

Please note that Quiz 11 will cover topics covered on Quizzes 1-10 and Exams 1 and 2. Review those topics even if they do not appear in this document.

1. Convert to radians.

- a) 45° b) -120° c) 10° d) 135° e) 270°

2. Convert to degrees.

- a) $\frac{\pi}{6}$ b) $-\frac{\pi}{9}$ c) $\frac{5\pi}{12}$ d) -3π e) $\frac{7\pi}{4}$

3. Find the exact value of each of the following expressions.

- a) $\sin 120^\circ$ c) $\tan \frac{7\pi}{4}$ e) $\cos(-90^\circ)$ g) $\sec\left(-\frac{5\pi}{3}\right)$ h) $\sin\left(-\frac{\pi}{2}\right)$
 b) $\cos 120^\circ$ d) $\sin \frac{5\pi}{6}$ f) $\cos \frac{5\pi}{6}$ i) $\tan(-270^\circ)$

4. Find all angles coterminal with 120° . Express your answer

- a) in degrees b) in radians
 c) Find all coterminal angles α such that $-500^\circ < \alpha < 500^\circ$.

5. True or false?

- a) If an angle α is co-terminal with 40° , then 2α is co-terminal with 80° .
 b) If twice an angle 2β is co-terminal with 20° , then β is co-terminal with 10° .

6. Solve each of the following equations.

- a) $4 + \log_2(5x) = 7$ b) $2 + \log_3((x-1)^2) = 6$ c) $\ln((x+1)^2) = 4$

7. Solve each of the following equations.

- a) $5^x = \frac{1}{25}$ b) $10^{1-5x} = 1000$ c) $3^{2x-1} = 243$ d) $3^{x^2-3x} = 81$

8. Graph each of the following functions.

- a) $f(x) = x^2 - 3x + 1$ c) $f(x) = |x|$ e) $f(x) = \sqrt[3]{x}$ g) $f(x) = \frac{1}{x^2}$
 b) $f(x) = x^3$ d) $f(x) = \sqrt{x}$ f) $f(x) = \frac{1}{x}$ h) $f(x) = 2^x$

9. Find the domain of each of the following functions.

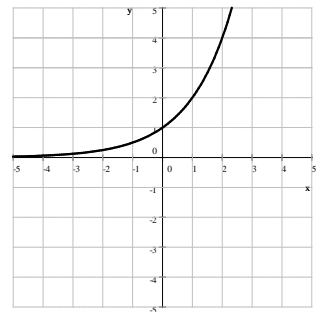
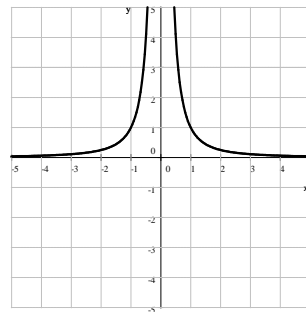
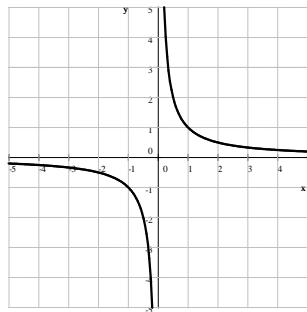
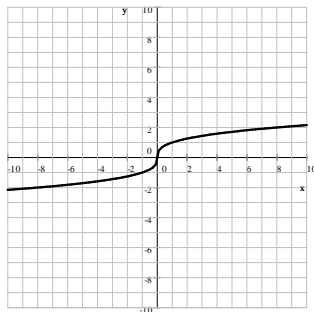
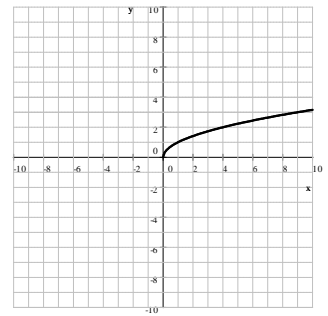
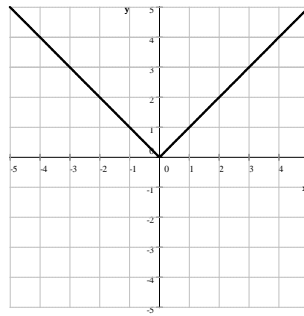
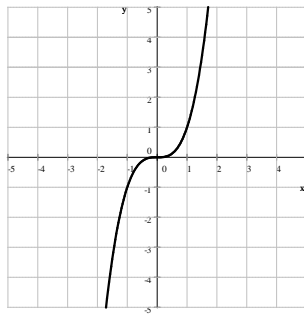
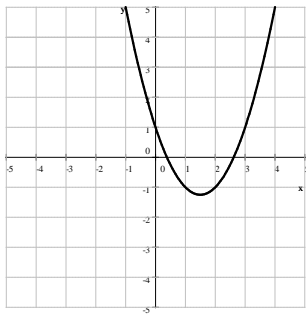
- a) $f(x) = \frac{1}{x-2}$ c) $f(x) = \frac{1}{6x-x^2}$ e) $f(x) = \frac{1}{x^2-7x-30}$
 b) $f(x) = \sqrt{x-2}$ d) $f(x) = \sqrt{6x-x^2}$ f) $f(x) = \sqrt{x^2-7x-30}$

