Please note that Quiz 4 will also cover all material covered on Quizzes 1-3.

1. Simplify each of the following expressions.

   a) $m^2 \cdot m^3$ b) $\left(m^2\right)^3$ c) $\dfrac{\left(a^3b^2\right)^3}{ab^4}$ d) $\dfrac{3^{2x-1}}{9^{x-1}}$ e) $\dfrac{\left(-p^2\right)^3 q^5}{(-p)^2 q^4}$ f) $\left(-\dfrac{2x^3y}{y^4}\right)^2 \left(\dfrac{2x^3y^2}{-8x^5y^2}\right)^3$

2. Let $M = 2^{100}$. Write each of the following expressions in terms of M.

   a) $2^{100} - 2^{101} + 2^{102} - 2^{103}$ b) $2^{103} - 5 \cdot 2^{102}$ c) 4^{100} d) 2^{200} e) 2^{500} f) 2^{99}

3. Solve each of the following systems of linear equations.

   a) $\begin{cases} 3x - 2y = 12 \\ 2x - y = 4 \end{cases}$ b) $\begin{cases} x + 5y = 3 \\ y = -\dfrac{1}{5}x + \dfrac{3}{5} \end{cases}$ c) $\begin{cases} 2x - y = 6 \\ x = \dfrac{1}{2}y + 1 \end{cases}$

4. Prove that the decimal $0.1\overline{74} = 0.174747474....$ represents a rational number by re-writing it as a fraction of two integers. You do not need to reduce the fraction.

5. Simplify each of the following expressions.

   a) $(5a - 1)^2$
   b) $\left(3x^5 + 4y\right)\left(3x^5 - 4y\right)$
   c) $\sqrt{125} - 3\sqrt{80} + \sqrt{45}$
   d) $\left(2 - \sqrt{3}\right)\left(5\sqrt{3} + 1\right)$
   e) $\left(\sqrt{7} - 2\right)^2$
   f) $\left(\sqrt{7} - 2\right)^3$
   g) $\left(\sqrt{5} - \sqrt{2}\right)^2$
   h) $\left(4 - \sqrt{11}\right)^3\left(4 + \sqrt{11}\right)^3$

6. Find the exact value of $-x^2 + 4x + 6$ if $x = 2 - \sqrt{3}$.

7. Completely factor each of the following **by completing the square as shown in class and the handouts**.

   a) $60x - 6x^2 + 2250$ b) $15a^3 - 8a^4 + a^5$ c) $6x^2 - 24x + 78$

8. Completely factor each of the following over the real numbers.

   a) $9x^2 - 16y^{10}$ b) $5x^2 + 10x$ c) $900x + 15x^2 - 3x^3$ d) $32x^4 - 2$

9. Graph the line $2x - 3y = -12$.

10. Solve each of the following equations. Make sure to check your solution(s).

    a) $2x^3 = 20x^2 + 1750x$
    b) $\dfrac{3x + 17}{2} = x - 1 + \dfrac{x + 19}{2}$
    c) $\dfrac{2}{3}(x - 7) = \dfrac{4}{5}(x + 1)$
    d) $7x^2 + (x + 3)(2x - 1) = (3x + 1)^2$
    e) $8a + 2a^2 = 42$
    f) $8x^3 = 50x^2$
    g) $8p^3 = 50p$
    h) $2 - (3 - x)(2x + 5) = (x - 1)(2x - 1)$
    i) $2x^2 = 2x$

11. Find the smallest value of each of the given expressions.

    a) $x^2 - 10x + 18$ b) $x^2 + 6x - 8$ c) $x^2 + 2x - 7$

12. a) One side of a rectangle is $4\,\text{ft}$ shorter than three times the other side. Find the sides if the perimeter is $64\,\text{ft}$.

    b) One side of a rectangle is $12\,\text{ft}$ shorter than three times the other side. Find the sides if the area is $63\,\text{ft}^2$.

13. Find the distance between $(7, -2)$ and $(3, 3)$.

14. Today a cab ride costs a flat fee of $\$3.25$ and then $\$1.8$ per mile. We take a ride that costs $\$23.05$. How far was our destination?

15. Compute the exact value of the area of the triangle with sides $10\,\text{cm}$, $10\,\text{cm}$, and $8\,\text{cm}$ long.

16. Find all numbers such that if we cube the number, we get back the same number.

17. The sides of a triangle are 8, 12, and x units long.
    a) What are the possible values for x?
    b) What are the possible values for x, given that x is the shortest side?
    c) What are the possible values for x, given that x is the longest side?

18. The hypotenuse of a right triangle is $74\,\text{cm}$. The difference between the other two sides is $46\,\text{cm}$. Find the sides of the triangle.

