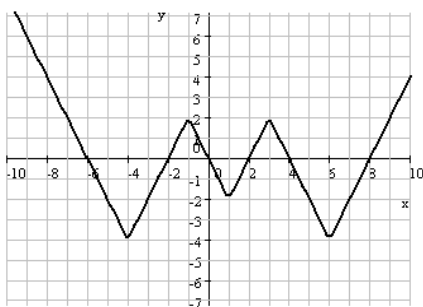


- Re-write $0.30\overline{72}$ as a fraction of integers.
- Simplify each of the following.
 - $(4\sqrt{3} - 2)^2$
 - $\sqrt{20} - 3\sqrt{180} + \sqrt{500}$
 - Rationalize the denominator in $\frac{6}{5 - \sqrt{7}}$
 - Find the exact value of $x^2 - 5x + 8$ if $x = 1 - 2\sqrt{3}$
- Simplify each of the following.
 - $\frac{2a^3(2a^2)^5(-4a)^2}{(-2a^2)(-4a^2)^3}$
 - $\frac{3^{2x-3} \cdot 2^{3x+1}}{6^{2x-1}}$
 - $\frac{3^x \cdot 6^{x+1}}{2^x \cdot 9^{x-1}}$
 - $\frac{2^{2018} + 5 \cdot 2^{2019}}{2^{2020}}$
- Let x denote the number 2^{143} . Simplify the expression $2^{143} + 3 \cdot 2^{144} + 2^{145}$ and write it in terms of x .
- Suppose that $M = 3^{100}$. Write each of the following in terms of M .
 - 3^{101}
 - 9^{100}
 - 3^{99}
 - 3^{50}
- Solve each of the following equations. Make sure to check your solution.
 - $x^3 = 24x^2 + 217x$
 - $\frac{3-x}{4} - \frac{10-3x}{5} = x+2$
 - $(3x-2)(9-x) = 5x - 2(x-3)^2$
 - $3(x-5) - 5(x-1) = -2x+1$
 - $18x^3 = 2x$
 - $18x^2 = 2x$
- Solve $3x^2 + 5x = 1$ by completing the square.
 - Check your solution(s), using exact values.
- Three sides of a triangle are 10, 15, and x units long.
 - What values are possible for x ?
 - What values are possible for x if it is the length of the shortest side?
 - What values are possible for x if it is the length of the longest side?
- Solve each of the following system of linear equations.
 - $\begin{cases} 3x - 5y = -12 \\ y - x = 4 \end{cases}$
 - $\begin{cases} 2x + 5y = -11 \\ 3x - y = -25 \end{cases}$
 - $\begin{cases} 2x + 3y = -1 \\ y = -\frac{2}{3}x + 2 \end{cases}$
- Solve each of the following system of equations.
 - $\begin{cases} y = x^2 - 2x - 2 \\ y = 2x - 5 \end{cases}$
 - $\begin{cases} (x-1)^2 + (y+4)^2 = 40 \\ y = \frac{1}{2}x + \frac{1}{2} \end{cases}$
 - $\begin{cases} y = x^2 - 6x + 27 \\ y = 4x - 7 \end{cases}$
 - $\begin{cases} x^2 + y^2 = 106 \\ x + y = 14 \end{cases}$
- Completely factor each of the following.
 - $4 - 6x^2 - 2x$
 - $2x^2 - 12x + 20$
 - $16y^4 - 1$
- Factor each of the following by completing the square.
 - $3x^2 - 4x - 319$
 - $3x^2 - 3x + 4$
 - $20x - 2x^2 - 46$
 - $-4x^2 + 3x + 7$
- Find all real numbers x with the following property: x is exactly 6 less than its own reciprocal.
- Graph each of the following equations.
 - $y = -\frac{2}{3}x + 1$
 - $y = x^2 + 4x - 56$
 - $2x - 3y + xy = 6$
- What is the smallest value of the expression $x^2 - 20x + 85$?
- A number is 20 less than another. What is the smallest possible value of their product?
- Find the distance between the points $A(-3, -5)$ and $B(3, 3)$.

