1. A point $P$ is located at a distance of 37 units from point $C$, the center of a circle. We drew the tangent lines from $P$ to the circle. On one tangent line, the point of tangency, $Q$ is at a distance of 35 units from point $P$. Find the exact value of the radius of the circle.

2. Suppose that $f$ is a function defined by $f(x) = -2x + 5$. Compute or simplify each of the following.

   a) $f(3) + f(4)$ c) $f(1) - f(2)$ e) $3 \cdot f(4)$ g) $f(f(3))$ i) $f(2x)$ k) $f(x^2)$

   b) $f(3+4)$ d) $f(1-2)$ f) $f(3 \cdot 4)$ h) $f(f(f(3)))$ j) $2f(x)$ l) $(f(x))^2$

3. Suppose that $g$ is a function defined by $g(x) = 2x^2 - 10$. Simplify each of the following.

   a) $g(2) + g(3)$ b) $g(2+3)$ c) $g(a+1)$ d) $g(a) + g(1)$ e) $g(2a)$ f) $2g(a)$

4. List all subsets of $A = \{1, 2, 3, 4\}$

5. List all two-element subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

6. How many 3$-$digit numbers can be formed using the digits $1, 3, 5, 7, 8$, and $9$, if

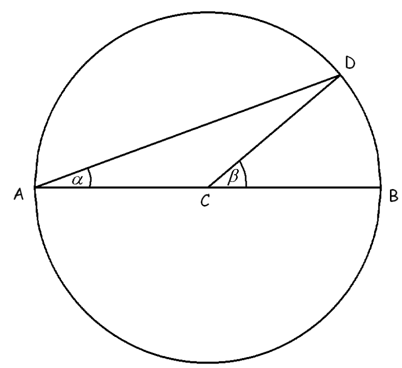
   a) repetition of digits is allowed

   b) repetition of digits is not allowed?

7. How many 4$-$digit numbers can be formed using the digits $1, 3, 5, 7, 8$, and $9$, if

   a) repetition of digits is allowed

   b) repetition of digits is not allowed?




8. Consider the circle shown on the picture. Prove that $\beta = 2\alpha$.

9. Rationalize the denominator in $\dfrac{\sqrt{x} - \sqrt{2}}{\sqrt{x} + \sqrt{2}}$.

10. Simplify each of the following. Present all answers using only positive exponents.

    a) $\dfrac{2^{-1} - 3^{-2}}{-2^{-2} + 1}$ b) $\dfrac{2b^{-2}\left(-a^3\right)^{-2} b^0}{\left(-b^2\right)^{-3} a^{-5}}$ c) $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$ d) $\left(\dfrac{1 - \sqrt{5}}{2}\right)^2 - 1$ e) $\dfrac{\sqrt{500} - \sqrt{20}}{\sqrt{45} - \sqrt{5}}$

11. Simplify each of the following.

    a) $2^{-3} + 5^{-2}$

    b) $\left(x^{-1}y^{-1}\right)^{-1}$

    c) $\left(x^{-1} + y^{-1}\right)^{-1}$

    d) $\left(\dfrac{2a^3b^{-2}\left(-ab^{-2}\right)^{-3} ba^0}{b^{-1}\left(-2b^2a^{-2}\right)^3 b}\right)$

    e) $\dfrac{2a^{-2}b^3}{a^5b^{-1}}$

    f) $\dfrac{2a^{-2} + b^3}{a^5 - b^{-1}}$

    g) $\left(\sqrt{3}\right)^6$

    h) $\left(\dfrac{1}{\sqrt{2}}\right)^{10}$

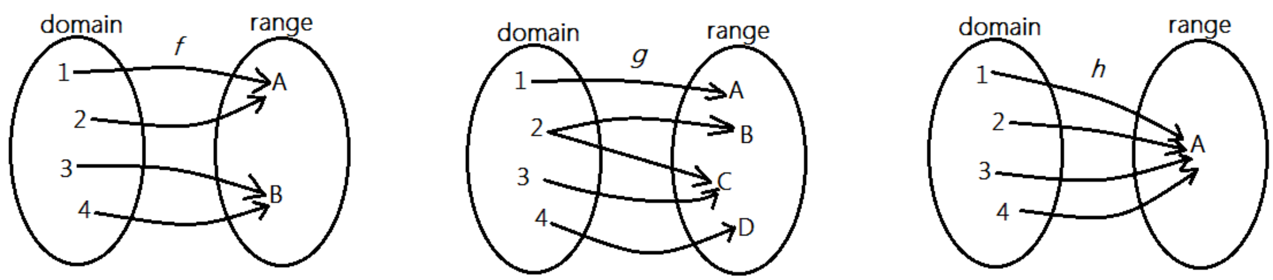
12. Solve the equation $5(x-2)^2 - 3x + 2 = x - 2$. Check your solution(s) using exact values.

