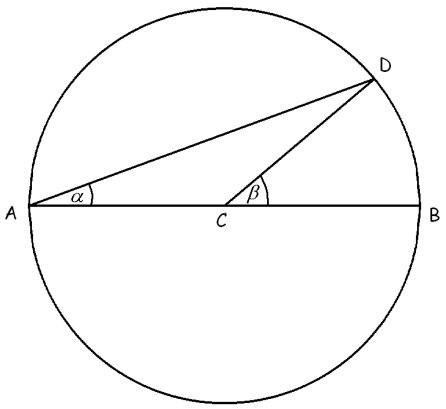
1. Suppose that $3^{400} = A$. Express each of the following in terms of A.

   a) 3^{402} b) 3^{399} c) 3^{200} d) 9^{400} e) $3^{400} + 3^{401} + 3^{402}$

2. List all three-digit numbers that can be formed using the numbers $1, 3, 6$ and 7 if repetition of digits is not allowed.

3. Prove that for every positive number x, $x + \dfrac{1}{x} \geq 2$.

4. Consider the circle shown on the picture. Prove that $\beta = 2\alpha$.



5. How many diagonals are there in a regular polygon of n sides?

6. Simplify each of the following.

   a) $\dfrac{3^{-2} - 2^{-3}}{3^{-2} + 2^{-3}}$ b) $\dfrac{a^{-2}b^3}{a^5 b^{-1}}$ c) $\dfrac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}}$ d) $\dfrac{\left(-2a^3b^{-1}\right)^{-2}\left(-ab^3a^{-5}\right)^2}{\left(-a^{-6}b^5\right)^3}$

7. A point P is located at a distance of 37 units from point C, the center of a circle. We drew the tangent lines from P to the circle. On one tangent line, the point of tangency, Q is at a distance of 35 units from point P.

   a) Find the exact value of the radius of the circle.

   b) Find an approximate value, accurate up to four or more decimal places, of the angle formed between the two tangent lines.

8. a) Find all real numbers with the following property: the number is 2 greater than its own reciprocal.

   b) Prove that the number(s) you found has that property.

9. Compute the exact value of each of the following. Simplify your answer.

   a) $\sin 60° - \tan 30°$ c) $\tan 30° \cdot \tan 45° \cdot \tan 60°$ e) $\dfrac{\tan 30° + \tan 45° + \tan 60°}{(\sin 30°)(\sin 60°)}$

   b) $\sec 45° - \sin 45°$ d) $\sin^2 30° + \cos^2 30°$ f) $\tan^2 60° - \sec^2 60° + 1$

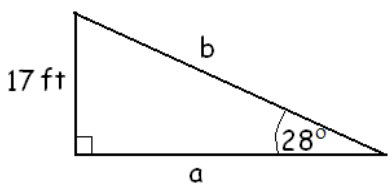
10. A right triangle has sides 5 ft, 12 ft, and 13 ft long.

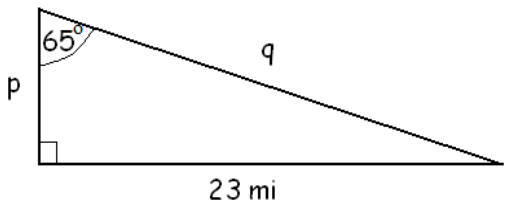
    a) State the value of all six trigonometric functions of α if α is the angle opposite the 5 ft long side.

    b) State the value of all six trigonometric functions of β if β is the angle opposite the 12 ft long side.

    c) Compute the approximate value (up to four or more decimal places) of the measure of the smallest angle in the triangle.

11. Compute the exact value and the approximate value for each of a, b, p, and q, based on the picture below.





12. Compute the perimeter and area of the 10-sided regular polygon that is written into a circle with radius 3 cm. Present both exact and approximate values for the answers.

13. Find the exact value and an approximate value for the angle that is formed between the line $y = \dfrac{1}{2}x$ and the positive part of the $x-$axis.

14. The hypotenuse of a right triangle is 74 units. The difference between the lengths of the other two sides is 46 units.

    a) How long are the sides of this triangle?

    b) Use your calculator to find an approximate value of the smallest angle in the triangle. Present your answer in degrees, accurate up to four or more decimals.

