Exam 3 will cover the following topics. All topics covered by Quizzes 1-6 and Exams 1 and 2. These topics include quadratic inequalities, functions and their graphs, inverse functions, exponents and logarithms, limits, differentiation by using the definition, techniques of differentiation, derivative of logarithmic and exponential functions, and applications of the derivative: increasing, decreasing functions, relative and absolute extrema, optimization problems, concavity behavior, points of inflection, tangent lines, and antiderivatives. Indefinite integrals, and using Riemann sums to approximate definite integrals. These include all handouts and the following sections from the book:
Chapter 1: all, Chapter 2: 2.1, 2.2, 2.3, 2.4 Chapter 3: 3.1, 3.2, 3.3, 3.4, Chapter 4: 4.1, 4.2, 4.3 Chapter 5: 5.1 and 5.3

Review Problems - Answers

1. Solve the inequalities.

   a) $\dfrac{2x-11}{3x+5} \geq 0$ b) $\dfrac{2x-11}{3x+5} \geq 15$

2. Find the domain of each of the following expressions.

   a) $\log_2(16-x^2) + \dfrac{1}{\log_2(x^2)}$ b) $\ln(x-2) - \ln(x+2)$ c) $\log_5(x^2-7x+2)$

3. Solve each of the following equations.

   a) $\log_6(-8-x) + \log_6(8-x) = 2$ c) $\log_2(2x-3) - \log_2(4x+10) = -3$

   b) $\log_3(7-x) + \log_3(1-x) = 3$ d) $\left[64^{\frac{2}{3}} \cdot 3^{-\log_{27} 8}\right]^{\frac{1}{3}} + \log_2 x^3 = 14$

4. Find an equation for the inverse of each of the following functions.

   a) $f(x) = 3^{5x-1}$ b) $f(x) = \dfrac{x+4}{3x-5}$ c) $f(x) = \ln(2x-1)$

5. Find each of the following limits.

   a) $\displaystyle\lim_{x\to 1^+} \frac{x^2-1}{x^2-3x+2}$ c) $\displaystyle\lim_{x\to\infty} \frac{2x^2+3x+1}{3x^2-5x+2}$ e) $\displaystyle\lim_{x\to 3} \frac{x^2-3x}{x^2-9}$

   b) $\displaystyle\lim_{x\to 2^+} \frac{x^2-1}{x^2-3x+2}$ d) $\displaystyle\lim_{x\to\infty} \frac{-x^3+2x+1}{x-3}$ f) $\displaystyle\lim_{x\to 3^+} \frac{x^2+3x}{x^2-9}$

6. Graph each of the following functions.

   a) $f(x) = \dfrac{x-3}{x-3}$ c) $f(x) = \dfrac{(x-3)^7}{(x-3)^5}$ e) $f(x) = \dfrac{(x-3)^2}{(x-3)^4}$

   b) $f(x) = \dfrac{(x-3)^4}{(x-3)}$ d) $f(x) = \dfrac{(x-3)}{(x-3)^4}$

7. Graph $f(x) = \dfrac{5x^2-20}{6x+3x^2-24}$

8. Find the derivative of each of the following functions.

a) $f(x) = 7^x - x^7 + \sqrt{7x} - e^7$

b) $f(x) = \dfrac{e^x - e^{-x}}{2}$

c) $f(x) = \log_5 (x^3 - 3x + 1)$

d) $f(x) = \dfrac{1}{4} x e^{4x} - \dfrac{1}{16} e^{4x}$

e) $f(x) = \dfrac{8x^2 - 20x + 1}{4x - 10}$

f) $f(x) = \ln\left(\dfrac{x^5 + 2}{x^4 - 3}\right)$

g) $f(x) = \ln(\ln(x^5))$

h) $f(x) = 2^{3x^2 - x + 5}$

9. Compute each of the following indefinite integrals.

a) $\displaystyle\int 12x^2 - 2x + 1\, dx$

b) $\displaystyle\int x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2}\, dx$

c) $\displaystyle\int x^5 - 2ax - a^2\, dx$

d) $\displaystyle\int x^5 - 2ax - a^2\, da$

e) $\displaystyle\int e^{2x} dx$

f) $\displaystyle\int 5^x - x^5\, dx$

g) $\displaystyle\int (5x - 3)^{20}\, dx$

h) $\displaystyle\int \frac{1}{2x + 1}\, dx$

i) $\displaystyle\int mx + b\, dx$

10. Find an equation of the tangent line drawn to the graph of $f(x) = \dfrac{1}{(3x - 1)^2}$ at $x = 1$.

11. For each of the following functions given, find the $x-$coordinate of all relative maximums and minimums. In each case, state whether the extrema is a maximum or minimum.

a) $f(x) = (x - 1)^{10} (4 - x)^5$

b) $f(x) = \dfrac{(2x - 7)^5}{x^2}$

c) $f(x) = \dfrac{x^2}{e^{2x}}$

12. Find the second derivative for the function $f(x) = \dfrac{5x - 1}{5x + 1}$. (Hint: although you may use the quotient rule, it can be avoided with a little algebraic transformation before the calculus).

