Review Problems

1. A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.4q^2 + 3q + 40$ thousand dollars, and all q units can be sold at a price of $p(q) = 22.2 - 1.2q$ thousand dollars per unit. Find the maximal profit.

2. Graph each of the following equations. In case of each of these equations, determine whether y is a function of x or not.

   a) $y = x^2 - 6x + 8$ c) $x = y^2$ e) $xy = 1$ g) $x^2y = 1$

   b) $x^2 = y^2$ d) $x^2 + y = 4$ f) $2x + 3y = 4$ h) $x^2 + y^2 = 9$

3. Simplify each of the following.

   a) $\log_{10}(8x^7) + \frac{1}{2}\log_{10}(25x^{16}) - 2\log_{10}(2x^3)$ c) $\ln(a^2 - 4) - \ln(a - 2)$

   b) $5^{-\log_{25} 11}$ d) $8^{\log_8 m - \log_2(3m)}$

4. Solve each of the following equations.

   a) $\sqrt{2x-1} + \sqrt{x-1} = 5$ d) $\frac{2}{3}\ln(2x-5) + 3 = 7$ g) $\log_3(x-4) + \log_3(2x-5) = 2$

   b) $3^{2x-1} = 2^{2-x}$ e) $\log_{3x+1}\left(\frac{1}{4}\right) = -1$ h) $\frac{1}{x} + \frac{1}{x-2} = \frac{3}{4}$

   c) $4^x - 5 \cdot 2^x = -4$ f) $\log_3(x-3) - \log_3(4x-2) = -2$

5. Solve each of the following inequalities.

   a) $\frac{5x-2}{2x+1} \geq 2$ b) $\frac{x}{2-x} > 4$

6. Perform each of the following divisions.

   a) $4x^5 - x^3 + 8$ by $x^2 + x - 2$ b) $x^5 - 1$ by $x - 1$

7. Suppose that $x = \log_5 10$ and $y = \log_9 5$. Express $\log_{24} 60$ in terms of x and y.

8. a) Find the points where the circles $(x-1)^2 + (y+7)^2 = 50$ and $(x-5)^2 + (y+4)^2 = 25$ intersect each other.

   b) Find an equation of the tangent line drawn to the graph of $4x - 2y + x^2 + y^2 = 20$ at the point $(-6, 4)$.

9. Compute an approximate value for b in a triangle given that $a = 3$, $c = 4$, and $\beta = 38°$.

10. Solve each of the following triangles.

    a) $a = 5$ $b = 12$ $c = 2$ b) $a = 7$ $b = 5$ $c = 6$

11. Graph each of the following functions.

    a) $f(x) = \frac{2}{x+1} + 3$ b) $f(x) = 2^{x-3} - 3$ c) $f(x) = |x+3| - 2$ d) $f(x) = (x-1)^3 - 1$

12. Sketch the graph of each of the following functions.

    a) $f(x) = (x+2)(x-1)(x-4)$ b) $f(x) = -x^3 + 4x$ c) $f(x) = -\frac{1}{2}x(x-3)^2$

13. Sketch the graph of each of the following functions.

a) $f(x) = x - 2$

b) $f(x) = (x-2)^2$

c) $f(x) = (x-2)^3$

d) $f(x) = \dfrac{1}{x-2}$

e) $f(x) = \dfrac{1}{(x-2)^2}$

f) $f(x) = \dfrac{1}{(x-2)^3}$

g) $f(x) = \dfrac{(x-2)^5}{(x-2)^7}$

h) $f(x) = \dfrac{(x-2)^2}{(x-2)^5}$

i) $f(x) = \dfrac{(x-2)^6}{(x-2)^6}$

j) $f(x) = \dfrac{(x-2)^5}{(x-2)^4}$

k) $f(x) = \dfrac{(x-2)^7}{(x-2)^5}$

l) $f(x) = \dfrac{(x-2)^5}{(x-2)^2}$

14. Let (a_n) be a geometric sequence with $a_1 = 50$ and $r = \dfrac{2}{3}$. Find an approximate value (up to 4 decimal places) for a) a_7. b) s_{10}

15. Find the exact value of the infinite sum of the geometric sequence $180, 60, 20, \ldots$.

16. a) Compute the exact value of $\sin \dfrac{x}{2}$ if $\cos x = \dfrac{1}{3}$.
b) Compute the exact value of $\tan 22.5°$.