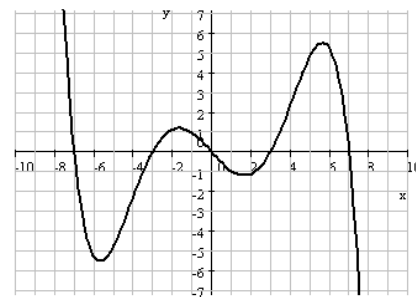
18. a) Find the set of all values of x for which $P(x, y)$ is on the graph shown and

- i) $y < 0$
- ii) $y \leq -6$
- iii) $y \geq 2$



- b) Find the set of all values of x for which $P(x, y)$ is on the graph shown and

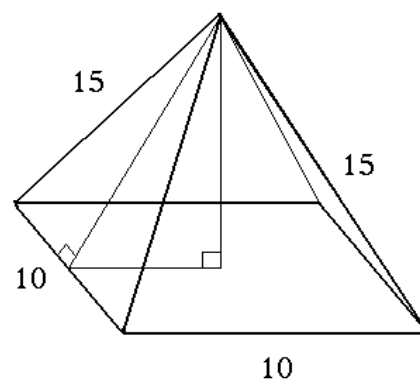
- a) $y \geq 0$
- b) $y < 0$



19. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens, how many cows?
20. We invested \$10000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if the combined interest from the two accounts is \$1238 after the first year?
21. In a hotel, the first night costs 45 dollars, and all additional nights cost 35 dollars. How long did Mr. Williams stay in the hotel if his bill was 325 dollars?
22. Graph $y = x^2 - 10x + 9$. State the coordinates of at least 5 points, including vertex and intercepts.
23. Use systematic listing to find each of the following.
- a) If 5 students competing and the first, second, and third places are recorded, how many different results are possible?
 - b) If we play lottery by pulling two marbles out of eight, how many outcomes are possible?
 - c) How many three-digit numbers can we form using only the digits 2, 5, 6, and 8?
 - d) How many three-digit numbers can we form using only the digits 2, 5, 6, and 8 if repetition of digits is not allowed?
24. a) One number is four less than three times another number. Find these numbers if their sum is 64.
b) One number is four less than three times another number. Find these numbers if their product is 160.
25. Compute the exact value of the area of the triangle with sides 6, 6, and 8 units long.
26. a) Find the exact value of the area of a triangle with sides 8 m, 7 m, and 7 m long.
b) Find the exact value of the area of a regular triangle with sides 5 meters long.

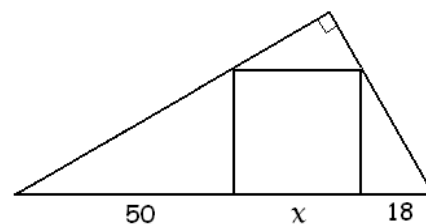
27. An arch is in the shape of a semicircle. At a point along the base 1 foot from an end of the arch, the height of the arch is 5 feet. Find the maximum height of the arch. Present exact value of the answer.

28. The picture shows a straight pyramid with a square base. The sides of the base are 10 in long. All other edges are 15 in long.

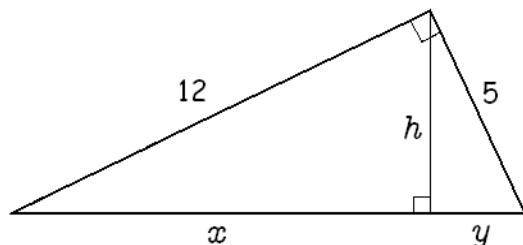


- a) Find the height of a triangular face.
 - b) Use part a) to find the height of the pyramid.
29. The conference had 150 attendees, 65 men and 85 women. When greeting each other, people shook each other's hand. How many handshakes took place if
- a) everyone shook hands with everyone else
 - b) men only shook every other men's hand, women only shook every other women's hand.
 - c) every man shook every women's hand.
 - d*) all 65 men came with their wives, everyone shook hands with everyone else, except for people they came together (i.e. no men shook hands with their own wife).

