Review Problems

Please note that Quiz 11 will also cover topics covered on Quizzes 1-10 and Exams 1 and 2. Please review those topics as well, even if they do not appear in this document.

1. Simplify each of the following expressions. Assume that all variables represent positive numbers. Present your answer containing only positive integers as exponents.

   a) $x^3 \cdot x^5$ b) $(x^3)^5$ c) $\dfrac{\left(x^{3/4}\right)^6}{x^{1/2}}$ d) $\dfrac{\left(x^{-2/3}\right)^{-5}}{x^{1/3}}$ e) $16^{-3/4}$ f) -2^{-2}

   g) $x^{3/2}$ h) $x^{-5/3}$ i) $\left(2ab^3c^{-5}\right)^{-2}$ j) $\dfrac{\left(x^{1/3}\right)^{2/3}}{x^{1/9}}$ k) $\dfrac{p^{-3}}{p^{-5}}$

2. Factor by completing the square. $-4x^2 + 6x - 1$

3. Simplify each of the following. Present exact values.

   a) $\sin 45° - 2\tan 30° + \sin 60°$

   b) $(\sin 1° + \sin 2° + ... + \sin 45°) - (\cos 46° + \cos 47° + ... + \cos 89°)$

4. State the domain for each of the following functions.

   a) $f(x) = \dfrac{x-2}{x^2-9}$

   b) $g(x) = \sqrt{x-8} - \sqrt{15-x} + \dfrac{1}{x-10}$

   c) $h(x) = \dfrac{2x+1}{5x-1} - \sqrt{x+2} - \dfrac{1}{x^2+2x+7}$

   d) $m(a) = \dfrac{2a-7}{-5+\sqrt{a+4}}$

   e) $f(x) = \sqrt{6x - x^2}$

5. Circle C_1 has a radius 5 unit long. Circle C_2 has a radius 11 unit long. The centers are at a distance of 12 units from each other. We draw the common tangent lines drawn to the circles.

   a) Find an approximation of the angle formed by the two tangent lines.

   b) Compute the distance between the two points of tangency on one of the common tangent lines.

6. Consider the parabola $y = 4x^2 - 3x - 8$.

   a) Find both coordinates of the vertex.

   b) Find all $x-$intercepts. Present exact values.

7. Seattle, WA and San Francisco, CA are located approximately on the same longitude. The latitude of these cities are 47.5° N and 37.4° N. Find the distance between the two cities assuming that the Earth is a sphere with radius 3960 miles. Round your answer to the nearest mile.

8. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

   a) $y = x + x^2 - 24$ and $y = 3x - 16$

   b) $(x+3)^2 + (y+2)^2 = 10$ and $(x-1)^2 + (y-2)^2 = 10$

   c) $(x+3)^2 + (y+2)^2 = 8$ and $x^2 + (y-1)^2 = 2$

9. Find all values of b for which the equation $x^2 - 8x - b^2 + 16 = 5b$ has exactly one real solution for x.

10. Consider the pyramid $ABCDE$ if its base is a square $ABCD$ with sides 6 m and side $AE = BE = CE = DE = 10$ m. Compute an approximate value for the angle that is formed between a triangular face and the base.

11. Prove each of the following identities.
    a) $\tan x + \dfrac{\cos x}{1 + \sin x} = \sec x$ b) $\dfrac{\cos x}{1 - \sin x} = \sec x + \tan x$

12. Compute an approximate value for each of the angles in a triangle with sides 7 cm, 7 cm, and 10 cm.

13. Compute the first element and common difference in a triangle if $s_{50} = 550$ and $s_{75} = -1050$.

14. Consider the arithmetic sequence determined by $a = -36$ and $d = 4$. Find n with
    a) $a_n = 360$ b) $s_n = -120$

15. One number a is 10 greater than twice another number b. Find each of the following.
    a) The minimal value of $a^2 + b^2$. c) The maximal value of $b^2 - a^2$
    b) The minimal value of ab

16. We drew an n-sided regular polygon into a circle with radius R. In terms of R and n, express
    a) the perimeter of the polygon b) the area of the polygon

Answers

1. a) x^8 b) x^{15} c) x^4 d) x^3 e) $16^{-3/4} = \dfrac{1}{8}$ f) $-\dfrac{1}{4}$
   g) $x\sqrt{x}$ h) $\dfrac{1}{x\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x}}{x^2}$ i) $\dfrac{c^{10}}{4a^2b^6}$ j) $\sqrt[9]{x}$ k) p^2

2. $-4\left(x - \dfrac{3}{4} - \dfrac{\sqrt{5}}{4}\right)\left(x - \dfrac{3}{4} + \dfrac{\sqrt{5}}{4}\right)$

3. a) $\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{6}$ b) $\dfrac{\sqrt{2}}{2}$

4. a) $x \neq \pm 3$ b) $8 \leq x \leq 15$ and $x \neq 10$ c) $x \geq -2$ and $x \neq \dfrac{1}{5}$ d) $a \geq -4$ and $a \neq 21$
   e) $0 \leq x \leq 6$

5. a) $60°$ b) $\sqrt{108} = 6\sqrt{3}$

6. a) $\left(\dfrac{3}{8}, -\dfrac{137}{16}\right)$ b) $\left(\dfrac{3 - \sqrt{137}}{8}, 0\right)$ and $\left(\dfrac{3 + \sqrt{137}}{8}, 0\right)$

7. 698 miles

8. a) $(-2, 22)$ and $(4, -4)$ b) $(-2, 1)$ and $(0, -1)$ c) $(-1, 0)$

9. $-5, 0$

10. $71.670\,246\,2°$

11. a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\begin{aligned} \text{RHS} &= \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x\,(1+\sin x) + \cos^2 x}{\cos x\,(1+\sin x)} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x + 1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \sec x = \text{LHS} \end{aligned}$$

b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} \\ &= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = \text{RHS} \end{aligned}$$

12. $44.415\,3°$, $44.415\,3°$, and $91.169\,4°$

13. $a = 60, d = -2$

14. a) 100 b) 4 and 15

15. a) 20 b) $-\dfrac{25}{2}$ c) $\dfrac{100}{3}$

16. a) $2nR\sin\left(\dfrac{180°}{n}\right)$ b) $n\left(R\sin\left(\dfrac{180°}{n}\right)\right)\left(R\cos\left(\dfrac{180°}{n}\right)\right) = nR^2\sin\left(\dfrac{180°}{n}\right)\cos\left(\dfrac{180°}{n}\right)$