17. a) Write $\sin 5x \cos 17x$ as a sum or difference. b) Write $\cos 8x - \cos 20x$ as a product.

18. Prove that if α and β are angles in a triangle such that

$$(\sin\alpha + \sin\beta)^2 + (\cos\alpha - \cos\beta)^2 = 2$$

then the triangle has a right angle.

19. Let l be the line $y = \dfrac{5}{12}x$. Let k be the line that bisects the angle formed by l and the positive part of the $x-$axis. Find the equation of k.

20. Graph each of the following functions on the interval $[-2\pi, 2\pi]$

a) $f(x) = \sin x$ b) $f(x) = \cos x$

21. Compute each of the following limits.

a) $\displaystyle\lim_{x\to-\infty} (-2x^5 + 8x^2)$

b) $\displaystyle\lim_{x\to\infty} (-2x^5 + 8x^2)$

c) $\displaystyle\lim_{x\to-\infty} (-2x^5 + 8x^6)$

d) $\displaystyle\lim_{x\to\infty} (-2x^5 + 8x^6)$

e) $\displaystyle\lim_{x\to-\infty} \frac{3x^2 - 1}{5x^2 - 3x + 2}$

f) $\displaystyle\lim_{x\to-\infty} \frac{100x - 1}{5x^2 - 3x + 2}$

g) $\displaystyle\lim_{x\to-\infty} \log_2 x$

h) $\displaystyle\lim_{x\to\infty} \frac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$

i) $\displaystyle\lim_{x\to\infty} \frac{-x^3 + 2x + 1}{x - 3}$

j) $\displaystyle\lim_{x\to-\infty} (2^x)$

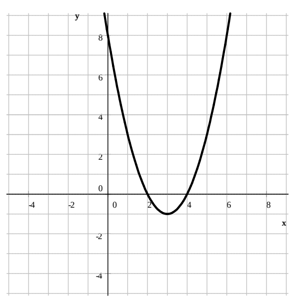
k) $\displaystyle\lim_{x\to\infty} \frac{2^{x+5}}{4^{x-1}}$

l) $\displaystyle\lim_{x\to\infty} \frac{3^{x+1} \cdot \left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

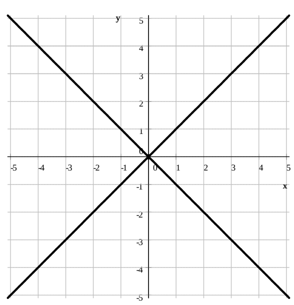
Review Problems - Answers

1. \$17 600

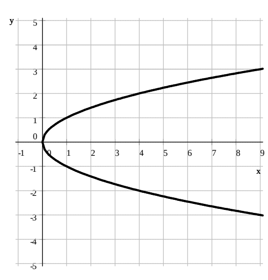
2. a) $y = x^2 - 6x + 8$
Solution: This is a function.
Algebraically: there is a unique formula to compute y in terms of x. Geometrically: its graph passes the vertical line test.



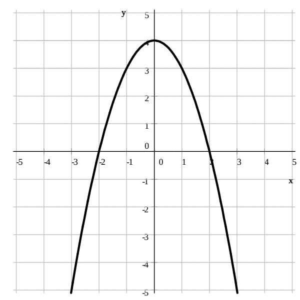
b) $x^2 = y^2$
Solution: Not a function. Algebraically: If we solve for y, we get $y = x$ or $y = -x$. Geometrically: its graph fails the vertical line test.



