1. Simplify each of the following.

a) $e^{\ln 2} - e^{2\ln 5} + e^{-\ln 3}$

b) $\log_5\left(2x^3\right) - 2\log_5(3x) - \dfrac{1}{\log_x 5}$

c) $\log_2\left(\sec^8\left(\frac{\pi}{4}\right)\right)$

d) $\dfrac{1}{2}\log_3 45x^2 + \log_3\sqrt{20} - \log_3 30x + \log_3 6 - \log_3 2$

e) $\ln(\tan 45°)\cdot\ln(\sin 60°)\cdot\ln(\cos 75°)$

2. Solve each of the following inequalities.

a) $\dfrac{2x-3}{x+1} < 5$ b) $\dfrac{3x}{x-2} \leq -1$ c) $\dfrac{p+1}{p-1} < 1$ d) $\dfrac{p+1}{p-1} \geq -4$

3. Perform the following division on polynomials.

a) $\left(5x^5 - x^4 - 7x^3 + x^2 - 3x\right) \div \left(x^2 - 2\right)$

b) $\left(x^4 - 5\right) \div (x+2)$

c) $\left(2x^3 - 5x^2 + 7x - 1\right) \div (x-3)$

d) $\left(x^7 - 1\right) \div \left(x^2 + 1\right)$

e) $(x+5) \div (x-3)$

f) $(2x-1) \div (x+1)$

4. Solve each of the following equations.

a) $\cos 2x = -\sin x$

b) $\sin 2x = -\sin x$

c) $\log_6(x+1) + \log_6(4x+4) = 2$

d) $\log_2(x-8) - \log_2(x-13) = -1$

e) $\sin 3x = -\dfrac{1}{\sqrt{2}}$

f) $\sin x = \cos x$

g) $\sin 2x = \cos 2x$

h*) $x\left(1 - \log_{21} 3\right) = \log_{21} 30 - \log_{21}\left(7^x + 1\right)$

i) $\log_2(x+3) + \log_2(x-3) = \log_2(x-9)$

j*) $4^{2\log_{10} x} \cdot 5^{\log_{10} x} = 6400$

5. Compute the exact value of each of the following.

a) $\sin 15°$

b) $\tan 75°$

c) $\sin 20° \cos 40° + \cos 20° \sin 40°$

d) $\dfrac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°}$

e) $\sin 10° \cos 130° - \sin 130° \cos 10°$

6. Suppose that α and β are angles with $\tan\alpha = -\dfrac{1}{2}$ and $\cos\beta = \dfrac{1}{3}$. We also know that neither α nor β belongs to the fourth quadrant. Compute the exact value of each of the following.

a) $\sin\alpha$ and $\cos\alpha$

b) $\sin\beta$ and $\tan\beta$

c) $\sin 2\alpha$

d) $\cos 2\alpha$

e) $\sin 4\alpha$

f) $\sin 3\alpha$

g) $\tan 2\beta$

h) $\sin(\alpha - \beta)$

i) $\cos(\alpha - \beta)$

j) $\tan 4\beta$

7. Find the exact value of $\tan y$ if we know that $\tan x = 2$ and $\tan(x+y) = -2$.

8. Find the exact value of $\tan x$ if we know that $\tan 2x = \dfrac{20}{21}$.

9. Find the exact value of $\cos x$ if we know that $\cos 2x = \dfrac{1}{3}$

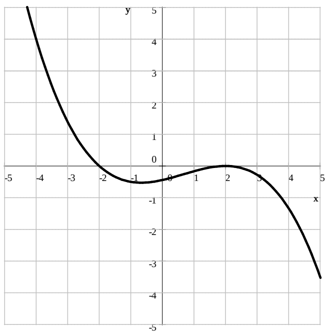
10. Find an equation for all tangent lines drawn to the graph of $y = -\dfrac{1}{2}x^2 - 2x + 5$ from the point $P(2,1)$.

