}{1.2^{0.5t_1}} &= 2 \\ 1.2^{0.5(t_2-t_1)} &= 2 \\ 0.5(t_2 - t_1)\ln 1.2 &= \ln 2 \\ t_2 - t_1 &= \frac{\ln 2}{0.5 \ln 1.2} = \frac{2 \ln 2}{\ln 1.2} \approx 7.603\,568 \text{ hours} \end{aligned}$$

4. a) $8 - 6i$ b) $18 - 26i$ c) $-5 + 2i$ d) 16 e) $-i$ f) i g) $\frac{13}{2}$ h) 1 i) 10 000

5. a) 172 b) $\sqrt[27]{a^{13}}$ c) 1 d) 72 e) 1 f) -2 g) 6 h) $-\frac{1}{2}$ i) $-\frac{\sqrt{3}}{3}$ j) 3 k) $\frac{1}{4}$ l) 32

6. a) $\log_{9/4}(30)$ b) 5 c) $3\log_{20} 10$ d) $\log_2 6$ e) ± 3 f) $2k\pi$, $\pm\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

   g) $\frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$ h) $k\pi$, $\pm\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$ i) 3 j) 2

   k) $-\frac{\pi}{10} + \frac{2\pi k}{5}$ $\frac{\pi}{60} + \frac{2\pi k}{5}$ $\frac{5\pi}{60} + \frac{2\pi k}{5}$ where $k \in \mathbb{Z}$ l) $\frac{\pi}{4} + \frac{2k\pi}{3}$ where k$\in \mathbb{Z}$

7. Claim: $\frac{\sqrt{2}+\sqrt{6}}{2} = \sqrt{\sqrt{3}+2}$

   Proof: We square both expressions.

$$\begin{aligned} \text{LHS}^2 &= \left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)^2 = \frac{\left(\sqrt{2}+\sqrt{6}\right)^2}{2^2} = \frac{8+2\sqrt{12}}{4} = \frac{8+2\left(2\sqrt{3}\right)}{4} = \frac{8+4\sqrt{3}}{4} = 2+\sqrt{3} \\ \text{RHS}^2 &= \sqrt{3}+2 \end{aligned}$$

   Since the two sides have the same square, tye are either equal or opposites. Since both sides are positive, they are equal.

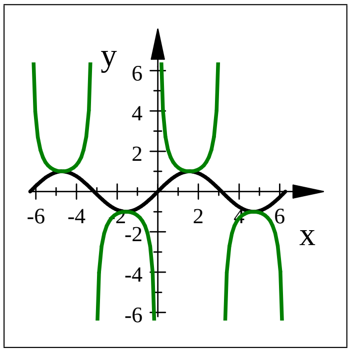
8. $y = x - 10$ and $y = -7x + 6$

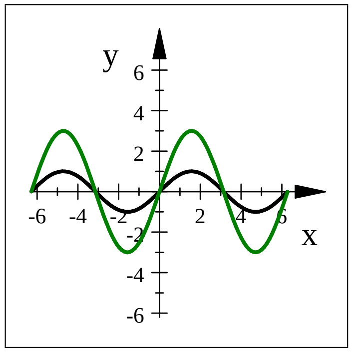
9. a) 72 b) 328 351

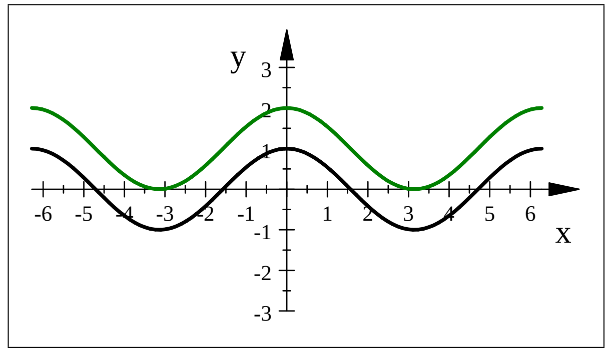
10. a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ b) $[-3, 3]$ c) $(-1, 7) \cup (7, \infty)$ d) $\left\{x \in \mathbb{R} : x \neq \frac{\pi}{2} + k\pi \text{ where } k \in \mathbb{Z}\right\}$ e) $(5, 6) \cup (6, \infty)$

    f) $(-\infty, -4) \cup (5, \infty)$ g) $x \neq \frac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$ h) $x \neq \frac{k\pi}{2}$ where $k \in \mathbb{Z}$