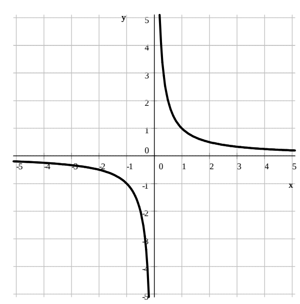
c) $x = y^2$
Solution: Not a function. Algebraically: when we solve for y, we get that $y = \pm\sqrt{x}$. Geometrically: the graph fails the vertical line test.



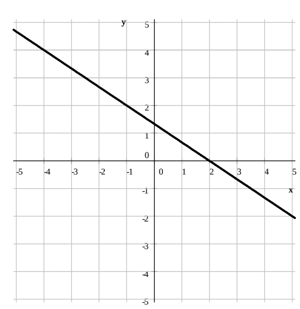
d) $x^2 + y = 4$
Solution: Function. We solve for y and see that it is a unique formula. $y = -x^2 + 4$. Its graph passes the vertical line test.



e) $xy = 1$
Solution: Function. Algebraically: We solve for y and see that it is a unique formula. $y = \dfrac{1}{x}$. Geometrically: Its graph passes the vertical line test.



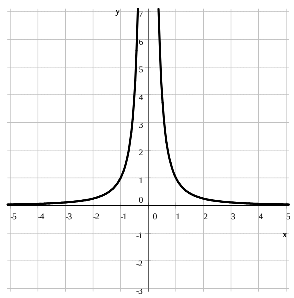
f) $2x + 3y = 4$
Solution: Function. We solve for y and see that it is a unique formula, $y = -\dfrac{2}{3}x + \dfrac{4}{3}$. Its graph passes the vertical line test.



g) $x^2y = 1$
Solution: Function. We solve for y and see that it is a unique formula.

$$\begin{aligned} x^2y &= 1 && \text{divide by } x^2 \\ y &= \frac{1}{x^2} \end{aligned}$$

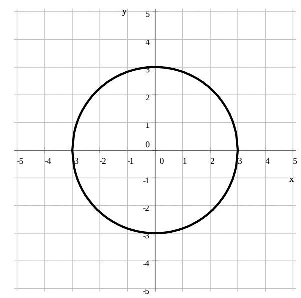
Its graph passes the vertical line test.



h) $x^2 + y^2 = 9$
Solution: Not a function. We solve for y and see that it is not a unique formula.

$$\begin{aligned} x^2 + y^2 &= 9 && \text{subtract } x^2 \\ y^2 &= 9 - x^2 \\ y &= \pm\sqrt{9 - x^2} \end{aligned}$$

Its graph fails the vertical line test.



3. a) $1 + 9\log_{10} x$ b) $\dfrac{1}{\sqrt{11}}$ c) $\ln(a+2)$ d) $\dfrac{1}{27m^2}$

4. a) 5 (145 doesn't work) b) $\log_{18} 12$ c) $0, 2$ d) $\dfrac{1}{2}(e^6 + 5)$ e) 1 f) 5 g) 7 h) $\dfrac{2}{3}, 4$

5. a) $\left(-\infty, -\dfrac{1}{2}\right) \cup [4, \infty)$ b) $\left(\dfrac{8}{5}, 2\right)$

6. a) $4x^3 - 4x^2 + 11x - 19$ R $41x - 30$ b) $x^4 + x^3 + x^2 + x + 1$

7. $x - 1 = \log_5 2$ and $\dfrac{1}{2y} = \log_5 3$

$$\log_{24} 60 = \frac{\log_5 60}{\log_5 24} = \frac{2\log_5 2 + \log_5 3 + 1}{3\log_5 2 + \log_5 3} = \frac{2(x-1) + \frac{1}{2y} + 1}{3(x-1) + \frac{1}{2y}} = \frac{4xy - 2y + 1}{6xy - 6y + 1}$$

