

Please note that Quiz 21 will also cover material from reviews from previous material. Please review those topics even if they do not appear in this document.

1. Graph each of the following. State the steps in graphing the functions.

a) $f(x) = 3 - \frac{2}{x+4}$ b) $g(x) = -\sqrt{-x+3} + 1$ c) $h(x) = 2\log_2(x-1) - 3$

2. Simplify each of the following.

a) $\frac{2\cos^2\frac{\pi}{8} - 1}{1 + 8\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}}$ e) $\log_2\log_3 81$
 b) $5^{(\log_{10} 625 / \log_{10} 25)}$ f) $\frac{\log_2 24 - \frac{1}{2}\log_2 72}{\log_3 18 - \frac{1}{3}\log_3 72}$
 c) $\log_8(12x^2) - \log_8(15x^3) + \log_8(20x)$ g) $\cos\frac{23\pi}{4} - \sin\frac{15\pi}{4}$
 d) $2\log_3 6 - \frac{1}{2}\log_3 400 + 3\log_3 \sqrt[3]{45}$

3. Prove each of the following.

a) $\sin 35^\circ + \sin 25^\circ = \cos 5^\circ$ b) $\cos 12^\circ - \cos 48^\circ = \sin 18^\circ$ c) $1 + \tan \alpha \tan \beta = \frac{\cos(\alpha - \beta)}{\cos \alpha \cos \beta}$

4. Solve each of the following.

a) $\frac{1}{x-2} \leq \frac{3}{4}$ d) $\log_3(x^3 - x) - \log_3 x = 1$ g) $\sin 2x - \cos x = 0$
 b) $\log_2(x+1) + \log_2(x+3) = 3$ e) $16^x - 4^{x+1} = 5$ h) $\sin 2x = \frac{\sqrt{3}}{2}$
 c) $\log_3 x + \log_x 3 = \frac{5}{2}$ f) $\frac{\sqrt{3-x} + \sqrt{3+x}}{\sqrt{3-x} - \sqrt{3+x}} = 2$

5. Suppose that $\log_7 2 = m$. Write $\log_{49} 28$ in terms of m .

6. Compute the inverse for each of the following functions.

a) $f(x) = \frac{2}{3}x - 6$ c) $f(x) = (2x-3)^3 + 8$ e) $f(x) = \frac{7x+10}{3x-7}$ f) $f(x) = e^{4x-1} - 3$
 b) $f(x) = \sqrt[3]{2x+1}$ d) $f(x) = \frac{3x-1}{7x-5}$ g) $f(x) = \log_3(5x-4)$

7. Graph the function given and graph the inverse relation in the same coordinate system. You do not have to find the equation for the inverse.

a) $f(x) = \log_2 x$ b) $f(x) = |x|$ c) $y = x(x^2 - 9)$

8. Perform the indicated operations on the complex numbers.

a) $(3-i)^2 - (3+i)^2$ b) $\frac{34+2i}{5+2i}$ c) $(1+i)^2$ d) $(1+i)^4$ e) i^{143}

9. Multiply the following polynomials with complex coefficients.

a) $(x+3i)(x-3i)$ b) $(x-1+2i)(x-1-2i)$ c) $(x^2-i)(x^2+i)$ d) $(x+3+2i)(x+3-2i)$

10. Find the exact value of the cosine of the largest angle in a triangle with sides 10 cm, 8 cm, and 7 cm long.

11. Compute each of the following.

- a) $\sin(37.5^\circ)$ (Hint: 37.5° is half of what angle?) c) $\cos\left(\frac{225^\circ}{2}\right)$
- b) $\tan \alpha$ if $\sin 2\alpha = \frac{2}{3}$ and 2α is in the first quadrant.

12. Express each of the following products as a sum or difference.

- a) $\cos 7a \sin 15a$ b) $\sin 7a \cos 15a$ c) $\cos 7a \cos 15a$ d) $\sin 7a \sin 15a$

13. Express each of the following sums or differences as a product.

- a) $\cos 16M + \cos 4M$ b) $\cos 16M - \cos 4M$ c) $\sin 16M + \sin 4M$ d) $\sin 16M - \sin 4M$