11. Sketch the graph for each of the following functions.

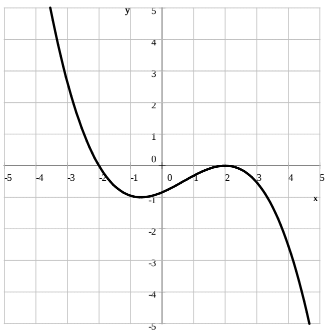
a) $f(x) = x^3 - 4x$ b) $f(x) = x^4 - 4x^2$ c) $f(x) = x^5 - 4x^3$

12. The picture below shows the graph of a function $y = f(x)$. Graph $y = \dfrac{1}{f(x)}$ in the same coordinate system.

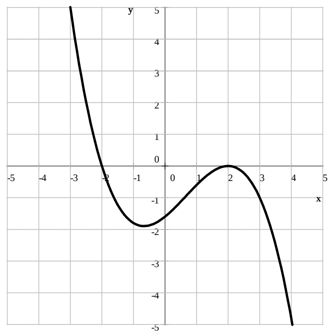
a)



b)



c)



Answers

1.) a) $-\dfrac{68}{3}$ b) $\log_5\left(\dfrac{2}{9}\right) = \log_5 2 - \log_5 9$ c) 4 d) 1 e) 0

2.) a) $\left(-\infty, -\dfrac{8}{3}\right) \cup (-1, \infty)$ b) $\left[\dfrac{1}{2}, 2\right)$ c) $(-\infty, 1)$ d) $\left(-\infty, \dfrac{3}{5}\right] \cup (1, \infty)$

3.) a) $5x^3 - x^2 + 3x - 1$ R $3x - 2$ b) $x^3 - 2x^2 + 4x - 8$ R 11 c) $2x^2 + x + 10$ R 29

d) $x^5 - x^3 + x$ R $-x - 1$ e) 1 R 8 f) 2 R -3

4.) a) $-\dfrac{\pi}{6} + 2k\pi, \quad -\dfrac{5\pi}{6} + 2k\pi, \quad \dfrac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$ b) $\pm\dfrac{2\pi}{3} + 2k\pi, \quad k\pi,$ where $k \in \mathbb{Z}$

c) 2 (-4 does not work) d) no solution (3 does not work) e) $-\dfrac{\pi}{12} + \dfrac{2}{3}k\pi \quad -\dfrac{\pi}{4} + \dfrac{2}{3}k\pi$ where $k \in \mathbb{Z}$

f) $\dfrac{\pi}{4} + k\pi$ g) $\dfrac{\pi}{8} + \dfrac{k\pi}{2}$ h) $\log_7 5$ i) no solution j) 100

5.) a) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$ b) $2 + \sqrt{3}$ c) $\dfrac{\sqrt{3}}{2}$ d) $\dfrac{\sqrt{3}}{3}$ e) $-\dfrac{\sqrt{3}}{2}$

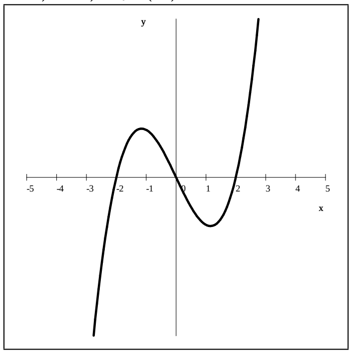
6.) a) $\sin\alpha = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$ and $\cos\alpha = -\dfrac{2}{\sqrt{5}} = -\dfrac{2\sqrt{5}}{5}$ b) $\sin\beta = \dfrac{\sqrt{8}}{3} = \dfrac{2\sqrt{2}}{3}$ and $\tan\beta = 2\sqrt{2}$

c) $\sin 2\alpha = -\dfrac{4}{5}$ d) $\cos 2\alpha = \dfrac{3}{5}$ e) $\sin 4\alpha = -\dfrac{24}{25}$ f) $\sin 3\alpha = \dfrac{11\sqrt{5}}{25}$ g) $\tan 2\beta = -\dfrac{4\sqrt{2}}{7}$

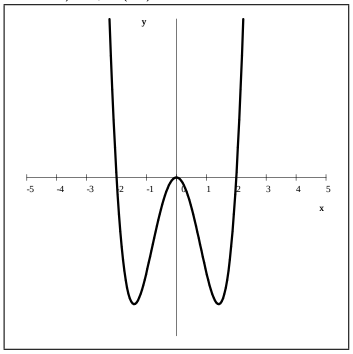
h) $\sin(\alpha - \beta) = \dfrac{\sqrt{5} + 4\sqrt{10}}{15}$ i) $\cos(\alpha - \beta) = \dfrac{\sqrt{10}}{30}$ j) $\tan 4\beta = -\dfrac{56\sqrt{2}}{17}$