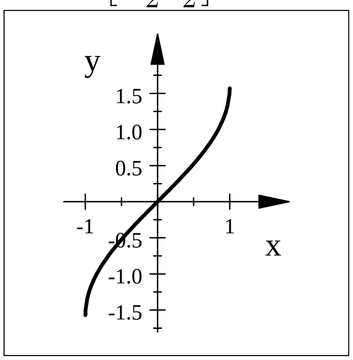
11. a) $f(x) = \sin x$ and $g(x) = \csc x$ b) $f(x) = \sin x$ and $g(x) = 3\sin x$ c) $f(x) = \cos x$ and $g(x) = \cos x + 1$







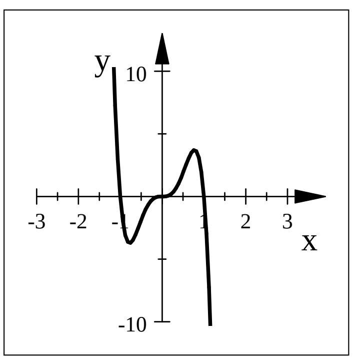
12. domain: $[-1, 1]$
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

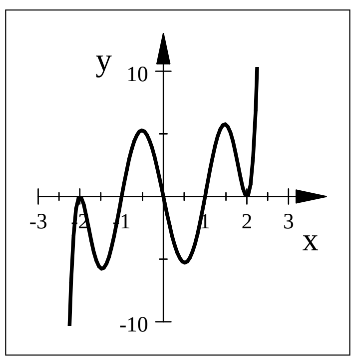


13. a) no such triangle
b) $\beta_1 \approx 34.44474654°$, $\gamma_1 \approx 116.55525346°$, $c_1 \approx 11.07037$ ft
$\beta_2 \approx 145.555\,253\,46°$, $\gamma_2 \approx 5.444\,746\,54°$, $c_2 \approx 1.174\,30582$ ft
c) $\beta \approx 19.8383°$, $\gamma \approx 140.161\,7°$, $c \approx 13.213\,91$ ft
d) $\alpha \approx 17.61244°$, $\beta \approx 133.43253°$, $\gamma \approx 28.9550°$
e) $\alpha_1 \approx 26.567\,838°$, $\gamma_1 \approx 133.432\,162°$, $c_1 \approx 27.602\,045$
$\alpha_2 \approx 153.432\,162°$, $\gamma_2 \approx 6.567\,838°$, $c_2 \approx 4.347\,503\,459$
f) $\beta \approx 15.161\,745\,79°$, $\gamma \approx 144.838\,254\,21°$, $c \approx 28.624\,256\,733\,067\,1$
g) $c = 6.531\,146$ $\alpha = 117.095\,52°$ $\beta = 42.904\,48°$

14. a) $f(x) = -2\left(x^5 - x^3\right)$
$= -2x^3(x-1)(x+1)$

b) $g(x) = x\left(x^2 - 1\right)\left(4 - x^2\right)^2$
$= (x+2)^2(x+1)\,x\,(x-1)(x-2)^2$

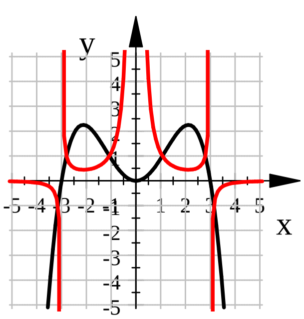


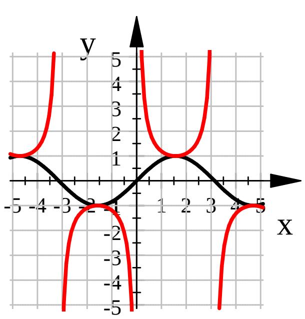


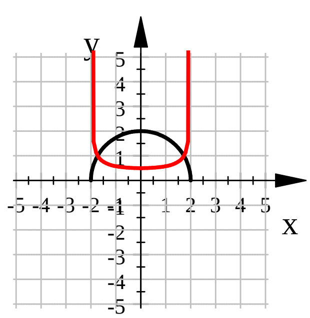
15. $f(x) = (x+4)(x+2)^2(x-1)(x-3)$

16. a) vertical asymptote at $x = 0$ hole at $x = 1$ b) vertical asymptote at $x = -3, 2, 5$ hole at $x = -1, 0$

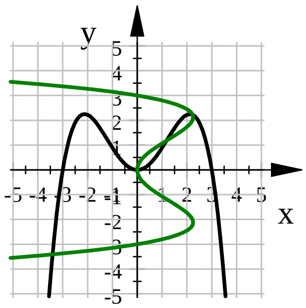
17.

