

1. Solve each of the following triangles.

a) $\beta = 78^\circ$ $\gamma = 49^\circ$ and $c = 15$ ft

c) $\alpha = 31^\circ$ $a = 4$ cm and $c = 5$ cm

b) $\alpha = 62^\circ$ $a = 7$ m and $b = 12$ m

2. Compute each of the following limits.

a) $\lim_{x \rightarrow -\infty} \left(-\frac{5}{3}x^7 \right)$

e) $\lim_{x \rightarrow -\infty} \left(-\frac{\sqrt{5}}{13}x^{12} \right)$

i) $\lim_{x \rightarrow -\infty} \sqrt{x-8}$

m) $\lim_{x \rightarrow -\infty} \frac{2^{3x-1}}{5^{2x+1}}$

b) $\lim_{x \rightarrow \infty} \left(-\frac{5}{3}x^7 \right)$

f) $\lim_{x \rightarrow \infty} \left(-\frac{\sqrt{5}}{13}x^{12} \right)$

j) $\lim_{x \rightarrow \infty} \sqrt{x-8}$

n) $\lim_{x \rightarrow \infty} \frac{2^{3x-1}}{5^{2x+1}}$

c) $\lim_{x \rightarrow -\infty} (4.3x^6)$

g) $\lim_{x \rightarrow -\infty} (\log_3 x)$

k) $\lim_{x \rightarrow -\infty} \frac{5}{x^3}$

o) $\lim_{x \rightarrow -\infty} \frac{2^{3x-1}}{5^{x-2}}$

d) $\lim_{x \rightarrow \infty} (4.3x^6)$

h) $\lim_{x \rightarrow \infty} (\log_3 x)$

l) $\lim_{x \rightarrow \infty} \frac{5}{x^3}$

p) $\lim_{x \rightarrow \infty} \frac{2^{3x-1}}{5^{x-2}}$

3. Find the exact value of $\log_2 \left(\log_2 \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}}}} \right)$.

4. Re-write $0.\overline{390} = 0.39090909090\dots$ as a ratio of two integers. You do not need to reduce the fraction.

5. Simplify each of the following.

a) $2^{\log_2 A} - 2^{\log_2 B} + 2^{\frac{1}{3}\log_2 C}$

b) $2^{\log_2 A - \log_2 B} + \frac{1}{3}\log_2 C$

c) $\tan \left(\frac{5\pi}{12} \right)$

d) $\sin 22.5^\circ$

6. Find $\tan x$ if we know that $\tan 2x = \frac{48}{55}$.

7. Solve each of the following inequalities.

a) $\frac{2x-5}{3x+1} < -2$

b) $2x + \frac{1}{3}x^2 \leq -3$

c) $\frac{3}{x-1} \leq \frac{x}{2}$

d) $(x+7)x(x-8) \leq 0$

e) $x^4 - x^2 \geq 0$

8. Solve each of the following equations.

a) $\sin 2x = \cos x$

b) $\cos 2x = \sin x$

c) $\sin 2x = \sin x$

d) $\cos 2x = \cos x$

9. Perform the following divisions on the polynomials.

a) $(x^4 - 2) \div (x + 1)$

c) $(2x^4 - x^3 + 2x - 4) \div (x^2 + 2x - 1)$

b) $(x^5 + 4x^3 - x^2 + 10x - 3) \div (x^2 + 1)$

10. Find an equation for all tangent lines drawn to the given functions.

a) to $f(x) = -\frac{1}{2}x^2 - 3x + 7$ from the point $(-1, 14)$

b) to $f(x) = \frac{1}{2}x^2 + x - 1$ from the point $(-1, -6)$

11. Suppose that $f(x) = x^2 - 3x + 1$ and $g(x) = -2x + 1$. Compute each of the following.

a) $f(g(-1))$

b) $g(f(-1))$

c) $f(2a)$

d) $g(2a)$

e) $f(g(x))$

f) $g(f(x))$

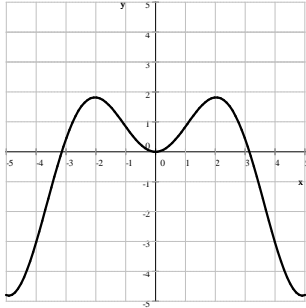
g) Is there any value of x for which $f(g(x)) = g(f(x))$?

12. Graph each of the following functions on the intervals indicated.

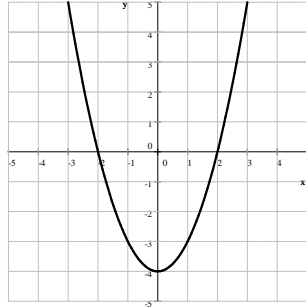
a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$ b) $g(x) = \csc x$ on $[-2\pi, 2\pi]$ c) $h(x) = \sqrt{9 - x^2}$ on its natural domain.

13. Graph $y = \frac{1}{f(x)}$ given the graph of $y = f(x)$.

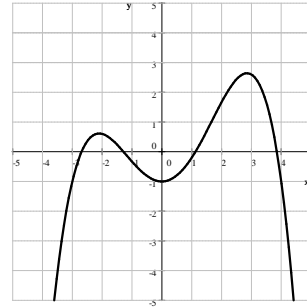
a)



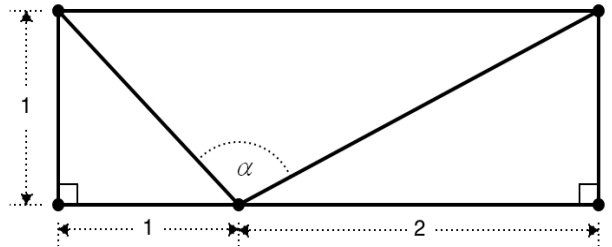
b)



c)



14. Compute the exact value of $\tan \alpha$ if α is the angle shown on the picture.



15. Suppose that x is an angle belonging to the second quadrant and y is an angle belonging to the fourth quadrant. We also know that $\sin x = \frac{2}{3}$ and $\cos y = \frac{3}{5}$. Compute each of the following.

a) $\cos x$ c) $\sin 2x$ e) $\sin(x - y)$ g) $\tan 2x$
 b) $\sin y$ d) $\cos 2x$ f) $\cos(x + y)$

16. Compute $\sin \alpha$ and $\cos \alpha$ in terms of M if we know that α is an acute angle and that $\tan \alpha = M$.

