Exam 1 will cover the following topics. All topics covered by Quizzes 1 and 2, exponents, limits, quadratic inequalities, and optimization 1. (Appendix A, Chapter 1, and handouts.)

Review Problems

1. Solve each of the following inequalities.

   a) $4x + x^2 < 21$ b) $2x^2 + 2 \geq 5x$ c) $x^2 - 6x + 13 \leq 0$

2. Simplify each of the following.

   a) $(-6)^{-2}$ b) -6^{-2} c) $16^{-3/4}$ e) $64^{2/3}$

3. Solve each of the following equations. Make sure to check your solution.

   a) $x^3 = 24x^2 + 217x$

   b) $\dfrac{3-x}{4} - \dfrac{10-3x}{5} = x + 2$

   c) $(x+4)(1-2x) = 3x - 2(x-3)^2$

   d) $3(x-5) - 5(x-1) = -2x + 1$

   e) $x^2 + 7 = 6x$

4. Factor $3x^2 - 4x - 319$ by completing the square.

5. In case of each polynomial given, determine (by completing the square) whether it factors or not. (You do not have to actually factor.)

   a) $20x + 2x^2 + 44$ b) $20x - 5x^2 - 25$

6. Find each of the following limits.

   a) $\lim_{x\to-\infty}\left(-2x^5 + 8x^2\right)$

   b) $\lim_{x\to\infty}\left(-2x^5 + 8x^2\right)$

   c) $\lim_{x\to-\infty}\left(-2x^5 + 8x^6\right)$

   d) $\lim_{x\to\infty}\left(-2x^5 + 8x^6\right)$

   e) $\lim_{x\to-\infty}\dfrac{3x^2-1}{5x^2-3x+2}$

   f) $\lim_{x\to\infty}\left(1 - \dfrac{2}{x}\right)$

   g) $\lim_{x\to\infty}\dfrac{2x^2+3x+1}{3x^2-5x+2}$

   h) $\lim_{x\to\infty}\dfrac{-x^3+2x+1}{x-3}$

   i) $\lim_{x\to\infty}\left(\dfrac{1}{3}\right)^x$

   j) $\lim_{x\to-\infty}(2^x)$

   k) $\lim_{x\to\infty}\dfrac{1}{\sqrt{x+2}-\sqrt{x}}$

   l) $\lim_{x\to\infty}\dfrac{3^{x-1}}{2^{2x-3}}$

7. Find each of the following limits.

   a) $\lim_{x\to-\infty}\dfrac{1}{x^2-9}$

   b) $\lim_{x\to2^-}\dfrac{1}{x^2-9}$

   c) $\lim_{x\to2^+}\dfrac{1}{x^2-9}$

   d) $\lim_{x\to2}\dfrac{1}{x^2-9}$

   e) $\lim_{x\to-3^-}\dfrac{1}{x^2-9}$

   f) $\lim_{x\to-3^+}\dfrac{1}{x^2-9}$

   g) $\lim_{x\to-3}\dfrac{1}{x^2-9}$

   h) $\lim_{x\to\infty}\dfrac{1}{x^2-9}$

8. Let $f(x) = \dfrac{x^2-2x-8}{x^2-4}$. Find each of the following limits.

   a) $\lim_{x\to-\infty} f(x)$

   b) $\lim_{x\to\infty} f(x)$

   c) $\lim_{x\to-2^-} f(x)$

   d) $\lim_{x\to-2^+} f(x)$

   e) $\lim_{x\to-2} f(x)$

   f) $\lim_{x\to0^-} f(x)$

   g) $\lim_{x\to0^+} f(x)$

   h) $\lim_{x\to0} f(x)$

   i) $\lim_{x\to2^-} f(x)$

   j) $\lim_{x\to2^+} f(x)$

   k) $\lim_{x\to2} f(x)$

9. Find each of the following limits.

a) $\lim_{x\to 5} \dfrac{\frac{1}{x}-\frac{1}{5}}{x-5}$ b) $\lim_{h\to 0} \dfrac{\sqrt{9+h}-3}{h}$ c) $\lim_{x\to 5} (2x-|x-5|)$ d) $\lim_{x\to 5} \dfrac{-3x}{(x-5)^2}$

10. Define $f(x)$ as follows:

$$f(x) = \begin{cases} 3x+m & \text{if } x<5 \\ -x^2+4x & \text{if } x\geq 5 \end{cases}$$

Find the value of m so that f is continuous on $\mathbb{R}$.

