

1. Perform the indicated operations on the complex numbers.

a)  $(3 - 2i)(7 + i)$    b)  $(2 - i)^2$    c)  $(3 + i)(3 - i)$    d)  $|2i - 1|$    e)  $\frac{5 - 5i}{2 + i}$    f)  $i^{143}$    g)  $(1 + i)^4$

2. Find all complex numbers  $z$  so that  $z^2 = 24 - 70i$

3. Let  $f$  be the function defined by  $f(x) = x^2 - 4x + 9$ .

a) Compute  $f(3)$ .   b) Is it true that  $f(2 + 3) = f(2) + f(3)$ ?   c) Simplify  $f(2a)$

4. Let  $f$  be the function defined by  $f(x) = \sqrt{x - 5}$ .

a) Compute  $f(3)$ .   b) Compute  $f(9) + f(21)$ .   c) Compute  $f(9 + 21)$ .

5. Given that  $f(x) = 2x - 3$  and  $g(x) = x^2 + 5$  compute each of the following.

a)  $(f + g)(5)$    c)  $\left(\frac{f}{g}\right)(4)$    e)  $g(f(2))$    g)  $g(f(0))$

b)  $(f + g)(-2)$    d)  $f(g(2))$    f)  $f(g(0))$

6. Two sides of a triangle are 8 cm and 15 cm long. Find the length of the third side so that the area of the triangle is greatest.

7. Find the smallest and largest value of  $3 \sin x + 4 \cos x$ .

8. Simplify each of the following.

a)  $\log_{10}(8x^7) + \frac{1}{2} \log_{10}(25x^{16}) - 2 \log_{10}(2x^3)$    c)  $\ln(a^2 - 4) - \ln(a - 2)$

b)  $5^{-\log_{25} 11}$    d)  $8^{\log_8 m - \log_2(3m)}$

9. Compute the exact value of each of the following expressions.

$A = (4\sqrt{18} - 5\sqrt{50} + 3\sqrt{98})(2\sqrt{2})$     $C = 2 \log_{10} 2 + \log_{10} 18 + 6 \log_{10} \sqrt{5} - 2 \log_{10} 3$

$B = 6^{-2} \cdot 16^{-1/2} \cdot 36^2 \cdot 4^{-3/2}$

10. Solve each of the following equations.

a)  $\sqrt{2x - 1} + \sqrt{x - 1} = 5$    d)  $\frac{2}{3} \ln(2x - 5) + 3 = 7$    f)  $\log_3(x - 3) - \log_3(4x - 2) = -2$

b)  $3^{2x-1} = 2^{2-x}$    g)  $\log_3(x - 4) + \log_3(2x - 5) = 2$

c)  $4^x - 5 \cdot 2^x = -4$    e)  $\log_{3x+1}\left(\frac{1}{4}\right) = -1$    h)  $\frac{1}{x} + \frac{1}{x-2} = \frac{3}{4}$

11. Solve each of the following inequalities.

a)  $\frac{5x - 2}{2x + 1} \geq 2$    b)  $\frac{x}{2 - x} > 4$

12. Perform each of the following divisions.

a)  $4x^5 - x^3 + 8$  by  $x^2 + x - 2$    b)  $x^5 - 1$  by  $x - 1$

13. Suppose that  $x = \log_5 10$  and  $y = \log_9 5$ . Express  $\log_{24} 60$  in terms of  $x$  and  $y$ .

14. a) Find the points where the circles  $(x - 1)^2 + (y + 7)^2 = 50$  and  $(x - 5)^2 + (y + 4)^2 = 25$  intersect each other.

b) Find an equation of the tangent line drawn to the graph of  $4x - 2y + x^2 + y^2 = 20$  at the point  $(-6, 4)$ .

15. Find an equation for all tangent lines drawn to  $y = \frac{1}{2}x^2 + 3x - 5$  from the point  $(-1, -12)$ .

16. Solve the triangle given that  $a = 10$ ,  $\alpha = 95^\circ$ , and  $\beta = 48^\circ$ .

17. a) Compute the exact value of  $\sin \frac{x}{2}$  if  $\cos x = \frac{1}{3}$ .      b) Compute the exact value of  $\tan 22.5^\circ$ .

