Review Problems

1. Convert to radians.

   a) $45°$ b) $-120°$ c) $10°$ d) $135°$ e) $270°$

2. Convert to degrees.

   a) $\frac{\pi}{6}$ b) $-\frac{\pi}{9}$ c) $\frac{5\pi}{12}$ d) -3π e) $\frac{7\pi}{4}$

3. Find the exact value of each of the following expressions.

   a) $\sin 120°$ c) $\tan \frac{7\pi}{4}$ e) $\cos(-90°)$ g) $\sec\left(-\frac{5\pi}{3}\right)$ i) $\tan(-270°)$

   b) $\cos 120°$ d) $\sin \frac{5\pi}{6}$ f) $\cos \frac{5\pi}{6}$ h) $\sin\left(-\frac{\pi}{2}\right)$

4. Find all angles coterminal with $120°$. Express your answer

   a) in degrees b) in radians

   c) Find all coterminal angles α such that $-500° < \alpha < 500°$.

5. True or false?

   a) If an angle α is co-terminal with $40°$, then 2α is co-terminal with $80°$.

   b) If twice an angle 2β is co-terminal with $20°$, then β is co-terminal with $10°$.

6. Solve each of the following equations. Present your answer in degrees and in radians.

   a) $\sin x = -\frac{1}{\sqrt{2}}$ c) $\tan x = -\sqrt{3}$ e) $\cos x = -1$

   b) $\cos x = -\frac{\sqrt{3}}{2}$ d) $\sin x = -\frac{3}{2}$ f) $\tan x = 0$

7. Graph each of the following functions.

   a) $f(x) = x^2 - 3x + 1$ b) $f(x) = x^3$ c) $f(x) = |x|$ d) $f(x) = \sqrt{x}$

   e) $f(x) = \sqrt[3]{x}$ f) $f(x) = \frac{1}{x}$ g) $f(x) = \frac{1}{x^2}$ h) $f(x) = x^3 - x$

8. Find the domain of each of the following functions.

   a) $f(x) = \frac{1}{x-2}$ c) $f(x) = \frac{1}{6x - x^2}$ e) $f(x) = \frac{1}{x^2 - 7x - 30}$

   b) $f(x) = \sqrt{x-2}$ d) $f(x) = \sqrt{6x - x^2}$ f) $f(x) = \sqrt{x^2 - 7x - 30}$

9. Prove each of the following identities.

   a) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$ c) $\frac{1 - \cos x}{1 + \cos x} = \frac{\sec x - 1}{\sec x + 1}$

   b) $\tan x - \csc x \sec x\,(1 - 2\cos^2 x) = \cot x$ d) $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

Answers

1. a) $\frac{\pi}{4}$ b) $-\frac{2\pi}{3}$ c) $\frac{\pi}{18}$ d) $\frac{3\pi}{4}$ e) $\frac{3\pi}{2}$

2. a) $30°$ b) $-20°$ c) $75°$ d) $-540°$ e) $315°$

3. a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) -1 d) $\frac{1}{2}$ e) 0 f) $-\frac{\sqrt{3}}{2}$ g) 2 h) -1 i) undefined

4. a) $120° + k \cdot 360°$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$

   b) $\frac{2\pi}{3} + 2k\pi$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$ c) $-240°, 120°, 480°$

5. True or false?

   a) If an angle α is co-terminal with $40°$, then 2α is co-terminal with $80°$.

   True: $\alpha = 40° + k \cdot 360° \implies 2\alpha = 80° + k \cdot 720°$

   b) If twice an angle 2β is co-terminal with $20°$, then β is co-terminal with $10°$.

   False: Let $\beta = 190°$. Clearly $10°$ and $190°$ are not co-terminal, but $2\beta = 380°$ which is co-terminal with $20°$.

6. a) $x = -45° + k \cdot 360°$ or $x = -135° + k \cdot 360°$ where $k \in \mathbb{Z}$

   $x = -\frac{\pi}{4} + 2k\pi$ or $x = -\frac{3\pi}{4} + 2k\pi$ where $k \in \mathbb{Z}$

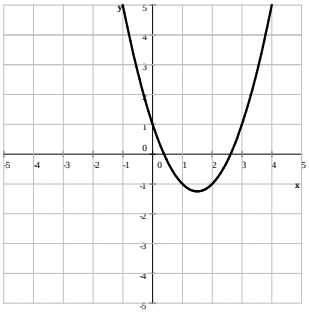
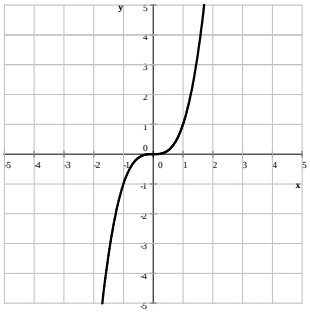
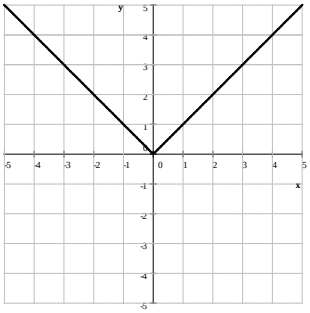
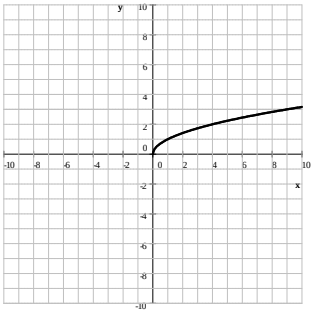
   b) $x = \pm 150° + k \cdot 360°$ where $k \in \mathbb{Z}$ $x = \pm\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