7.) $\dfrac{4}{3}$ 8.) $\dfrac{2}{5}$ or $-\dfrac{5}{2}$ 9.) $\pm\dfrac{\sqrt{6}}{3}$ 10.) $y = -2x + 5$ and $y = -6x + 13$

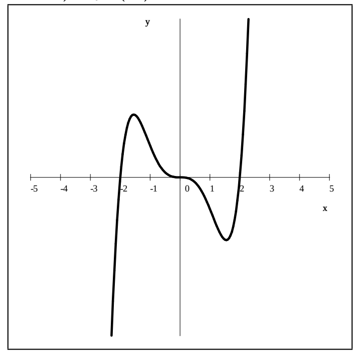
11.) a) $f(x) = x^3 - 4x$



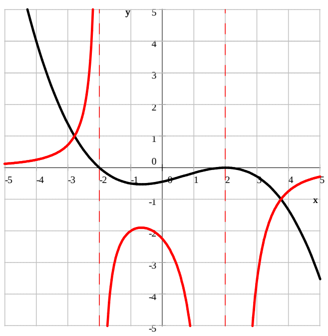
b) $f(x) = x^4 - 4x^2$



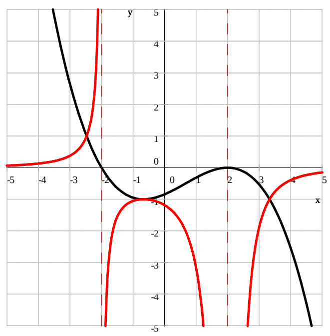
c) $f(x) = x^5 - 4x^3$



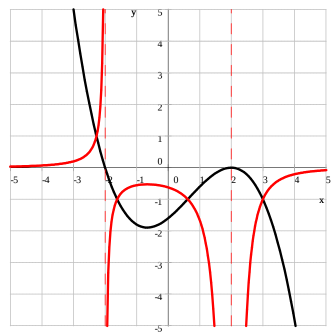
12.) a)



b)



c)



Solutions

4. h) $x(1 - \log_{21} 3) = \log_{21} 30 - \log_{21}(7^x + 1)$

$$\begin{aligned} x(1 - \log_{21} 3) &= \log_{21} 30 - \log_{21}(7^x + 1) \\ x(\log_{21} 21 - \log_{21} 3) &= \log_{21} 30 - \log_{21}(7^x + 1) \\ x \log_{21} \frac{21}{3} &= \log_{21} 30 - \log_{21}(7^x + 1) \\ x \log_{21} 7 &= \log_{21} 30 - \log_{21}(7^x + 1) \\ \log_{21}(7^x) &= \log_{21}\left(\frac{30}{7^x + 1}\right) \end{aligned}$$

Since $f(x) = \log_{21} x$ is a one-to-one function, we can conclude that

$$\begin{aligned} 7^x &= \frac{30}{7^x + 1} \\ 7^x(7^x + 1) &= 30 \end{aligned}$$

Let $a = 7^x$

$$\begin{aligned} a(a + 1) &= 30 \\ a^2 + a - 30 &= 0 \\ (a + 6)(a - 5) &= 0 \end{aligned}$$

$$a_1 = -6 \quad \text{and} \quad a_2 = 5$$

If $a = -6$, then we have $7^x = -6$. This equation has no solution. If $a = 5$, then we have $7^x = 5$ and so $x = \log_7 5$.

4. j) $4^{2\log_{10} x} \cdot 5^{\log_{10} x} = 6400$

$$\begin{aligned}
4^{2\log x} \cdot 5^{\log x} &= 6400 \\
\left(4^2\right)^{\log x} \cdot 5^{\log x} &= 6400 \\
(16)^{\log x} \cdot 5^{\log x} &= 6400 \\
(16 \cdot 5)^{\log x} &= 6400 \\
80^{\log x} &= 80^2 \\
\log x &= 2 \\
x &= 100
\end{aligned}$$