8. a) $(8, -8)$ and $(2, 0)$ b) $\dfrac{4}{3}(x+6) = y - 4$ or $y = \dfrac{4}{3}x + 12$

9. $2.467\,335$ unit

10. a) no such triangle b) $\alpha \approx 78.463\,041°$ $\beta \approx 44.415\,308\,6°$ $\gamma \approx 57.121\,65°$

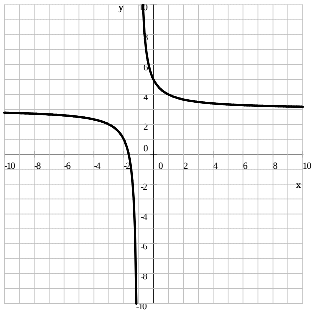
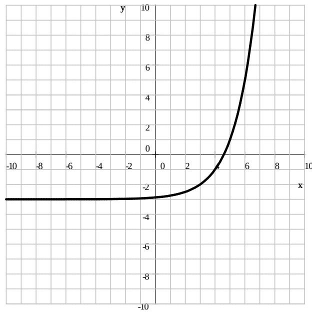
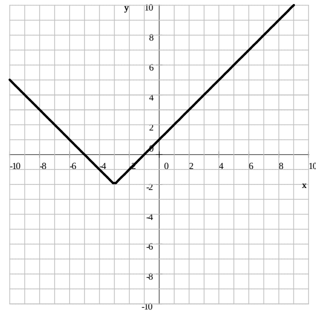
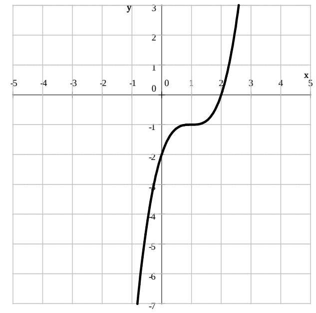
11. a) $f(x) = \dfrac{2}{x+1} + 3$ b) $f(x) = 2^{x-3} - 3$ c) $f(x) = |x+3| - 2$ d) $f(x) = (x-1)^3 - 1$

a) start with $y = \dfrac{1}{x}$
shift by 1 to the left,
stretch by 2 along $y-$axis
shift up by 3 units

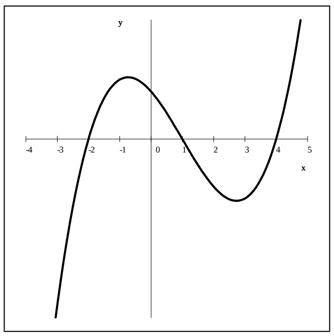
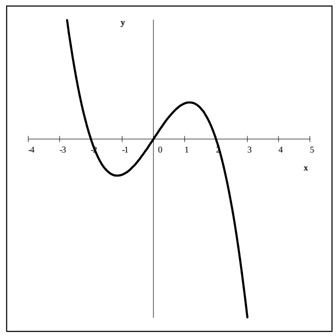
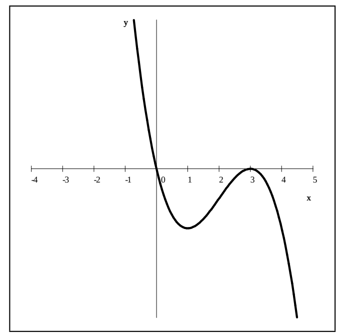
b) start with $y = 2^x$
shift by 3 to the right
shift down by 3 units