18. Simplify the expression  $\frac{\sin 16A - \sin 4A}{\cos 4A - \cos 16A}$  (Hint: use the sum-product identities!)

19. Compute the domain of each of the following expressions.

a)  $\ln(x^2 + 4x)$       b)  $\ln x + \ln(x + 4)$       c)  $\sqrt{x^2 - 4} + \sqrt{4 - x^2}$       d)  $2^x - 5^{x+1}$

20. For each of the following functions given, determine its discontinuities and whether they are vertical asymptotes or holes.

a)  $f(x) = \frac{(x+5)^2(x+1)^4x^2(x-2)^6(x-3)}{(x+5)(x+4)^2(x+1)^4x^4(x-2)(x-3)^2}$       b)  $g(x) = \frac{(2x+4)(x+3)^2}{(x-2)(x^3-9x)}$

21. Express  $\tan 3x$  in terms of  $\tan x$ .

22. a) Solve the equation  $\cos 4x = -\frac{1}{2}$ .      b) List all solutions that fall between  $0^\circ$  and  $360^\circ$ .

23. Suppose that  $\sin \alpha = \frac{3}{5}$ . Compute  $\sin 4\alpha$  if  $\alpha$  is      a) in the first quadrant.      b) in the second quadrant.

24. Prove each of the following identities.

a)  $\tan^2 x = \sec^2 x - 1$       b)  $\frac{\tan x}{\sec x - 1} = \frac{\sec x + 1}{\tan x}$       c)  $\tan(x + \pi) = \tan x$

25. Prove that if  $\alpha$  and  $\beta$  are angles in a triangle such that

$$(\sin \alpha + \sin \beta)^2 + (\cos \alpha - \cos \beta)^2 = 2$$

then the triangle has a right angle.

26. Let  $l$  be the line  $y = \frac{5}{12}x$ . Let  $k$  be the line that bisects the angle formed by  $l$  and the positive part of the  $x$ -axis. Find the equation of  $k$ .

27. Graph each of the following functions on the interval given.

a)  $f(x) = -\sin\left(\pi x - \frac{\pi}{2}\right) + 1$  on  $[-5, 5]$       b)  $g(x) = 2 \cos\left(\frac{x}{3} - \frac{\pi}{4}\right)$  on  $[-6\pi, 6\pi]$

28. Compute each of the following limits.

a)  $\lim_{x \rightarrow -\infty} \frac{2^x + 1}{3 \cdot 2^x - 1}$       f)  $\lim_{x \rightarrow -\infty} \frac{100x - 1}{5x^2 - 3x + 2}$       k)  $\lim_{x \rightarrow \infty} \frac{2^{x+5}}{4^{x-1}}$   
 b)  $\lim_{x \rightarrow \infty} \frac{2^x + 1}{3 \cdot 2^x - 1}$       g)  $\lim_{x \rightarrow -\infty} \log_2 x$       l)  $\lim_{x \rightarrow \infty} \frac{3^{x+1} \cdot \left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$   
 c)  $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^6)$       h)  $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$   
 d)  $\lim_{x \rightarrow \infty} (-2x^5 + 8x^6)$       i)  $\lim_{x \rightarrow \infty} \frac{-x^3 + 2x + 1}{x - 3}$   
 e)  $\lim_{x \rightarrow -\infty} \frac{3x^2 - 1}{5x^2 - 3x + 2}$       j)  $\lim_{x \rightarrow -\infty} (2^x)$

29. Sketch the graph of each of the following functions.

a)  $f(x) = x^4 - 5x^2$       b)  $f(x) = (x^2 - 4)(x - 2)^2$       c)  $f(x) = -\frac{1}{2}x(x - 3)^2$