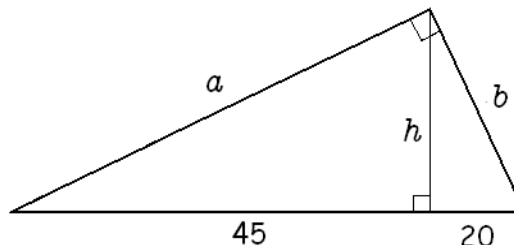
30. A person is standing 15 feet away from a street light that is 31 feet tall. How long is his shadow if he is 6 feet tall?
31. A square is drawn into a right triangle as shown on the picture. Find the exact value of x .



32. Find the exact values on the right triangles shown on the picture.



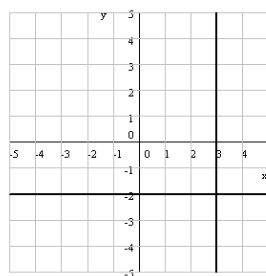
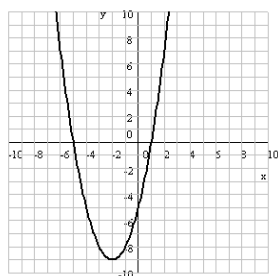
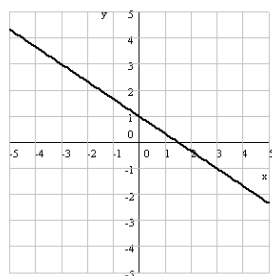
a)



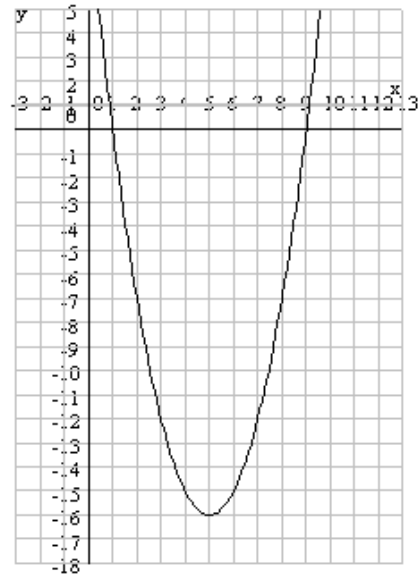
b)

Answers

1. $\frac{3042}{9900}$ 2. a) $52 - 16\sqrt{3}$ b) $-6\sqrt{5}$ c) $\frac{\sqrt{7}+5}{3}$ d) $6\sqrt{3}+16$ 3. a) $8a^7$ b) $\frac{4}{9}2^x$ c) 54 d) $\frac{11}{4}$
4. $11x$ 5. a) $3M$ b) M^2 c) $\frac{M}{3}$ d) \sqrt{M}
6. a) $-7, 0, 31$ b) -5 c) $0, 12$ d) no solution e) $-\frac{1}{3}, 0, \frac{1}{3}$ f) $\frac{1}{9}, 0$
7. a) $\frac{-5 \pm \sqrt{37}}{6}$ b) if $x = \frac{-5 + \sqrt{37}}{6}$, then
- $$\begin{aligned} \text{LHS} &= 3x^2 + 5x = 3 \left(\frac{-5 + \sqrt{37}}{6} \right)^2 + 5 \left(\frac{-5 + \sqrt{37}}{6} \right) = 3 \cdot \frac{25 + 37 - 10\sqrt{37}}{36} + 5 \cdot \frac{-5 + \sqrt{37}}{6} \\ &= \frac{62 - 10\sqrt{37}}{12} + \frac{-25 + 5\sqrt{37}}{6} = \frac{31 - 5\sqrt{37}}{6} + \frac{-25 + 5\sqrt{37}}{6} = \frac{31 - 5\sqrt{37} + 25 + 5\sqrt{37}}{6} = \frac{6}{6} = 1 = \text{RHS} \end{aligned}$$
8. a) $5 < x < 25$ b) $5 < x < 10$ c) $15 < x < 25$ 9. a) $x = -4, y = 0$ b) $x = -8, y = 1$ c) no solution
10. a) $(1, -3)$ and $(3, 1)$ b) $(-5, -2)$ and $(3, 2)$ c) no solution d) $(5, 9)$ and $(9, 5)$
11. a) $-2(3x - 2)(x + 1)$ b) $2(x^2 - 6x + 10)$ c) $(4y^2 + 1)(2y - 1)(2y + 1)$
12. a) $3 \left(x + \frac{29}{3} \right) (x - 11)$ b) can not be factored c) $-2(x - 5 + \sqrt{2})(x - 5 - \sqrt{2})$ d) $-4(x + 1) \left(x - \frac{7}{4} \right)$
13. $-3 + \sqrt{10}$ and $-3 - \sqrt{10}$
14. a) $y = -\frac{2}{3}x + 1$ b) $y = x^2 + 4x - 5$ c*) $2x - 3y + xy = 6$