11. Graph the parabola $y = 6-2x^2-4x$. Clearly label the coordinates of five points on the parabola, including vertex and intercepts.

12. A citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. Find the total number of trees the grower should plant to maximize yield.

13. The cost of manufacturing q units of a product is given by $C(q) = 6q^2+10q$. Suppose we can sell all q units for a total of $142q+1674$ dollars. Find the maximum profit we can achieve.

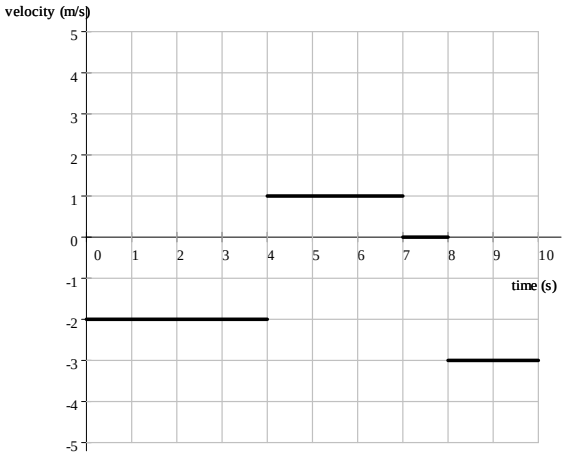
14. An object's height (measured in feet) is defined by $s(t) = t^3-12t$ where t is the time, measured in seconds.

a) Find the location of the object at $t=3$ seconds.

b) Find the average velocity of the object between

i) $t=0$ and $t=2$ s ii) $t=1$ s and $t=2$ s iii) $t=1.5$ s and $t=2$ s

15. The picture below shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward.).

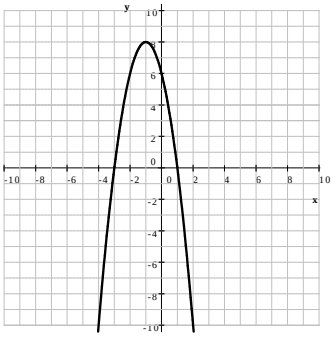


a) Suppose that the object starts at a height of 5 m. Where is it at $t=10$ seconds?.

b) Suppose that the object starts at a height of 9 m. Where is it at $t=10$ seconds?

Review Problems - Answers

1. See handout quadratic inequalities.
   a) $-7 < x < 3$ b) $x \le \frac{1}{2}$ or $x \ge 2$ c) no solutions
2. a) $\frac{1}{36}$ b) $-\frac{1}{36}$ c) $\frac{1}{8}$ e) 16
3. a) $-7, 0, 31$ b) -5 c) 1 d) no solution e) $3+\sqrt{2}$ and $3-\sqrt{2}$
4. $3\left(x+\frac{29}{3}\right)(x-11) = (3x+29)(x-11)$ For detailed solutions and practice, see the handout Completing the Square, Part 3. This problem is Example 3.
5. a) factors b) does not factor
6. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 1 g) $\frac{2}{3}$ h) $-\infty$ i) 0 j) 0 k) ∞ l) 0
7. a) 0 b) $-\frac{1}{5}$ c) $-\frac{1}{5}$ d) $-\frac{1}{5}$ e) ∞ f) $-\infty$ g) undefined h) 0
8. a) 1 b) 1 c) $\frac{3}{2}$ d) $\frac{3}{2}$ e) $\frac{3}{2}$ f) 2 g) 2 h) 2 i) ∞ j) $-\infty$ k) undefined
9. a) $-\frac{1}{25}$ b) $\frac{1}{6}$ c) 10 d) $-\infty$
10. -20
11. vertex: $(-1, 8)$ $x-$intercepts: $(-3, 0)$ and $(1, 0)$ $y-$intecept: $(0, 6)$



12. 80 trees (for 25600 oranges) See Optimization 1
13. \$2400. See Optimization 1
14. a) $-9\,\text{ft}$ b) i) $-8\frac{\text{ft}}{\text{s}}$ ii) $-5\frac{\text{ft}}{\text{s}}$ iii) $-2.75\frac{\text{ft}}{\text{s}}$
15. a) $-6\,\text{m}$ b) $-2\,\text{m}$