30. Sketch the graph of each of the following functions.

$$\text{a) } f(x) = \frac{(x+2)(x-1)(x-4)}{(x+2)(x^2+x)}$$

$$\text{b) } f(x) = \frac{-2(x+5)^4(x+1)^4x^3(x-2)^5(x-3)}{(x+5)^2(x+4)(x+1)^4x^5(x-2)^2(x-3)^2}$$

$$\text{c) } f(x) = \frac{3x-10}{x+3}$$

## Answers

1.) a)  $23 - 11i$     b)  $3 - 4i$     c)  $10$     d)  $\sqrt{5}$     e)  $1 - 3i$     f)  $-i$     g)  $-4$

2.)  $7 - 5i$  and  $-7 + 5i$     3.) a)  $f(3) = 6$     b)  $f(5) = 14$  and  $f(2) + f(3) = 11$     c)  $f(2a) = 4a^2 - 8a + 9$

4.) a)  $f(3) = \text{undefined}$     b)  $f(9) + f(21) = 6$     c)  $f(30) = 5$

5.) a)  $37$     b)  $2$     c)  $\frac{5}{21}$     d)  $15$     e)  $6$     f)  $7$     g)  $14$     6.)  $17 \text{ cm}$

7.) smallest value:  $-5$     largest value:  $5$     8.) a)  $1 + 9 \log_{10} x$     b)  $\frac{1}{\sqrt{11}}$     c)  $\ln(a+2)$     d)  $\frac{1}{27m^2}$

9.)  $A = 32$      $B = \frac{9}{8}$      $C = 3$     10.) a)  $5$  ( $145$  doesn't work)    b)  $\log_{18} 12$     c)  $0, 2$

d)  $\frac{1}{2}(e^6 + 5)$     e)  $1$     f)  $5$     g)  $\frac{11}{2}$     h)  $\frac{2}{3}, 4$     11.) a)  $\left(-\infty, -\frac{1}{2}\right) \cup [4, \infty)$     b)  $\left(\frac{8}{5}, 2\right)$

12.) a)  $4x^3 - 4x^2 + 11x - 19$  R  $41x - 30$     b)  $x^4 + x^3 + x^2 + x + 1$

13.)  $x - 1 = \log_5 2$     and     $\frac{1}{2y} = \log_5 3$

$$\log_{24} 60 = \frac{\log_5 60}{\log_5 24} = \frac{2 \log_5 2 + \log_5 3 + 1}{3 \log_5 2 + \log_5 3} = \frac{2(x-1) + \frac{1}{2y} + 1}{3(x-1) + \frac{1}{2y}} = \frac{4xy - 2y + 1}{6xy - 6y + 1}$$

14.) a)  $(8, -8)$  and  $(2, 0)$     b)  $\frac{4}{3}(x+6) = y - 4$  or  $y = \frac{4}{3}x + 12$     15.)  $y = 5x - 7$  and  $y = -x - 13$

16.)  $\gamma = 37^\circ$      $b \approx 7.4598$      $c \approx 6.0411386$     17.) a)  $\pm \frac{\sqrt{3}}{3}$     b)  $\sqrt{2} - 1$     18.)  $\cot 10A$

19.) a)  $(-\infty, -4) \cup (0, \infty)$     b)  $(0, \infty)$     c)  $\{-2, 2\}$     d)  $\mathbb{R}$

20.) a) discontinuities at  $x = -5, -4, -1, 0, 2, 3$     vertical asymptotes at  $x = -4, 0, 3$  and holes at  $x = -5, -1, 2$     b) discontinuities at  $x = -3, 0, 2, 3$     vertical asymptotes at  $x = 0, 2, 3$  and

hole at  $x = -3$     21.)  $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

$$\begin{aligned} \tan 3x &= \tan(x+2x) = \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = \frac{\tan x + \frac{2 \tan x}{1 - \tan^2 x}}{1 - \tan x \frac{2 \tan x}{1 - \tan^2 x}} = \frac{\tan x + \frac{2 \tan x}{1 - \tan^2 x}}{1 - \tan x \frac{2 \tan x}{1 - \tan^2 x}} \cdot \frac{1 - \tan^2 x}{1 - \tan^2 x} \\ &= \frac{\tan x(1 - \tan^2 x) + 2 \tan x}{1(1 - \tan^2 x) - \tan x(2 \tan x)} = \frac{\tan x - \tan^3 x + 2 \tan x}{1 - \tan^2 x - 2 \tan^2 x} = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \end{aligned}$$