14. Perform the following divisions between the polynomials given.

- a) $x^4 + 5x^3 - x^2 + 10$ by $x + 2$ e) $x^4 + x^2 - 3$ by $x^2 - x + 2$
- b) $5x^4 - 7x^3 + 3x^2 - 9x + 8$ by $x - 1$ d) $x^4 - 3x^3 + 5x^2 - 6x + 1$ by $x^2 - 2$
- c) $x^4 - 6x^3 + 3x^2 - 2x$ by $x - 5$ f) $x^6 - 1$ by $x^2 - 1$

15. Classify the discontinuities of each of the following functions as a hole or a vertical asymptote.

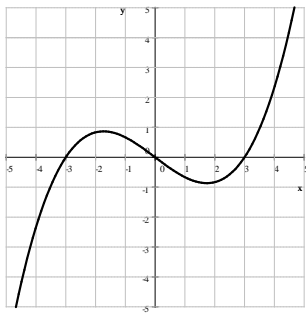
- a) $f(x) = \frac{x^3 - x}{x^2(x - 1)}$ b) $g(x) = \frac{(x + 3)^2(x + 1)^3x^4(x - 2)^5(x - 5)^8}{(x + 3)^3(x + 1)^2x^4(x - 2)^8(x - 5)^9}$

16. Graph each of the following.

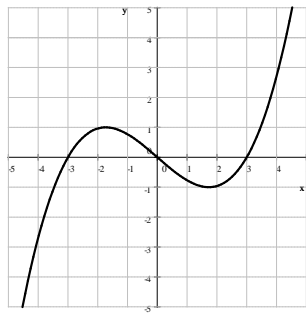
- a) $f(x) = x^4 - 4x^2$ b) $g(x) = -x^3 + 6x$ c) $h(x) = -(x + 2)^2x^3(x - 2)^2$

17. Given the graph of $y = f(x)$, plot the graph of $y = \frac{1}{f(x)}$ in the same coordinate system.

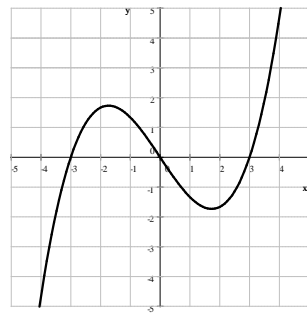
a)



b)



c)



18. Find $\tan \beta$ if we know that $\tan \alpha = \frac{1}{2}$ and $\tan(\alpha + \beta) = \frac{13}{11}$.

19. Find an equation for the tangent line drawn to the circle $(x - 4)^2 + (y + 5)^2 = 25$ at the point $(7, -1)$.

20. Find the domain of each of the following functions.

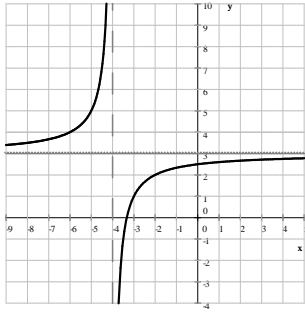
- a) $f(x) = \ln(\sin^2 x)$ b) $f(x) = \frac{1}{\sin x - \cos x}$ c) $f(x) = \log_5(\sin x + \cos x)^2$

21. Two sides of a triangle are 8 ft and 15 ft long. Find the exact value of the third side if we know that the area of the triangle is 48 ft^2 .

Answers

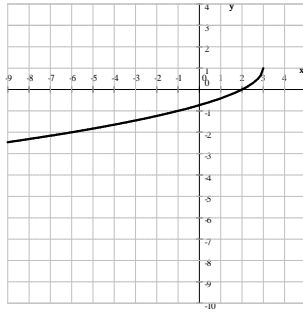
1. a) $f(x) = 3 - \frac{2}{x+4}$

Start with $y = \frac{1}{x}$
 shift to the left by 4 units
 stretch along by the y -axis by 2
 reflect to the x -axis
 shift up by 3 units



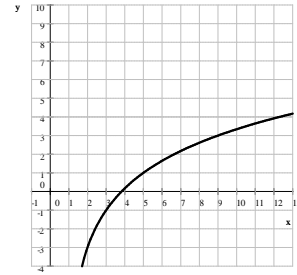
b) $g(x) = -\sqrt{-x+3} + 1$

Start with $y = \sqrt{x}$
 reflect to the y -axis
 shift to the right by 3 units
 reflect to the x -axis
 shift up by 1 unit



c) $h(x) = 2 \log_2(x-1) - 3$

Start with $y = \log_2 x$
 shift to the right by 1 unit
 stretch along by the y -axis by 2
 reflect to the x -axis
 shift down by 3 units