13. a) Solve $3x^2 + x = 3x + 2$.

    b) Check your solutions using exact values.

14. A person is standing 3 ft away from a street light that is 15.6 ft tall. How long is his shadow if he is 5.2 ft tall?

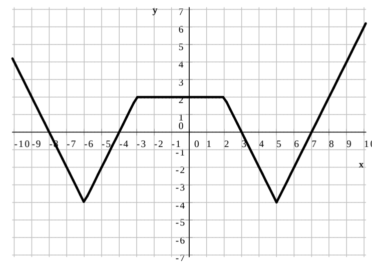
15. The picture below shows three relations. Which of them are also functions?




16. Find all values of $x$ for which $P(x,y)$ is on the graph given and

a) $y \leq 0$ c) $y \geq 2$ e) $y < -2$

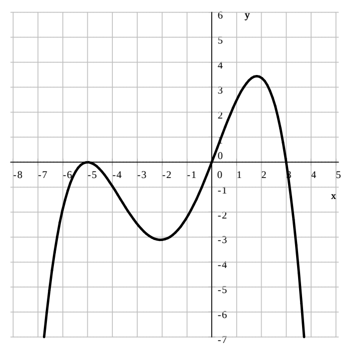
b) $y > 2$ d) $y \leq 2$ f) $y < -5$



17. Find all values of $x$ for which $P(x,y)$ is on the graph given and

a) $y > 0$ c) $y \geq 0$

b) $y < 0$ d) $y \leq 0$



18. A company finds that if it prices its product at \$30, then it can sell 1000 items. For every dollar increase in the price, the company will sell 5 less items. What is the maximum revenue possible, and what price guarantees that maximal revenue?

19. Solve each of the following systems over the real numbers.

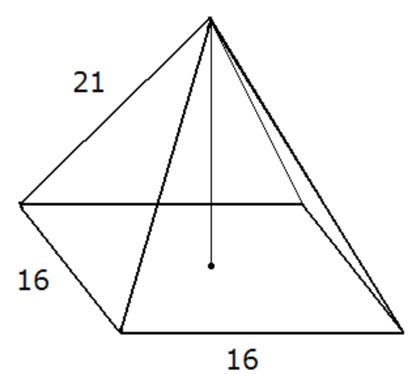
a) $\begin{cases} x+y=8 \\ 2xy=30 \end{cases}$ b) $\begin{cases} x+y=-1 \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{6} \end{cases}$ c) $\begin{cases} x^2+y^2=10 \\ 2x+y=5 \end{cases}$ d) $\begin{cases} x^2+4x+4=-2 \\ x+y=3 \end{cases}$

20. An arch is in the shape of a semicircle. At a point along the base 4 feet from an end of the arch, the height of the arch is 10 feet. Find the maximum height of the arch.

21. The hypotenuse of a right triangle is 50 feet long. Find the other two sides, given that the perimeter of the triangle is 112 feet.

22. Prove that there are no real values for $x$ and $y$ that would satisfy the equation $x^2-4x+5=-6y-y^2-11$.

23. Consider a square based straight pyramid as shown on the picture. The base is a square with sides $16\,\mathrm{m}$ long, and all other edges are $21\,\mathrm{m}$ long. Find the exact value of the height of the pyramid.



# Answers

1. 12 units 2. a) $-4$ b) $-9$ c) $2$ d) $7$ e) $-9$ f) $-19$ g) $7$ h) $-9$ i) $-4x+5$ j) $-4x+10$ k) $-2x^2+5$ l) $4x^2-20x+25$ 3. a) $6$ b) $40$ c) $2a^2+4a-8$ d) $2a^2-18$ e) $8a^2-10$ f) $4a^2-20$