22.) a)  $\pm \frac{\pi}{6} + \frac{k\pi}{2}$  where  $k \in \mathbb{Z}$

b)  $60^\circ, 150^\circ, 240^\circ, 330^\circ$  (from  $-30^\circ + k \cdot 90^\circ$ ) and  $30^\circ, 120^\circ, 210^\circ, 300^\circ$  (from  $30^\circ + k \cdot 90^\circ$ )

23.) a)  $\frac{336}{625}$     b)  $-\frac{336}{625}$

24.) a)

$$\text{RHS} = \sec^2 x - 1 = \frac{1}{\cos^2 x} - 1 = \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} = \text{LHS}$$

b)

$$\text{LHS} = \frac{\tan x}{\sec x - 1} = \frac{\tan x}{\sec x - 1} \cdot \frac{\sec x + 1}{\sec x + 1} = \frac{\tan x (\sec x + 1)}{\sec^2 x - 1} = \frac{\tan x (\sec x + 1)}{\tan^2 x} = \frac{\sec x + 1}{\tan x} = \text{RHS}$$

c)

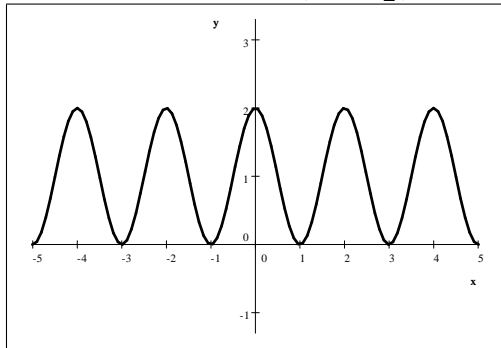
$$\text{LHS} = \tan(x + \pi) = \frac{\tan x + \tan \pi}{1 - \tan x \cdot \tan \pi} = \frac{\tan x + 0}{1 - \tan x \cdot 0} = \tan x = \text{RHS}$$

25.)

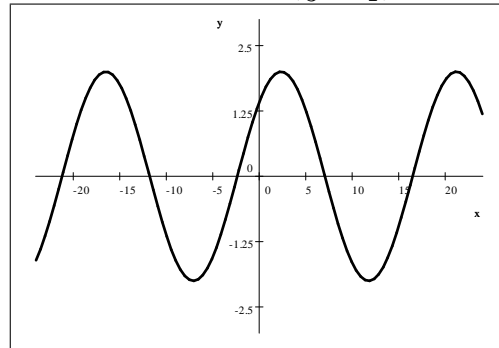
$$\begin{aligned} (\sin \alpha + \sin \beta)^2 + (\cos \alpha - \cos \beta)^2 &= 2 \\ \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta - 2 \cos \alpha \cos \beta &= 2 \\ 2 + 2 \sin \alpha \sin \beta - 2 \cos \alpha \cos \beta &= 2 \\ 2 \sin \alpha \sin \beta - 2 \cos \alpha \cos \beta &= 0 \\ -2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) &= 0 \\ -2 \cos(\alpha + \beta) &= 0 \\ \alpha + \beta &= 90^\circ \implies \gamma = 90^\circ \end{aligned}$$