15. Consider a circle of radius 12 units and with a center C. P is a point located 30 units away from C. We draw a tangent line from P to the circle. The point of tangency is Q.

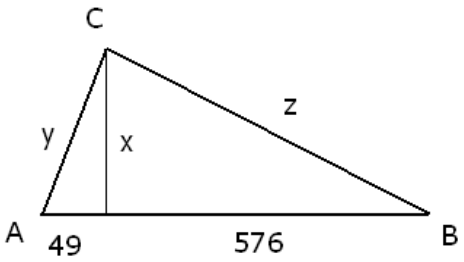
    a) Compute the exact value of $d(P, Q)$ (that is the distance between P and Q).

    b) Let α denote the angle CPQ. Compute the exact value of all six trigonometric functions of α.

16. Solve the equation $5(x-2)^2 - 3x + 2 = x - 2$. Check your solution(s) using exact values.

17. a) Solve $3x^2 + x = 3x + 2$.
    b) Check your solutions using exact values.

18. Find the exact value of x, y, and z based on the picture.



19. Solve each of the given inequalities.

    a) $\dfrac{2x+3}{5} - \dfrac{x-1}{2} \geq 11 - x$ b) $(x-3)^2 > (x+1)^2$ c) $\dfrac{2}{3}x - \dfrac{3}{10} < -\dfrac{1}{30}$

20. The conference was attended by 220 people.

    a) If everyone shook hands with everyone, how many handshakes took place?

    b) 120 of the attendees were men and 100 women. How many hadshakes took place if men only shook hands with men and woman only with women?

    c) 120 of the attendees were men and 100 women. How many hadshakes took place if men only shook hands with women?

    d) Out of the 220 people, 200 were married couples, the other 20 were single men. How many handshakes took place if everyone shook hands with everyone else, except for their spouse?

21. A company finds that if it prices its product at $\$50$, then it can sell 800 items. For every dollar increase in the price, the company will sell 4 less items.

    a) What is the maximum revenue possible, and what price guarantees that maximal revenue?

    b) What price range will guarantee a revenue greater than $\$50\,400$?

22. The hypotenuse of a right triangle is three feet shorter than four times its shortest side. The other side is three feet longer than three times its shortest side.

    a) Find the sides of the triangle.

    b) Find an approximate value of the smallest angle in the triangle. Present your answer accurate up to three or more decimal places.

23. Graph $y = -x^2 + 2x + 3$. State the coordinates of at least five points, includeing vertex and intercepts.

24. Solve each of the given systems of equations.

a) $\begin{cases} (x-1)^2+(y+3)^2=10 \\ 3y=-x-8 \end{cases}$

b) $\begin{cases} (x+8)^2+(y-2)^2=50 \\ y+7x=-4 \end{cases}$

c) $\begin{cases} (x-4)^2+(y+6)^2=20 \\ x+2y=7 \end{cases}$

d) $\begin{cases} (x+8)^2+(y-2)^2=50 \\ 7y=x+22 \end{cases}$

e) $\begin{cases} x+y=8 \\ 2xy=30 \end{cases}$

f) $\begin{cases} x+y=-1 \\ \frac{1}{x}+\frac{1}{y}=\frac{1}{6} \end{cases}$

g) $\begin{cases} x^2+y^2=10 \\ 2x+y=5 \end{cases}$

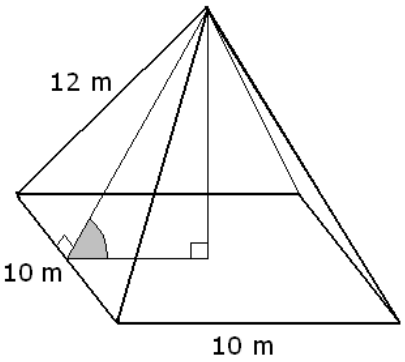
h) $\begin{cases} x^2+4x+4=-2 \\ x+y=3 \end{cases}$

25. A person is standing 3 ft away from a street light that is 15.6 ft tall. How long is his shadow if he is 5.2 ft tall?