15. -15 (when $x = 10$) 16. -100 17. 10 units
18. a) i) $(-6, -2) \cup (0, 2) \cup (4, 8)$ ii) there is no such x iii) $(-\infty, -7] \cup \{-1, 3\} \cup [9, \infty)$
 b) i) $(-\infty, -7] \cup [-3, 0] \cup [3, 7]$ ii) $(-7, -3) \cup (0, 3) \cup (7, \infty)$ 19. 19 chickens and 34 cows
20. \$7300 at 14% and \$2700 at 8% 21. 8 nights
22. $y = x^2 - 10x + 9 = (x - 5)^2 - 16 = (x - 1)(x - 9)$
 y -intercept: $(0, 9)$ vertex: $(5, -16)$
 x -intercepts: $(1, 0)$ and $(9, 0)$
23. a) 60 b) 28 c) 64 d) 24
24. a) 17 and 47 b) 8 with 20 and $-\frac{20}{3}$ with -24 25. $8\sqrt{5}$
26. a) $4\sqrt{33} \text{ m}^2$ b) $\frac{25}{4}\sqrt{3} \text{ m}^2$ 27. 13 ft
28. a) $10\sqrt{2} \text{ in} \approx 14.1421 \text{ in}$ b) $5\sqrt{7} \text{ in} \approx 13.2288 \text{ in}$
29. a) 11 175 b) 5650 c) 5525 d) 11 110 30. 3.6 ft 31. 30 units
32. a) $h = \frac{60}{13}$, $x = \frac{144}{13}$, $y = \frac{25}{13}$ b) $h = 30$, $a = 15\sqrt{13}$, $b = 10\sqrt{13}$



22. $y = x^2 - 10x + 9$

References

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|--|---|
| 1. Fractions and Decimals | 17. Graphical Solutions |
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| 3. Rules of Exponents | 19. Systems of Linear Equations (substitution, elimination) |
| 4. Rules of Exponents | 20. Linear Word Problems |
| 5. Rules of Exponents | 21. Graphing Parabolas 1 |
| 6. Linear Equations, Factoring 1 | 22. a) Linear Word Problems, b) Factoring 1 |
| 7. Completing the Square Part 4, Radical Expressions | 23. Introduction to Combinatorics |
| 8. Triangle Inequalities | 24. Pythagorean Theorem |
| 9. Systems of Linear Equations (substitution, elimination) | 25. Pythagorean Theorem |
| 10. Non-linear systems | 26. Pythagorean Theorem |
| 11. Factoring A, Factoring 1 | 27. Pythagorean Theorem |
| 12. Completing the Square 1, 2, 3, 4 | 28. Pythagorean Theorem |
| 13. Completing the Square 4 | 29. Introduction to Combinatorics |
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| 15. Smallest Value of a Quadratic Expression | 31. Similar Triangles |
| 16. Smallest Value of a Quadratic Expression | 32. Similar Triangles |