2. a) $\frac{\sqrt{2}}{4}$ b) 25 c) $\frac{4}{3}$ d) 4 e) 2 f) $\frac{9}{8}$ g) $\sqrt{2}$

3. a) $\sin 35^\circ + \sin 25^\circ = \cos 5^\circ$

$$\begin{aligned} \sin 35^\circ + \sin 25^\circ &= \sin(30^\circ + 5^\circ) + \sin(30^\circ - 5^\circ) \\ &= \sin 30^\circ \cos 5^\circ + \cos 30^\circ \sin 5^\circ + \sin 30^\circ \cos 5^\circ - \cos 30^\circ \sin 5^\circ \\ &= 2 \sin 30^\circ \cos 5^\circ = 2 \left(\frac{1}{2}\right) \cos 5^\circ = \cos 5^\circ \end{aligned}$$

b) $\cos 12^\circ - \cos 48^\circ = \sin 18^\circ$

$$\begin{aligned} \cos 12^\circ - \cos 48^\circ &= \cos(30^\circ - 18^\circ) - \cos(30^\circ + 18^\circ) \\ &= \cos 30^\circ \cos 18^\circ + \sin 30^\circ \sin 18^\circ - (\cos 30^\circ \cos 18^\circ - \sin 30^\circ \sin 18^\circ) \\ &= \cos 30^\circ \cos 18^\circ + \sin 30^\circ \sin 18^\circ - \cos 30^\circ \cos 18^\circ + \sin 30^\circ \sin 18^\circ \\ &= 2 \sin 30^\circ \sin 18^\circ = 2 \left(\frac{1}{2}\right) \sin 18^\circ = \sin 18^\circ \end{aligned}$$

c) $1 + \tan \alpha \tan \beta = \frac{\cos(\alpha - \beta)}{\cos \alpha \cos \beta}$

$$\begin{aligned} \text{LHS} &= 1 + \tan \alpha \tan \beta = 1 + \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \alpha \cos \beta} \\ &= \frac{\cos(\alpha - \beta)}{\cos \alpha \cos \beta} = \text{RHS} \end{aligned}$$

4. a) $(-\infty, 2) \cup \left[\frac{10}{3}, \infty\right)$ b) 1 c) $9, \sqrt{3}$ d) 2 e) $\log_4 5$ f) $-\frac{12}{5}$

g) $\frac{\pi}{2} + k\pi$ $\frac{\pi}{6} + 2k\pi$ $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ h) $x = \frac{\pi}{6} + k\pi$ or $x = \frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$

5. $m + \frac{1}{2}$

$$\log_{49} 28 = \frac{\log_7 28}{\log_7 49} = \frac{\log_7 7 + \log_7 4}{2} = \frac{1 + 2\log_7 2}{2} = \frac{1 + 2m}{2} = m + \frac{1}{2}$$

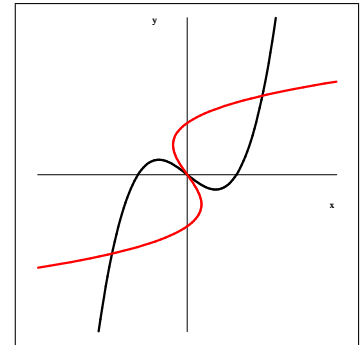
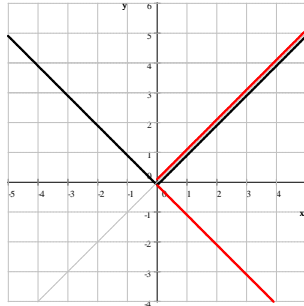
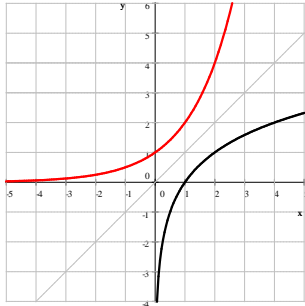
6. a) $f^{-1}(x) = \frac{3}{2}x + 9$ b) $f^{-1}(x) = \frac{1}{2}x^3 - \frac{1}{2}$ c) $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-8} + \frac{3}{2}$ d) $f^{-1}(x) = \frac{5x-1}{7x-3}$

e) $f^{-1}(x) = \frac{7x+10}{3x-7}$ f) $f^{-1}(x) = \frac{1}{4}(\ln(x+3)+1)$ g) $f^{-1}(x) = \frac{1}{5}(3^x+4)$