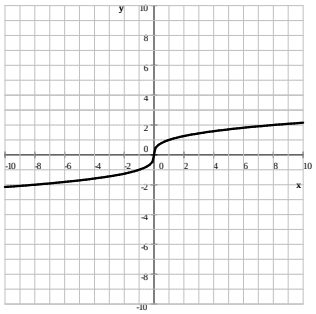
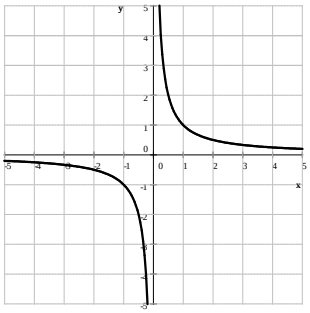
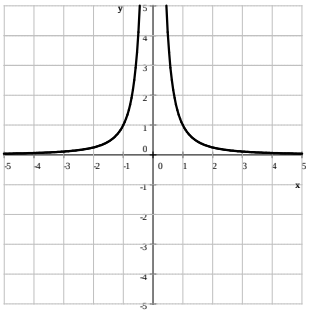
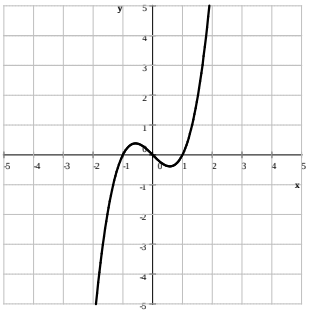
   c) $x = -60° + k \cdot 180°$ where $k \in \mathbb{Z}$ $x = -\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$

   d) no solution e) $x = 180° + k \cdot 360°$ where $k \in \mathbb{Z}$ $x = \pi + 2k\pi$ where $k \in \mathbb{Z}$

   f) $x = k \cdot 180°$ where $k \in \mathbb{Z}$ $x = k\pi$ where $k \in \mathbb{Z}$

7. a) $f(x) = x^2 - 3x + 1$ b) $f(x) = x^3$ c) $f(x) = |x|$ d) $f(x) = \sqrt{x}$

e) $f(x) = \sqrt[3]{x}$ f) $f(x) = \dfrac{1}{x}$ g) $f(x) = \dfrac{1}{x^2}$ h) $f(x) = x^3 - x$

8. a) $x \neq 2$ b) $x \geq 2$ c) $x \neq 0, 6$ d) $[0, 6]$ e) $x \neq -3, 10$
f) $(-\infty, -3] \cup [10, \infty)$

9. a) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

$$\text{LHS} = \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\left(\dfrac{1}{\sin x}\right)} + \frac{\cos x}{\left(\dfrac{1}{\cos x}\right)} = \sin^2 x + \cos^2 x = 1 = \text{RHS}$$

b) $\tan x - \csc x \sec x \left(1 - 2\cos^2 x\right) = \cot x$

$$\begin{aligned}
\text{LHS} &= \tan x - \csc x \sec x \left(1 - 2\cos^2 x\right) = \frac{\sin x}{\cos x} - \frac{1}{\sin x}\left(\frac{1}{\cos x}\right)\left(1 - 2\cos^2 x\right) \\
&= \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2\cos^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2\cos x}{\sin x} \\
&= \frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x} + \frac{2\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x - 1 + 2\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x - 1 + \cos^2 x}{\sin x \cos x} \\
&= \frac{1 - 1 + \cos^2 x}{\sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x = \text{RHS}
\end{aligned}$$

c) $\dfrac{1 - \cos x}{1 + \cos x} = \dfrac{\sec x - 1}{\sec x + 1}$

$$\text{RHS} = \frac{\sec x - 1}{\sec x + 1} = \frac{\dfrac{1}{\cos x} - 1}{\dfrac{1}{\cos x} + 1} = \frac{\dfrac{1}{\cos x} - \dfrac{\cos x}{\cos x}}{\dfrac{1}{\cos x} + \dfrac{\cos x}{\cos x}} = \frac{\dfrac{1 - \cos x}{\cos x}}{\dfrac{1 + \cos x}{\cos x}} = \frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} = \text{LHS}$$

d) $\dfrac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

$$\begin{aligned}
\text{LHS} &= \frac{\sec x + \csc x}{\tan x + \cot x} = \frac{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}} = \frac{\dfrac{\sin x}{\cos x \sin x} + \dfrac{\cos x}{\sin x \cos x}}{\dfrac{\sin^2 x}{\cos x \sin x} + \dfrac{\cos^2 x}{\sin x \cos x}} = \frac{\dfrac{\sin x + \cos x}{\cos x \sin x}}{\dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}} = \frac{\dfrac{\sin x + \cos x}{\cos x \sin x}}{\dfrac{1}{\cos x \sin x}} \\
&= \frac{\sin x + \cos x}{\cos x \sin x} \cdot \frac{\cos x \sin x}{1} = \sin x + \cos x = \text{RHS}
\end{aligned}$$