Answers

1. a) $\alpha = 53^\circ$ $b \approx 19.44087$ ft $a \approx 15.8730364$ ft b) no solution
 c) $\gamma_1 \approx 40.075583^\circ$, $\beta_1 \approx 108.924417^\circ$, $b_1 \approx 7.34662$ cm or
 $\gamma_2 \approx 139.924417^\circ$, $\beta_2 \approx 9.075583^\circ$, $b_2 \approx 1.2250532$ cm
2. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $-\infty$ f) $-\infty$ g) undefined h) ∞ i) undefined
 j) ∞ k) 0 l) 0 m) ∞ n) 0 o) 0 p) ∞
3. -7
4. $\frac{387}{990}$
5. a) $A - B + \sqrt[3]{C}$ b) $\frac{A\sqrt[3]{C}}{B}$ c) $2 + \sqrt{3}$ d) $\frac{1}{2}\sqrt{2 - \sqrt{2}}$

6. $\frac{3}{8}$ or $-\frac{8}{3}$

7. a) $\left(-\frac{1}{3}, \frac{3}{8}\right)$ b) $\{-3\}$ c) $[-2, 1) \cup [3, \infty)$ d) $(-\infty, -7] \cup [0, 8]$ e) $(-\infty, -1] \cup \{0\} \cup [1, \infty)$

8. a) $\frac{\pi}{2} + k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ b) $-\frac{\pi}{2} + 2k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

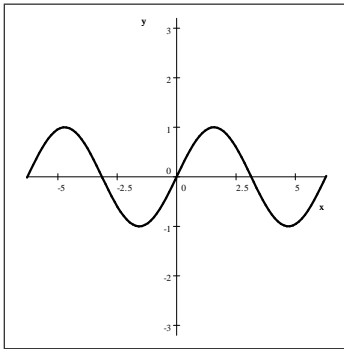
c) $k\pi, \pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ d) $2k\pi, \pm\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

9. a) $x^3 - x^2 + x - 1$ R -1 b) $x^3 + 3x - 1$ R $7x - 2$ c) $2x^2 - 5x + 12$ R $-27x + 8$

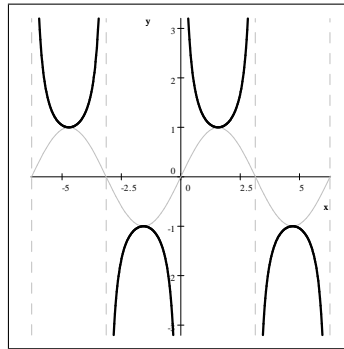
10. a) $y = -5x + 9$ and $y = x + 15$ b) $y = -3x - 9$ and $y = 3x - 3$

11. a) 1 b) -9 c) $4a^2 - 6a + 1$ d) $-4a + 1$ e) $4x^2 + 2x - 1$ f) $-2x^2 + 6x - 1$ g) 0 and $\frac{2}{3}$

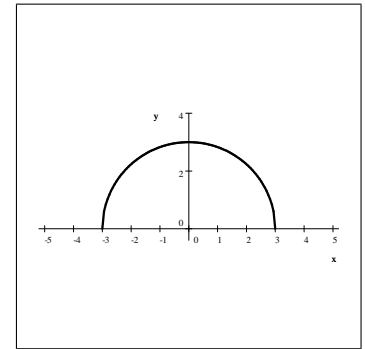
12. a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$



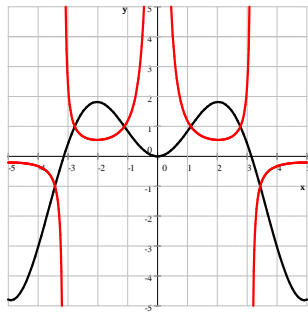
b) $g(x) = \csc x$ on $[-2\pi, 2\pi]$



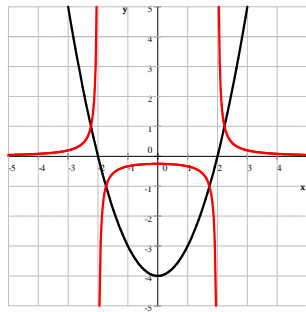
c) $h(x) = \sqrt{9 - x^2}$ on $[-3, 3]$



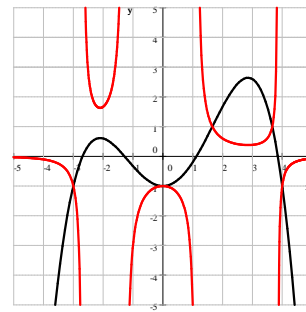
13. a)



b)



c)



14. -3

15. a) $-\frac{\sqrt{5}}{3}$ b) $-\frac{4}{5}$ c) $-\frac{4\sqrt{5}}{9}$ d) $\frac{1}{9}$ e) $\frac{6 - 4\sqrt{5}}{15}$ f) $-\frac{3\sqrt{5} - 8}{15}$ g) $4\sqrt{5}$

16. $\sin \alpha = \frac{M}{\sqrt{M^2 + 1}}$ and $\cos \alpha = \frac{1}{\sqrt{M^2 + 1}}$