Review Problems

1. The sum of a number a and seven times a number b is 100.

   a) Find the smallest value of $a^2 + b^2$. b) Find the greatest value of ab.

2. We are driving toward a tower. The angle of elevation is $35°$. Then we drive 50 ft toward the tower. Now the angle of elevation is $40°$. How tall is the tower? Present your answer as an approximation, accurate up to three decimal places.

3. Simplify each of the following expressions. Assume that x represents a positive number.

   a) $x^3 \cdot x^5$ b) $(x^3)^5$ c) $\dfrac{(x^{3/4})^6}{x^{1/2}}$ d) $\dfrac{(x^{-2/3})^{-5}}{x^{1/3}}$ e) $16^{-3/4}$ f) -2^{-2}

4. A number is 3 greater than its own reciprocal. Find this number.

5. Find the first element a and the common difference d in the arithmetic sequence if it is given that $a_8 = -28$ and $s_8 = 28$.

6. Three sides of a right triangle, in increasing order, are also the fourth, twenty-fifth, and twenty-eightth elements of an arithmetic sequence. The the first element of the sequence is 4. Compute (up to four decimal places) the smallest angle in the triangle.

7. Consider the right triangle ABC. Find all sides if $\gamma = 90°$, $\beta = 24°$ and $a = 17$. Present your answer as an approximation, accurate up to at least four decimal places.

8. True or false? For all angles α, $\sin 2\alpha = 2\sin\alpha$.

9. Find the exact value of each of the following expressions.

   a) $3\sin 30°\cos 60° - 2\sin 60°\cos 30°$

   b) $2\sin 60° + 2\sin 30° - 2\tan 45°$

   c) $\sin^2 16° + \cos^2 16°$

   d) $\dfrac{\cos 40°}{\sin 50°}$

   e) $\dfrac{\sec 45°\cos 30° - \csc 60°}{\sin 60°}$

   f) $\dfrac{\cot 30° - \tan^2 60°}{\cot 30° + \tan^2 60°}$

   g) $\sin 30°\cos 60° - \cos 30°\sin 60°$

   h) $\cot 30° + \cot 45° + \cot 60°$

10. Find the exact value of $\sin\beta$ if we know that $\sin\alpha = \dfrac{2}{3}$ and $\alpha + \beta = 90°$.

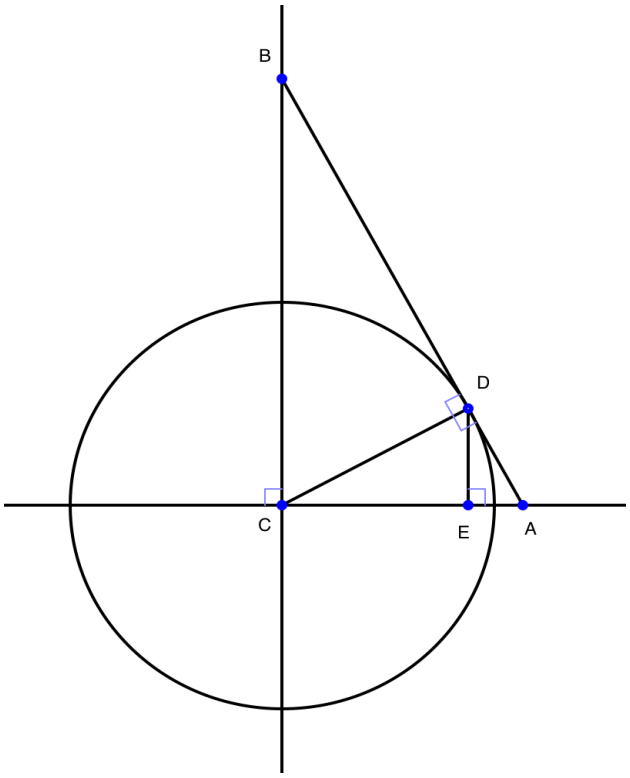
11. Find $\cos\alpha$ if $\tan\alpha = \dfrac{4}{7}$.

12. Find the area of a regular 9−sided polygon written into a circle of radius 10 cm.

13. Suppose that circle C_1 has radius 4 and circle C_2 has radius 7. The two centers are 15 units apart. Find an approximate value of the angle formed by the two common tangent lines drawn to the circles.

14. Consider the picture shown below. Line AB is tangent to the unit circle, where D is the point of tangency. Let α denote angle DCE. Match each of the six trigonometric functions with the length of each of the line segments given.

$\sin\alpha,\ \cos\alpha,\ \tan\alpha,\ \csc\alpha, \sec\alpha, \cot\alpha$ and $AC,\ AD,\ BC,\ BD,\ CE,\ DE$



Answers

1.) a) 200 b) $\frac{2500}{7}$ 2.) 211.510 92 ft

3.) a) x^8 b) x^{15} c) x^4 d) x^3 e) $\frac{1}{8}$ f) $-\frac{1}{4}$ 4.) $\frac{3-\sqrt{13}}{2}, \frac{3+\sqrt{13}}{2}$

5.) $a = 35, d = -9$ 6.) 22.619 865° 7.) $c = 18.608\,816\,7$ $b = 7.568\,888$

8.) False. For example, consider $\alpha = 30°$.
Then $\sin 2\alpha = \sin 60° = \frac{\sqrt{3}}{2}$ and $2\sin\alpha = 2\sin 30° = 2\left(\frac{1}{2}\right) = 1$.

9.) a) $-\frac{3}{4}$ b) $\sqrt{3}-1$ c) 1 d) 1 e) $\sqrt{2}-\frac{4}{3}$ f) $\sqrt{3}-2$ g) $-\frac{1}{2}$ h) $\frac{4\sqrt{3}}{3}+1$

10.) $\frac{\sqrt{5}}{3}$ 11.) $\frac{7}{\sqrt{65}} = \frac{7\sqrt{65}}{65}$ 12.) $900\cos 20°\sin 20°\ \text{cm}^2 \approx 289.254\,424\,36\ \text{cm}^2$ 13.) 23.073 92°

14.) $\sin\alpha = DE$ $\cos\alpha = CE$ $\tan\alpha = AD$ $\csc\alpha = BC$ $\sec\alpha = AC$ $\cot\alpha = BD$