c) start $y = |x|$
shift by 3 to the left
shift down by 2 units

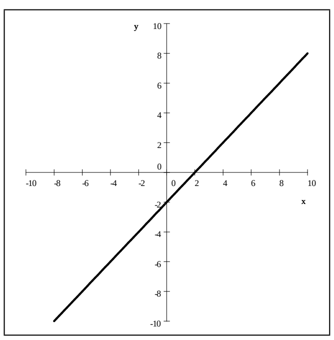
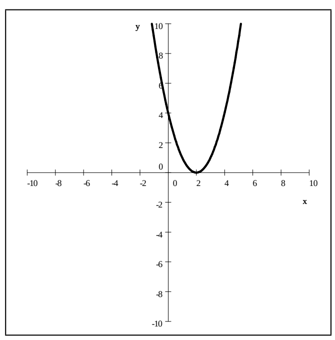
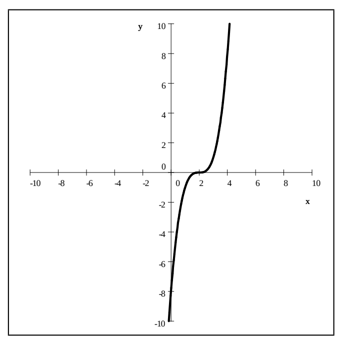
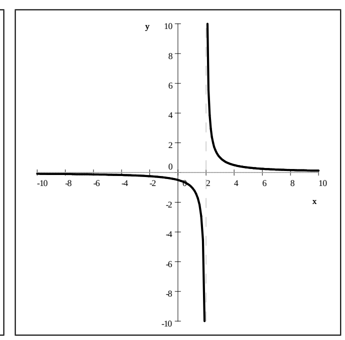
d) start with $y = x^3$
shift by 1 to the right
shift down by 1 unit

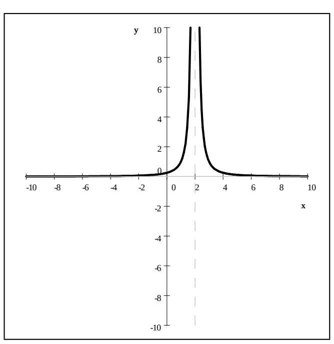
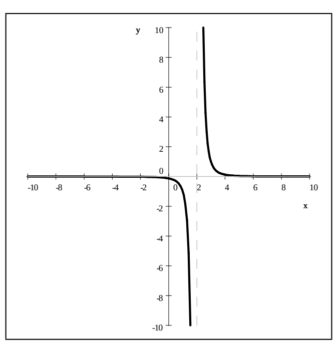
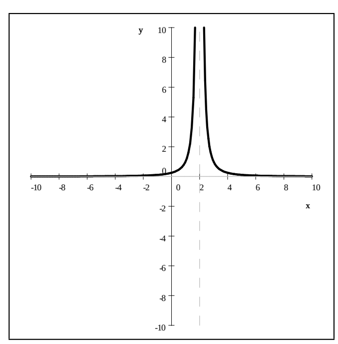
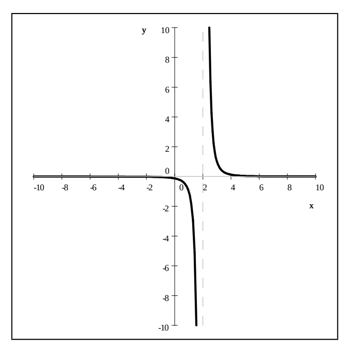
12. a) $f(x) = (x+2)(x-1)(x-4)$ b) $f(x) = -x^3 + 4x$ c) $f(x) = -\frac{1}{2}x(x-3)^2$