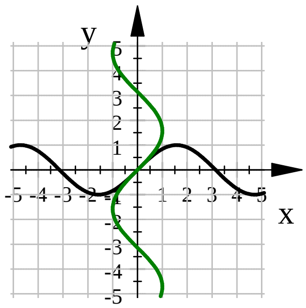


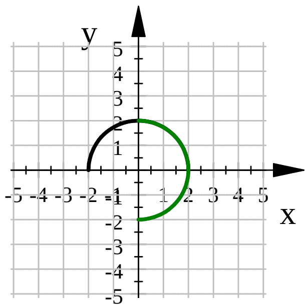




18.







19. $\sqrt{145}$ ft or $\sqrt{433}$ ft 20. $\dfrac{3\sqrt{15}}{4}$ 21. $\dfrac{13}{15}$ 22. $\dfrac{47}{32}$

23. $B = \dfrac{\ln 3}{\ln 2} A$ or $B = A \log_2 3$

Solution: Let $f(x) = c \cdot d^x$ $(d > 1)$ be the exponential function expressing the population. Let x express the years passed, starting at 2007. For A, we write the equation

$$\begin{aligned} 2c &= c \cdot d^A \\ 2 &= d^A \\ \ln 2 &= A \ln d \\ \frac{\ln 2}{\ln d} &= A \end{aligned}$$

For B, we write

$$\begin{aligned} 3c &= c \cdot d^B \\ 3 &= d^B \\ \ln 3 &= B \ln d \\ \frac{\ln 3}{\ln d} &= B \end{aligned}$$

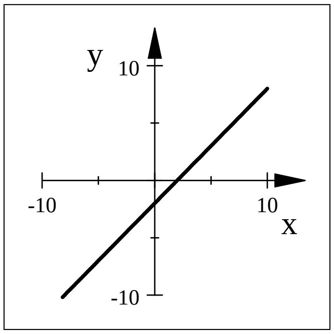
So $A = \dfrac{\ln 2}{\ln d}$ and $B = \dfrac{\ln 3}{\ln d}$. Then

$$\begin{aligned} \frac{B}{A} &= \frac{\frac{\ln 3}{\ln d}}{\frac{\ln 2}{\ln d}} = \frac{\ln 3}{\ln d} \cdot \frac{\ln d}{\ln 2} = \frac{\ln 3}{\ln 2} \\ \frac{B}{A} &= \frac{\ln 3}{\ln 2} \\ B &= A \frac{\ln 3}{\ln 2} = A \log_2 3 \end{aligned}$$

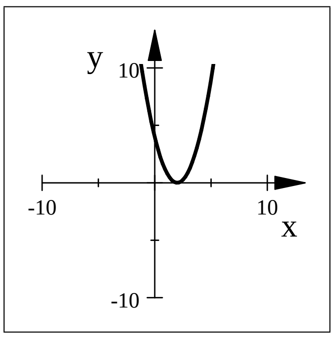
24. a) $f^{-1}(x) = \dfrac{1}{2}(e^x + 1)$ b) $f^{-1}(x) = -\dfrac{2}{x} + 3$ c) $f^{-1}(x) = \sqrt[3]{\dfrac{x}{5} + 4}$ 25. $2 - \sqrt{3}$

26. a) $9 \cdot 10^4$ b) $9 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 27\,216$ c) $9 \cdot 10^4 - 9 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 62\,784$