10. Prove each of the following identities.

- a) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$ c) $\frac{1 - \cos x}{1 + \cos x} = \frac{\sec x - 1}{\sec x + 1}$
 b) $\tan x - \csc x \sec x (1 - 2\cos^2 x) = \cot x$ d) $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

Answers

1. a) $\frac{\pi}{4}$ b) $-\frac{2\pi}{3}$ c) $\frac{\pi}{18}$ d) $\frac{3\pi}{4}$ e) $\frac{3\pi}{2}$
2. a) 30° b) -20° c) 75° d) -540° e) 315°
3. a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) -1 d) $\frac{1}{2}$ e) 0 f) $-\frac{\sqrt{3}}{2}$ g) 2 h) -1 i) undefined
4. a) $120^\circ + k \cdot 360^\circ$ where $k = 0, 1, -1, 2, -2, 3, -3, \dots$
 b) $\frac{2\pi}{3} + 2k\pi$ where $k = 0, 1, -1, 2, -2, 3, -3, \dots$ c) $-240^\circ, 120^\circ, 480^\circ$
5. True or false?
 a) If an angle α is co-terminal with 40° , then 2α is co-terminal with 80° .
 True: $\alpha = 40^\circ + k \cdot 360^\circ \implies 2\alpha = 80^\circ + k \cdot 720^\circ$
 b) If twice an angle 2β is co-terminal with 20° , then β is co-terminal with 10° .
 False: Let $\beta = 190^\circ$. Clearly 10° and 190° are not co-terminal, but $2\beta = 380^\circ$ which is co-terminal with 20° .
6. a) $\frac{8}{5}$ b) $10, -8$ c) $-e^2 - 1, e^2 - 1$
7. a) -2 b) $-\frac{2}{5}$ c) 3 d) $-1, 4$
8. a) $f(x) = x^2 - 3x + 1$ b) $f(x) = x^3$ c) $f(x) = |x|$ d) $f(x) = \sqrt{x}$



- e) $f(x) = \sqrt[3]{x}$ f) $f(x) = \frac{1}{x}$ g) $f(x) = \frac{1}{x^2}$ h) $f(x) = 2^x$
9. a) $x \neq 2$ b) $x \geq 2$ c) $x \neq 0, 6$ d) $[0, 6]$ e) $x \neq -3, 10$ f) $(-\infty, -3] \cup [10, \infty)$

$$10. \text{ a) } \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$$

$$\text{LHS} = \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\left(\frac{1}{\sin x}\right)} + \frac{\cos x}{\left(\frac{1}{\cos x}\right)} = \sin^2 x + \cos^2 x = 1 = \text{RHS}$$

$$\text{b) } \tan x - \csc x \sec x (1 - 2 \cos^2 x) = \cot x$$

$$\begin{aligned} \text{LHS} &= \tan x - \csc x \sec x (1 - 2 \cos^2 x) = \frac{\sin x}{\cos x} - \frac{1}{\sin x} \left(\frac{1}{\cos x}\right) (1 - 2 \cos^2 x) \\ &= \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos x}{\sin x} \\ &= \frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x - 1 + 2 \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x - 1 + \cos^2 x}{\sin x \cos x} \\ &= \frac{1 - 1 + \cos^2 x}{\sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x = \text{RHS} \end{aligned}$$

$$\text{c) } \frac{1 - \cos x}{1 + \cos x} = \frac{\sec x - 1}{\sec x + 1}$$

$$\text{RHS} = \frac{\sec x - 1}{\sec x + 1} = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1} = \frac{\frac{1}{\cos x} - \frac{\cos x}{\cos x}}{\frac{1}{\cos x} + \frac{\cos x}{\cos x}} = \frac{\frac{1 - \cos x}{\cos x}}{\frac{1 + \cos x}{\cos x}} = \frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} = \text{LHS}$$

$$\text{d) } \frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$$

$$\begin{aligned} \text{LHS} &= \frac{\sec x + \csc x}{\tan x + \cot x} = \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{\sin x}{\cos x \sin x} + \frac{\cos x}{\sin x \cos x}}{\frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\sin x \cos x}} = \frac{\frac{\sin x + \cos x}{\cos x \sin x}}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}} = \frac{\sin x + \cos x}{\frac{1}{\cos x \sin x}} \\ &= \frac{\sin x + \cos x}{\cos x \sin x} \cdot \frac{\cos x \sin x}{1} = \sin x + \cos x = \text{RHS} \end{aligned}$$