13. Polynomial Approximations.

a) Find the equation(s) of the tangent line(s) drawn to the graph of $f(x) = \dfrac{1}{1 + x^2}$ at $x = -1$.

b) Let $f(x) = \dfrac{1}{x + 1}$. Find a polynomial function $P(x)$ such that P is of degree four and

$$f(0) = P(0), \quad f'(0) = P'(0), \quad f''(0) = P''(0), \quad f'''(0) = P'''(0), \text{ and } f^{(4)}(0) = P^{(4)}(0)$$

14. Find the equation of $F(x)$ if $F'(x) = 10x^4 - 6x^2 + 4x - 5$ and $F(-1) = 8$.

15. Suppose that f is a function with derivative

$$f'(x) = \left(16 - x^2\right)\left(4 - x^2\right)(2 - x)$$

a) Graph f'

b) Find all x for which f has a relative maximum.

c) Find all x for which f has a relative minimum.

d) How many points of inflection does f have?

e) Plot the graph of f in the same coordinate system with f'.

f) Is it possible that f does not have any $x-$intercept?

16. We shoot a small object upward, from the top of a tower. The acceleration function of the object is $a(t) = -10$. (Location is measured in meters, velocity in $\frac{\text{m}}{\text{s}}$, acceleration in $\frac{\text{m}}{\text{s}^2}$.)

    a) Given that $v(0) = 160$, find $v(t)$, the velocity function of the object.

    b) Given that $h(0) = 525$, find $h(t)$, the location function of the object.

    c) Find the maximum height that the object reaches.

17. Consider the function $f(x) = \sqrt{1-x^2}$ on the interval $[-1, 1]$.

    a) Compute the left Riemann sum with a uniform partition of 5 subintervals.

    b) Compute the right Riemann sum with a uniform partition of 5 subintervals.

18. Consider the function $f(x) = \ln x$ on the interval $[1, 6]$.

    a) Compute the left Riemann sum with a uniform partition of 5 subintervals.

    b) Compute the right Riemann sum with a uniform partition of 5 subintervals.

Review Problems - Answers

1. a) $x < -\frac{5}{3}$ or $x \geq \frac{11}{2}$ b) $-2 \leq x < -\frac{5}{3}$

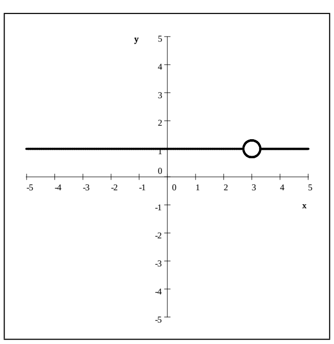
2. a) $-4 < x < 4$ and $x \neq -1, 0, 1$ b) $x > 2$ c) $x < \frac{7-\sqrt{41}}{2}$ or $x > \frac{7+\sqrt{41}}{2}$

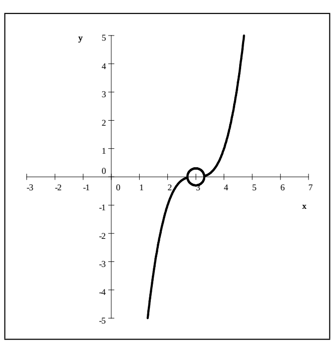
3. a) -10 b) -2 c) $\frac{17}{6}$ d) 16

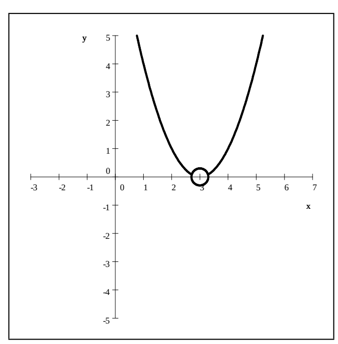
4. a) $f^{-1}(x) = \frac{1}{5}(\log_3 x + 1)$ b) $f^{-1}(x) = \frac{5x+4}{3x-1}$ c) $f^{-1}(x) = \frac{1}{2}(e^x + 1)$

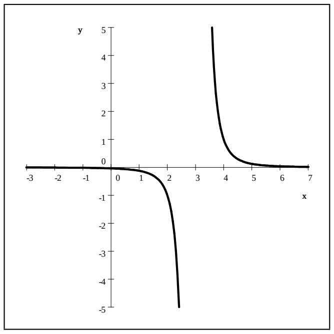
5. a) -2 b) ∞ c) $\frac{2}{3}$ d) $-\infty$ e) $\frac{1}{2}$ f) ∞

6. a) $f(x) = \frac{x-3}{x-3}$ b) $f(x) = \frac{(x-3)^4}{(x-3)}$ c) $f(x) = \frac{(x-3)^7}{(x-3)^5}$