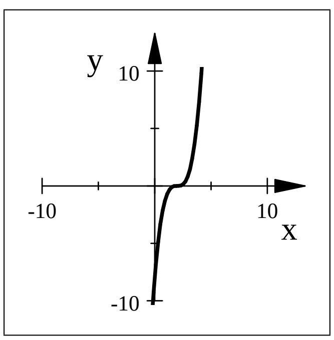
27. a) $f(x) = x - 2$



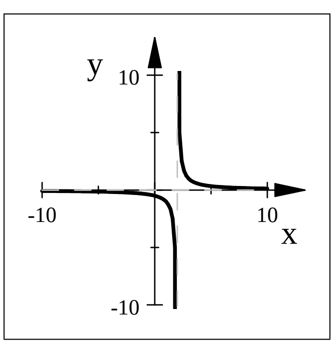
b) $f(x) = (x-2)^2$



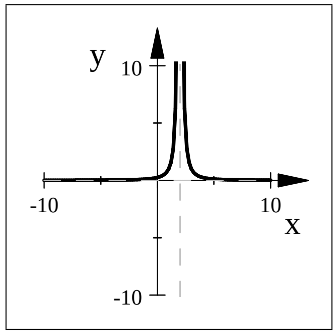
c) $f(x) = (x-2)^3$



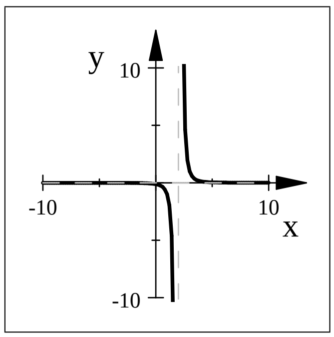
d) $f(x) = \dfrac{1}{x-2}$



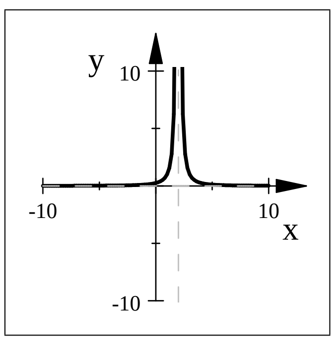
e) $f(x) = \dfrac{1}{(x-2)^2}$



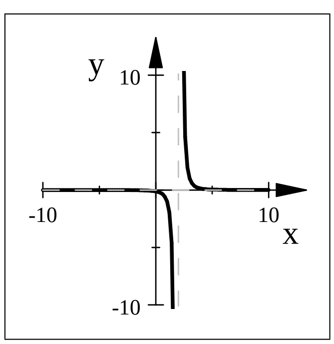
f) $f(x) = \dfrac{1}{(x-2)^3}$



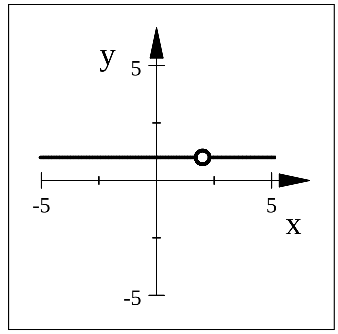
g) $f(x) = \dfrac{(x-2)^5}{(x-2)^7}$



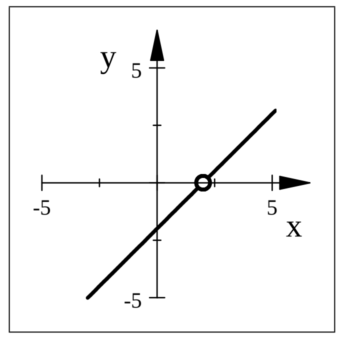
h) $f(x) = \dfrac{(x-2)^2}{(x-2)^5}$



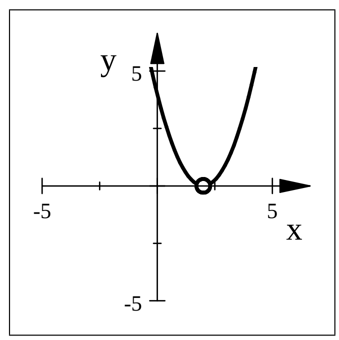
i) $f(x) = \dfrac{(x-2)^6}{(x-2)^6}$



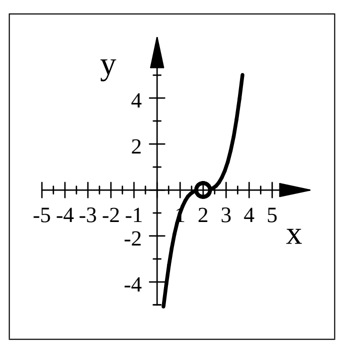
j) $f(x) = \dfrac{(x-2)^5}{(x-2)^4}$



k) $f(x) = \dfrac{(x-2)^7}{(x-2)^5}$



l) $f(x) = \dfrac{(x-2)^5}{(x-2)^2}$



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