26. Prove that for any acute angle α, $\sin\alpha+\cos\alpha>1$.

27. a) Find the distance between the points $A(3,-7)$ and $B(-2,5)$.
b) Find the distance between the points $P(x,y)$ and $Q(3,8)$.

28. Consider a square based straight pyramid as shown on the picture. The base is a square with sides 10 m long, and all other edges are 12 m long. Find the exact and approximate value of the angle that is formed between a triangular face and the square base. The angle is marked on the picture.



Answers

1. a) $9A$ b) $\frac{A}{3}$ c) $\sqrt{A}$ d) A^2 e) $13A$

2.

136	316	613	713
137	317	617	716
163	361	631	731
167	367	637	736
173	371	671	761
176	376	673	763

3. Proof: Let x be any positive number. Then $(x-1)^2\geq 0$ since it is a square.

$$\begin{aligned} (x-1)^2 &\geq 0 \\ x^2-2x+1 &\geq 0 && \text{add } 2x \\ x^2+1 &\geq 2x && \text{divide by } x;\ \text{recall that } x>0 \\ x+\frac{1}{x} &\geq 2 \end{aligned}$$

4. Line segments AC, BC, and CD are all radii in the circle, and so they are equal. So ACD triangle is isosceles, and the angles opposite AC and CD are also equal to each other. Thus $\measuredangle ADC=\alpha$. The third angle in triangle ACD is $180^\circ-2\alpha$. Angles ACD and DCB are supplementary angles beause together they form a straight angle. Thus

$$\begin{aligned} \measuredangle ACD+\measuredangle DCB &= 180^\circ \\ 180^\circ-2\alpha+\beta &= 180^\circ && \text{subtract } 180^\circ \\ -2\alpha+\beta &= 0 && \text{add } 2\alpha \\ \beta &= 2\alpha \end{aligned}$$

5. $\frac{n(n-3)}{2}$ 6. a) $-\frac{1}{17}$ b) $\frac{b^4}{a^7}$ c) $\frac{xy}{y-x}$ d) $-\frac{a^4}{4b^7}$ 7. a) 12 unit b) 24.973 78°

8. a) $1-\sqrt{2}$ and $1+\sqrt{2}$

b) If $x=1-\sqrt{2}$, then its reciprocal is $\dfrac{1}{1-\sqrt{2}}=\dfrac{1}{1-\sqrt{2}}\cdot\dfrac{1+\sqrt{2}}{1+\sqrt{2}}=\dfrac{1+\sqrt{2}}{1-2}=\dfrac{1+\sqrt{2}}{-1}=-1-\sqrt{2}$

If we add 2 to this number, we get $-1-\sqrt{2}+2=1-\sqrt{2}$, and so x is indeed 2 greater than its reciprocal. And if $x=1+\sqrt{2}$, then its reciprocal is $\dfrac{1}{1+\sqrt{2}}=\dfrac{1}{\sqrt{2}+1}\cdot\dfrac{\sqrt{2}-1}{\sqrt{2}-1}=\dfrac{\sqrt{2}-1}{1}=\sqrt{2}-1$, which is indeed 2 less than $\sqrt{2}+1$. 9. a) $\dfrac{\sqrt{3}}{6}$ b) $\dfrac{\sqrt{2}}{2}$ c) 1 d) 1 e) $\dfrac{4\sqrt{3}+16}{3}$ f) 0

10. a) $\sin\alpha=\dfrac{5}{13}$ $\cos\alpha=\dfrac{12}{13}$ $\tan\alpha=\dfrac{5}{12}$ $\csc\alpha=\dfrac{13}{5}$ $\sec\alpha=\dfrac{13}{12}$ $\cot\alpha=\dfrac{12}{5}$

b) $\sin\beta=\dfrac{12}{13}$ $\cos\beta=\dfrac{5}{13}$ $\tan\beta=\dfrac{12}{5}$ $\csc\beta=\dfrac{13}{12}$ $\sec\beta=\dfrac{13}{5}$ $\cot\beta=\dfrac{5}{12}$ c) $22.619\,865°$