4. All subsets of $A=\{1,2,3,4\}$

   0-element subsets: $\varnothing$

   1-element subsets: $\{1\}$,$\{2\}$,$\{3\}$,$\{4\}$

   2-element subsets: $\{1,2\}$,$\{1,3\}$,$\{1,4\}$,$\{2,3\}$,$\{2,4\}$,$\{3,4\}$

   3-element subsets: $\{1,2,3\}$,$\{1,2,4\}$,$\{1,3,4\}$,$\{2,3,4\}$

   4-element subsets: $\{1,2,3,4\}$

5. All two-element subsets of $U=\{1,2,3,4,5,6,7,8,9,10\}$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\{1,2\}$ | | | | | | | | |
| $\{1,3\}$ | $\{2,3\}$ | | | | | | | |
| $\{1,4\}$ | $\{2,4\}$ | $\{3,4\}$ | | | | | | |
| $\{1,5\}$ | $\{2,5\}$ | $\{3,5\}$ | $\{4,5\}$ | | | | | |
| $\{1,6\}$ | $\{2,6\}$ | $\{3,6\}$ | $\{4,6\}$ | $\{5,6\}$ | | | | |
| $\{1,7\}$ | $\{2,7\}$ | $\{3,7\}$ | $\{4,7\}$ | $\{5,7\}$ | $\{6,7\}$ | | | |
| $\{1,8\}$ | $\{2,8\}$ | $\{3,8\}$ | $\{4,8\}$ | $\{5,8\}$ | $\{6,8\}$ | $\{7,8\}$ | | |
| $\{1,9\}$ | $\{2,9\}$ | $\{3,9\}$ | $\{4,9\}$ | $\{5,9\}$ | $\{6,9\}$ | $\{7,9\}$ | $\{8,9\}$ | |
| $\{1,10\}$ | $\{2,10\}$ | $\{3,10\}$ | $\{4,10\}$ | $\{5,10\}$ | $\{6,10\}$ | $\{7,10\}$ | $\{8,10\}$ | $\{9,10\}$ |

6. a) $6^3=216$ b) $6\cdot5\cdot4=120$ 7. a) $6^4=1296$ b) $6\cdot5\cdot4\cdot3=360$

8. Line segments $AC$, $BC$, and $CD$ are all radii in the circle, and so they are equal. So $ACD$ triangle is isosceles, and the angles opposite $AC$ and $CD$ are also equal to each other. Thus $\measuredangle ADC=\alpha$. The third angle in triangle $ACD$ is $180°-2\alpha$. Angles $ACD$ and $DCB$ are supplementary angles beause together they form a straight angle. Thus

$$\begin{aligned} \measuredangle ACD+\measuredangle DCB &= 180° \\ 180°-2\alpha+\beta &= 180° && \text{subtract } 180° \\ -2\alpha+\beta &= 0 && \text{add } 2\alpha \\ \beta &= 2\alpha \end{aligned}$$

9. $\dfrac{x+2-2\sqrt{2x}}{x-2}$ 10. a) $\dfrac{14}{27}$ b) $-\dfrac{2b^4}{a}$ c) $2-\sqrt{3}$ d) $\dfrac{1-\sqrt{5}}{2}$ e) $4$

11. a) $\dfrac{1}{3}$ b) $\sqrt[3]{7}$ c) $4$ d) $\dfrac{1}{4}$ e) $-4$ f) undefined g) $-\dfrac{1}{4}$ h) $\sqrt{5}$ i) $\dfrac{1}{2}$ j) $1$ k) undefined l) $-\dfrac{1}{2}$ m) $\dfrac{1}{8}$ n) $1$ o) $\dfrac{1}{8}$ p) undefined q) $10\sqrt{10}$ r) $\dfrac{1}{\sqrt{6}}=\dfrac{\sqrt{6}}{6}$ 12. $x_{1,2}=\dfrac{12\pm2\sqrt{6}}{5}$

13. a) $\dfrac{1\pm\sqrt{7}}{3}$ b) If $x=\dfrac{1-\sqrt{7}}{3}$, then

$$\text{LHS}=3\left(\frac{1-\sqrt{7}}{3}\right)^2+\frac{1-\sqrt{7}}{3}=3\cdot\frac{8-2\sqrt{7}}{9}+\frac{1-\sqrt{7}}{3}=\frac{8-2\sqrt{7}}{3}+\frac{1-\sqrt{7}}{3}=\frac{9-3\sqrt{7}}{3}=\frac{3\left(3-\sqrt{7}\right)}{3}=3-\sqrt{7}$$

$$\text{RHS}=3\left(\frac{1-\sqrt{7}}{3}\right)+2=1-\sqrt{7}+2=3-\sqrt{7}$$

Checking the other solution goes similarly.

14. $1.5\,\text{ft}$ 15. $f$ and $h$ are functions.

16. a) $[-8,-4]\cup[3,7]$ b) $(-\infty,-9)\cup(8,\infty)$ c) $(-\infty,-9]\cup[-3,2]\cup[8,\infty)$ d) $[-9,8]$ e) $(-7,-5)\cup(4,6)$ f) no solution 17. a) $(0,3)$ b) $(-\infty,-5)\cup(-5,0)\cup(3,\infty)$ c) $[0,3]\cup\{-5\}$ d) $(-\infty,0]\cup[3,\infty)$

18. a price of $\$115$ for a revenue of $\$66\,125$

19. a) $(3,5)$ and $(5,3)$ b) $(2,-3)$ and $(-3,2)$ c) $(1,3)$ and $(3,-1)$ d) no real solution

20. $14.5$ feet 21. 14 and 48 feet

22. We complete the square on both sides.

$$(x-2)^2+1=-(y+3)^2-2$$

Then we see that the value of the right-hand side is 1 or greater for all values of $x$. Meanwhile, the value of the other side is $-2$ or less. So the two sides can never be equal.

23. $\sqrt{313}\,\text{m}$

Last revised: September 18, 2018