7. a) $f(x) = \log_2 x$

b) $f(x) = |x|$

c) $y = x(x^2 - 9)$



8. a) $-12i$ b) $6 - 2i$ c) $2i$ d) -4 e) $-i$

9. a) $x^2 + 9$ b) $x^2 - 2x + 5$ c) $x^4 + 1$ d) $x^2 + 6x + 13$

10. $\frac{13}{112}$

11. a) $\sqrt{\frac{\sqrt{2}+4-\sqrt{6}}{8}}$ b) $\frac{3-\sqrt{5}}{2}$ c) $-\frac{1}{2}\sqrt{2-\sqrt{2}}$

12. a) $\frac{1}{2}\sin 8a + \frac{1}{2}\sin 22a$ b) $\frac{1}{2}\sin 22a - \frac{1}{2}\sin 8a$ c) $\frac{1}{2}\cos 8a + \frac{1}{2}\cos 22a$ d) $\frac{1}{2}\cos 8a - \frac{1}{2}\cos 22a$

13. a) $2\cos 10M \cos 6M$ b) $-2\sin 10M \sin 6M$ c) $2\sin 10M \cos 6M$ d) $2\cos 10M \sin 6M$

14. a) $x^3 + 3x^2 - 7x + 14$ R -18 b) $5x^3 - 2x^2 + x - 8$ c) $x^3 - x^2 - 2x - 12$ R -60

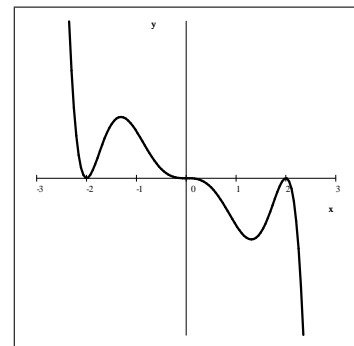
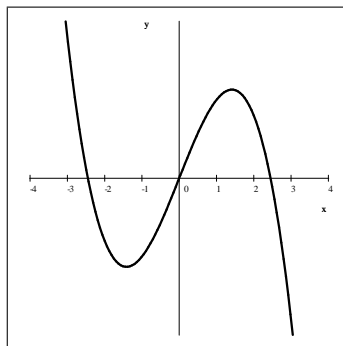
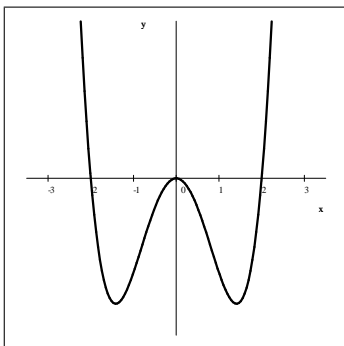
d) $x^2 - 3x + 7$ R $-12x + 15$ e) $x^2 + x$ R $-2x - 3$ f) $x^4 + x^2 + 1$

15. a) vertical asymptote at $x = 0$ hole at $x = 1$ b) vertical asymptote at $x = -3, 2, 5$ hole at $x = -1, 0$

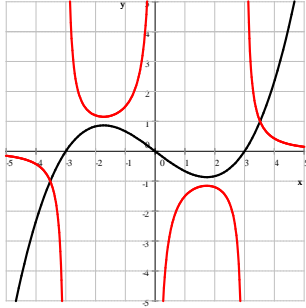
16. a) $f(x) = x^4 - 4x^2$

b) $g(x) = -x^3 + 6x$

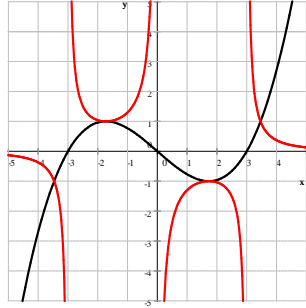
c) $h(x) = -(x+2)^2 x^3 (x-2)^2$



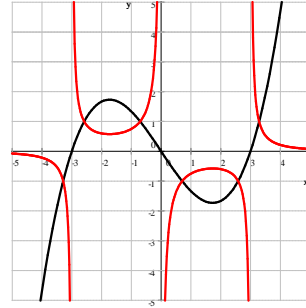
17. a)



b)



c)

18. $\frac{3}{7}$

19. $y = -\frac{3}{4}x + \frac{17}{4}$

20. a) $x \neq k\pi \quad k \in \mathbb{Z}$

b) $x \neq \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$

c) $x \neq -\frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$

21. $\sqrt{145}$ ft or $\sqrt{433}$ ft