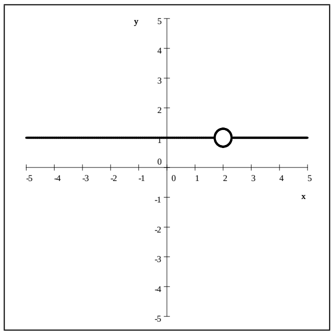
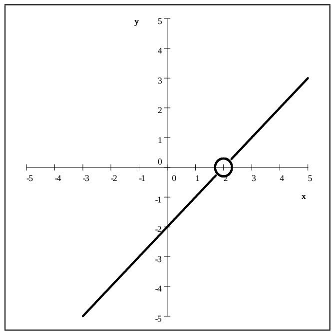
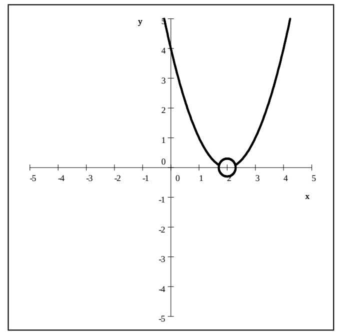
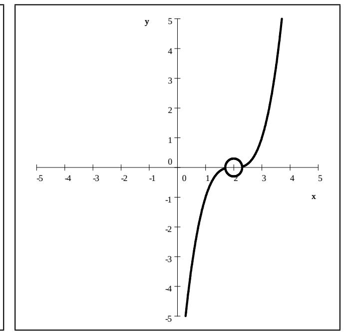
13. a) $f(x) = x - 2$ b) $f(x) = (x-2)^2$ c) $f(x) = (x-2)^3$ d) $f(x) = \dfrac{1}{x-2}$

e) $f(x) = \dfrac{1}{(x-2)^2}$ f) $f(x) = \dfrac{1}{(x-2)^3}$ g) $f(x) = \dfrac{(x-2)^5}{(x-2)^7}$ h) $f(x) = \dfrac{(x-2)^2}{(x-2)^5}$

i) $f(x) = \dfrac{(x-2)^6}{(x-2)^6}$ j) $f(x) = \dfrac{(x-2)^5}{(x-2)^4}$ k) $f(x) = \dfrac{(x-2)^7}{(x-2)^5}$ l) $f(x) = \dfrac{(x-2)^5}{(x-2)^2}$

14. a) $50\left(\dfrac{2}{3}\right)^6 \approx 4.389\,574\,76$ b) $50\dfrac{1-\left(\dfrac{2}{3}\right)^{10}}{1-\left(\dfrac{2}{3}\right)} \approx 147.398\,77$

15. 270

16. a) $\pm\dfrac{\sqrt{3}}{3}$ b) $\sqrt{2}-1$

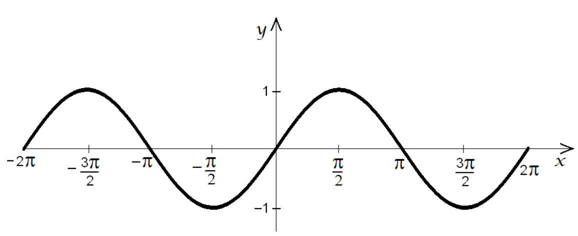
17. a) $\dfrac{1}{2}(\sin 22x - \sin 12x)$ b) $2\sin 14x \sin 6x$

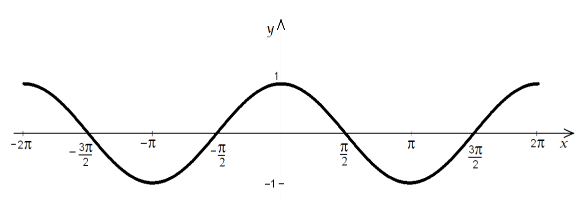
18.

$$\begin{aligned}
(\sin\alpha + \sin\beta)^2 + (\cos\alpha - \cos\beta)^2 &= 2 \\
\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta &= 2 \\
2 + 2\sin\alpha\sin\beta - 2\cos\alpha\cos\beta &= 2 \\
2\sin\alpha\sin\beta - 2\cos\alpha\cos\beta &= 0 \\
-2(\cos\alpha\cos\beta - \sin\alpha\sin\beta) &= 0 \\
-2\cos(\alpha+\beta) &= 0 \\
\alpha + \beta &= 90^\circ \implies \gamma = 90^\circ
\end{aligned}$$

19. $y = \dfrac{1}{5}x$ or $y = -5x$

20. a) $f(x) = \sin x$ b) $f(x) = \cos x$





21. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\dfrac{3}{5}$ f) 0 g) undefined h) $\dfrac{2}{3}$ i) $-\infty$ j) 0 k) 0 l) 3