26.)  $y = \frac{1}{5}x$  or  $y = -5x$

27.) a)  $f(x) = -\sin\left(\pi x - \frac{\pi}{2}\right) + 1$  on  $[-5, 5]$

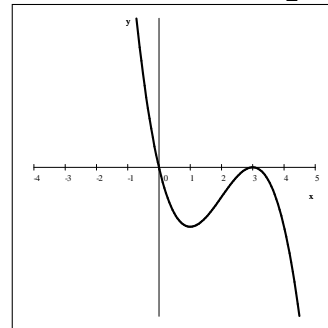
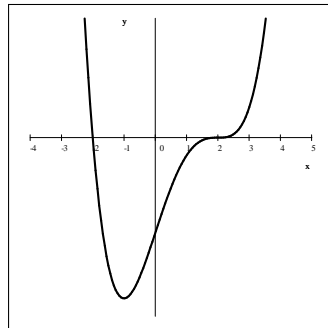
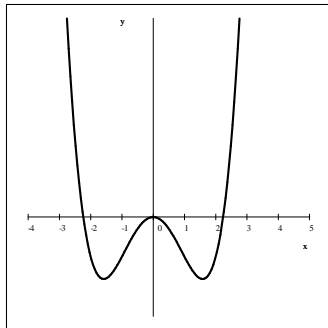


b)  $g(x) = 2 \cos\left(\frac{x}{3} - \frac{\pi}{4}\right)$  on  $[-6\pi, 6\pi]$

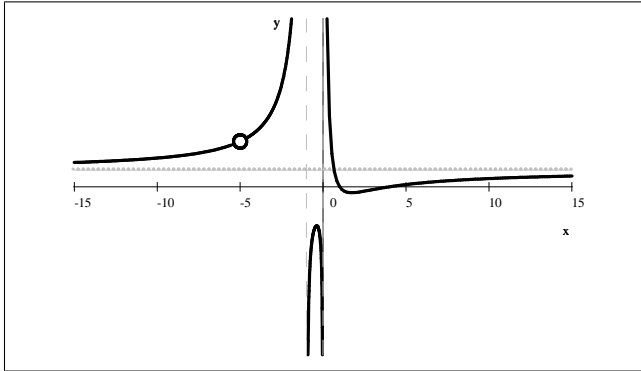


28.) a)  $-1$     b)  $\frac{1}{3}$     c)  $\infty$     d)  $\infty$     e)  $\frac{3}{5}$     f)  $0$     g) undefined    h)  $\frac{2}{3}$     i)  $-\infty$     j)  $0$     k)  $0$     l)  $3$

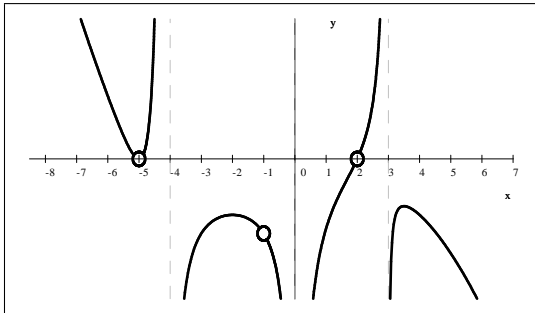
29.) a)  $f(x) = (x + \sqrt{5})x^2(x - \sqrt{5})$     b)  $f(x) = (x + 2)(x - 2)^3$     c)  $f(x) = -\frac{1}{2}x(x - 3)^2$



$$30.) \text{ a) } f(x) = \frac{(x+5)(x-1)(x-4)}{(x+5)(x^2+x)} = \begin{cases} \frac{(x-1)(x-4)}{x(x+1)} & \text{if } x \neq -5 \\ \text{undefined} & \text{if } x = -5 \end{cases}$$



$$\text{b) } f(x) = \frac{-2(x+5)^4(x+1)^4x^3(x-2)^5(x-3)}{(x+5)^2(x+4)(x+1)^4x^5(x-2)^2(x-3)^2} = \begin{cases} \frac{-2(x+5)^2(x-2)^3}{(x+4)x^2(x-3)} & \text{if } x \neq -5, -1, 2 \\ \text{undefined} & \text{if } x = -5, -1, 2 \end{cases}$$



$$\text{c) } f(x) = \frac{3x-10}{x+3}$$