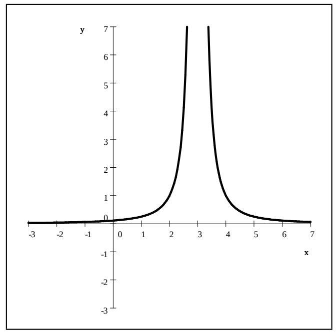






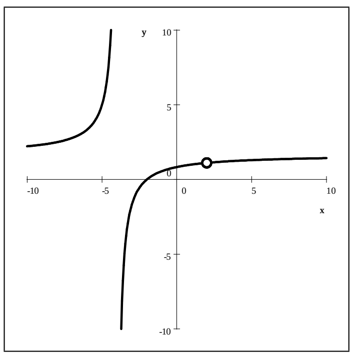


d) $f(x) = \dfrac{(x-3)}{(x-3)^4}$



e) $f(x) = \dfrac{(x-3)^2}{(x-3)^4}$

7. $f(x) = \dfrac{5x^2 - 20}{6x + 3x^2 - 24} = \begin{cases} \dfrac{5(x+2)}{3(x+4)} & \text{if } x \neq 2 \\ \text{undefined} & \text{if } x = 2 \end{cases}$



8. a) $f'(x) = 7^x \ln 7 - 7x^6 + \dfrac{7}{2\sqrt{7x}}$ b) $f'(x) = \dfrac{e^x + e^{-x}}{2}$ c) $f'(x) = \dfrac{3x^2 - 3}{(x^3 - 3x + 1)\ln 5}$

d) $f'(x) = xe^{4x}$ e) $f'(x) = \dfrac{8x^2 - 40x + 49}{4x^2 - 20x + 25} = 2 - \dfrac{1}{(2x-5)^2}$ f) $f'(x) = \dfrac{5x^4}{x^5 + 2} - \dfrac{4x^3}{x^4 - 3}$

g) $f'(x) = \dfrac{1}{x \ln x}$ h) $f'(x) = (6x - 1)\left(2^{3x^2 - x + 5}\right)(\ln 2)$

9. a) $4x^3 - x^2 + x + C$ b) $\dfrac{x^3}{3} + \dfrac{x^2}{2} + x + \ln|x| - \dfrac{1}{x} + C$ c) $\dfrac{x^6}{6} - ax^2 - a^2x + C$

d) $x^5a - xa^2 - \dfrac{a^3}{3} + C$ e) $\dfrac{1}{2}e^{2x} + C$ f) $\dfrac{5^x}{\ln 5} - \dfrac{x^6}{6} + C$ g) $\dfrac{1}{105}(5x-3)^{21} + C$

h) $\dfrac{1}{2}\ln|2x + 1| + C$ i) $\dfrac{m}{2}x^2 + bx + C$

10. $y - \dfrac{1}{4} = -\dfrac{3}{4}(x - 1)$ or $y = -\dfrac{3}{4}x + 1$

11. a) a relative minimum at $x = 1$, a relative maximum at $x = 3$

b) no relative minimum, a relative maximum at $x = -\dfrac{7}{3}$

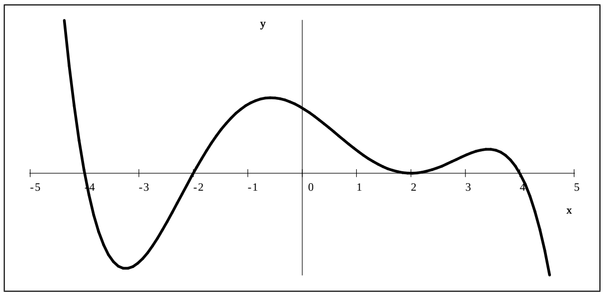
c) a relative minimum at $x = 0$, a relative maximum at $x = 1$

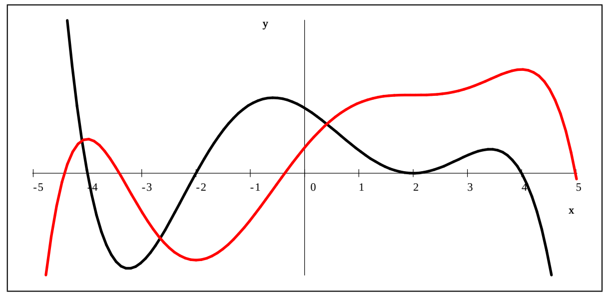
12. $-\dfrac{100}{(5x+1)^3}$

13. a) $y = \dfrac{1}{2}x + 1$ b) $x^4 - x^3 + x^2 - x + 1$

14. $F(x) = 2x^5 - 2x^3 + 2x^2 - 5x + 1$

15. a) see below b) $-4, 4$ c) -2 d) 4 e) see below f) yes





16. a) $v(t) = -10t + 160$ b) $h(t) = -5t^2 + 160t + 525$ c) $h_{\max} = 1805\,\mathrm{m}$

17. a) 1.4238 b) 1.4238

18. a) 4.78749 b) 6.5792