19. We throw an object upward from the top of a $1024\,\text{ft}$ tall building. The height of the object, (measured in feet) t seconds after we threw it is
$$h(t) = -16t^2 + 192t + 1024$$
    a) Where is the object 2 seconds after we threw it?
    b) How long does it take for the object to hit the ground?

20. A farmer has some chickens and cows. One day he was asked: "*How many chickens and how many cows do you have?*" His answer was: *"All together, there are* 73 *heads and* 188 *legs"*. How many chickens and how many cows does the farmer have?

21. A total of $\$20\,000$ is to be invested in bonds and stocks. If the amount invested in bonds is to be $\$4500$ more than the amount invested in stocks, how much money is invested in each category?

22. Sally worked 50 hours last week and made $\$660$ for the week. For every hour worked over 40 her job pays time and a half. What is Sally's regular hourly pay rate?

23. An arch is in the shape of a semicircle. At a point along the base 4 feet from an end of the arch, the height of the arch is 12 feet. Find the maximum height of the arch.

24. a) Compute the exact value of the area of an equilateral triangle with sides 12 meters.
    b) Express the area of an equilateral triangle in terms of x, where each side is x units long.

25. We have 200 coins in a jar, all dimes and quarters. How many dimes do we have if the total value of all coins is $\$40.55$?

Answers

1. a) m^5 b) m^6 c) a^8b^2 d) 3 e) $-p^4q$ f) $-\dfrac{1}{16y^6}$ See handout Exponents

2. a) $-5M$ b) $-12M$ c) M^2 d) M^2 e) M^5 f) $\dfrac{M}{2}$ See handout Exponents

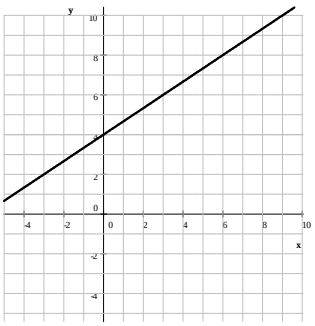
3. See handout on system of equations. a) $(-4,-12)$ b) this system is dependent, there are infinitely many solutions
    c) this system is inconsistent, there is no solution

4. See handout Decimals and Fractions. $\dfrac{173}{990}$

5. See handout Radical Expressions. a) $25a^2-10a+1$ b) $9x^{10}-16y^2$ c) $-4\sqrt{5}$ d) $9\sqrt{3}-13$
    e) $11-4\sqrt{7}$ f) $-50+19\sqrt{7}$ g) $7-2\sqrt{10}$ h) 125

6. See handout Radical Expressions. 7

7. See handout on completing the square 1 and 2.
   a) $-6\left(x-25\right)\left(x+15\right)$ b) $a^3\left(a-3\right)\left(a-5\right)$ c) $6\left(x^2-4x+13\right)$
8. See handout Factoring 1.
   a) $\left(3x-4y^5\right)\left(3x+4y^5\right)$ b) $5x\left(x+2\right)$ c) $-3x\left(x+15\right)\left(x-20\right)$ d) $2\left(4x^2+1\right)\left(2x+1\right)\left(2x-1\right)$
9. See handout Graphing Lines.



10. See handouts Linear Equations, The Zero Product Rule, Factoring A, and Factoring 1.
    a) $35, 0$, and -25 b) identity, all real numbers are solution c) -41
    d) -4 e) $-7, 3$ f) $\frac{25}{4}, 0$ g) $-\frac{5}{2},\ 0,\ \frac{5}{2}$ h) 7 i) $0, 1$
11. See handout Smallest Value of a Quadratic Expression a) -7 b) -17 c) 6
12. a) $9\,\text{ft}$ by $23\,\text{ft}$ See handout linear word problems
    b) $7\,\text{ft}$ by $9\,\text{ft}$ See handout factoring 1
13. $\sqrt{41}$ see handout Pythagorean Theorem
14. 11 miles see handout linear word problems
15. $8\sqrt{21}\,\text{cm}^2$ see handout Pythagorean Theorem
16. a) $-1, 0, 1$ see handout Factoring 1
17. a) $4 < x < 20$ b) $4 < x < 8$ c) $12 < x < 20$ see handout triangle inequality
18. $24\,\text{cm}, 70\,\text{cm}, 74\,\text{cm}$ see handout Pythagorean Theorem
19. a) $1344\,\text{ft}$ b) 16 seconds see handout Factoring 1
20. 52 chickens and 21 cows See handout on system of equation
21. $\$7750$ in stocks and $\$12\,250$ in bonds See handout on system of equations
22. $\$12$ see handout linear word problems
23. $20\,\text{ft}$ see handout Pythagorean Theorem
24. a) $36\sqrt{3}$ b) $\frac{\sqrt{3}}{4}x^2$ see handout Pythagorean Theorem
25. 63 See handout on system of equations

Last revised: January 24, 2019