11. $a=\dfrac{17}{\tan 28°}\,\text{ft}\approx 31.9724\,\text{ft}$ $b=\dfrac{17}{\sin 28°}\,\text{ft}\approx 36.21093\,\text{ft}$

$p=\dfrac{23}{\tan 65°}\,\text{mi}\approx 10.72507614\,\text{mi}$ $q=\dfrac{23}{\sin 65°}\,\text{mi}\approx 25.377\,692\,\text{mi}$

12. $P=60\sin 18°\,\text{cm}\approx 18.541\,02\,\text{cm}$ $A=90\sin 18°\cos 18°\,\text{cm}^2\approx 26.450\,336\,\text{cm}^2$

13. exact value: $\tan^{-1}\left(\dfrac{1}{2}\right)$ approximation: $26.565\,051\,18°$ 14. a) 24, 70, and 74 units long b) $18.924\,644°$

15. a) $\sqrt{756}=6\sqrt{21}$ b) $\sin\alpha=\dfrac{2}{5}$ $\cos\alpha=\dfrac{\sqrt{21}}{5}$ $\tan\alpha=\dfrac{2}{\sqrt{21}}$ $\csc\alpha=\dfrac{5}{2}$ $\sec\alpha=\dfrac{5}{\sqrt{21}}$ $\cot\alpha=\dfrac{\sqrt{21}}{2}$

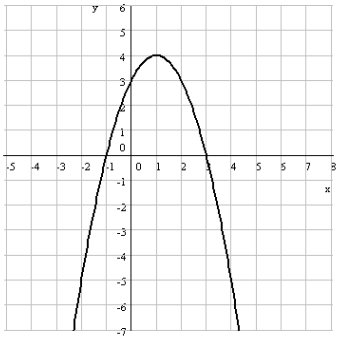
16. $x_{1,2}=\dfrac{12\pm 2\sqrt{6}}{5}$ 17. a) $\dfrac{1\pm\sqrt{7}}{3}$ b) If $x=\dfrac{1-\sqrt{7}}{3}$, then

$\text{LHS}=3\left(\dfrac{1-\sqrt{7}}{3}\right)^2+\dfrac{1-\sqrt{7}}{3}=3\cdot\dfrac{8-2\sqrt{7}}{9}+\dfrac{1-\sqrt{7}}{3}=\dfrac{8-2\sqrt{7}}{3}+\dfrac{1-\sqrt{7}}{3}=\dfrac{9-3\sqrt{7}}{3}=\dfrac{3\left(3-\sqrt{7}\right)}{3}=3-\sqrt{7}$

and $\text{RHS}=3\left(\dfrac{1-\sqrt{7}}{3}\right)+2=1-\sqrt{7}+2=3-\sqrt{7}$. Checking the other solution goes similarly.

18. $x=168, y=175, z=600$ 19. a) $[11,\infty)$ b) $(-\infty,1)$ c) $\left(-\infty,\dfrac{2}{5}\right)$ 20. a) 24 090 b) 12 090 c) 12 000 d) 23 990

21. a) \$62 500, if the price is \$125 b) between \$70 and \$180 22. a) 7 ft, 24 ft, and 25 ft b) $\sin^{-1}\left(\dfrac{7}{25}\right)\approx 16.260\,2°$

23. 

$y=-(x-2)^2+4=(x+1)(x-3)$ Vertex: $(2,4)$, $y-$intercept: $(0,3)$, $x-$intercepts: $(-1,0)$ and $(3,0)$, additional point: $(5,-5)$

24. a) $(-2,-2)$ and $(4,-4)$ b) $(-1,3)$ c) no solution

d) $(-1,3)$ and $(-15,1)$ e) $(3,5)$ and $(5,3)$ f) $(2,-3)$ and $(-3,2)$

g) $(1,3)$ and $(3,-1)$ h) no real solution 25. 1.5 ft

26. The triangle inequality states that $a+b>c$. Divide both sides by c. $c>0$

27. a) 13 unit b) $\sqrt{(x-3)^2+(y-8)^2}$ 28. exact value: $\cos^{-1}\left(\dfrac{5}{\sqrt{119}}\right)$ approximation: $62.719\,36°$