

1. Let $a = \log_2 6$ and $b = \log_2 10$. Express each of the following in terms of a and b .

a) $\log_2 3$ b) $\log_2 15$ c) $\log_5 3$

2. Solve the equation $\log_3 (2x - 1) + \log_3 (x - 5) = 5$.

3. Find each of the following limits.

a) $\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{x^2 - 3x + 2}$ b) $\lim_{x \rightarrow 3} \frac{x - 3}{\frac{1}{3} - \frac{1}{x}}$ c) $\lim_{x \rightarrow \infty} \frac{-x^3 + 2x + 1}{x - 3}$

d) $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16}$ e) $\lim_{x \rightarrow \infty} \frac{\log_3 x + 1}{\log_4 (x^2)}$ f) $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\ln x}$

4. Sketch the graph of $f(x) = \frac{-6(x+2)^3(4x)^2(x-5)}{3(-2-x)^5 x^2(10-2x)^4}$.

5. Find the derivative for each of the following functions.

a) $f(x) = 2x^2 \ln x - x^2$ d) $f(x) = -2 \ln(4e^{-3x})$ g) $f(x) = \frac{8x^2 - 20x + 1}{4x - 10}$

b) $f(x) = \frac{e^{4x^5 - 7x}}{e^{x^5}}$ e) $f(x) = 7^x - x^7 + \sqrt{7x} - e^7 + \ln 7$

c) $f(x) = \frac{5x^3 - 1}{x^3}$ f) $f(x) = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$

6. Find the equation of the tangent line drawn to the graph of $f(x) = x^3 - 7x + 6$ at $x = -1$.

7. Find the equation of the tangent line drawn to the graph of $-3x^2 - 16xy - 2y^2 + 3y = 178$ at the point $(-3, 5)$.

8. Consider the relation determined by the equation $xy^2 - 5x = 2(y^2 + x^2y - 16)$. Find an equation for all tangent line(s) drawn to the graph of the relation at $x = 3$.

9. Let $f(x) = 3e^{2x}$.

a) Compute $\int f(x) dx$. b) Find the average value of f on the interval $[-2, 2]$.

c) Find the equation for $P(x)$ if P is a polynomial function satisfying the following conditions. $P(x)$ is of degree three, and $P(0) = f(0)$, $P'(0) = f'(0)$, $P''(0) = f''(0)$, and $P'''(0) = f'''(0)$.

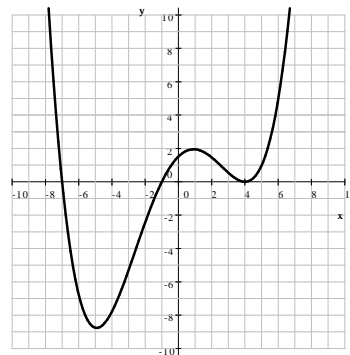
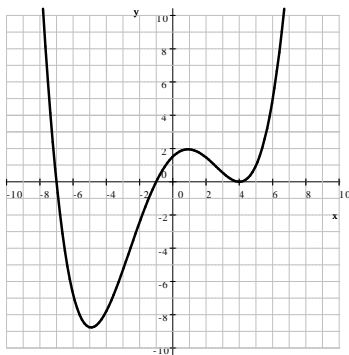
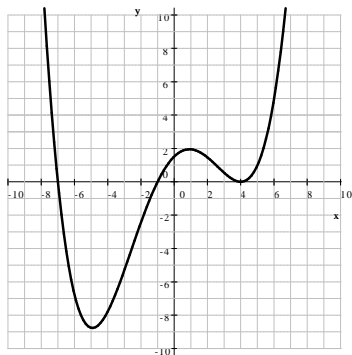
10. We know the following things about a function f . $f'(x) = 20x^3 - 3$ and $f(-1) = 16$. Find f .

11. The figures below all show the graph of $f(x)$. Sketch the graph of $g(x)$ into the same coordinate system if

a) $g(x) = \frac{1}{f(x)}$

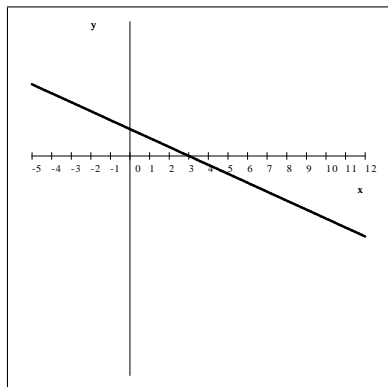
b) $g(x) = f'(x)$

c) $g(x)$ where $g'(x) = f(x)$

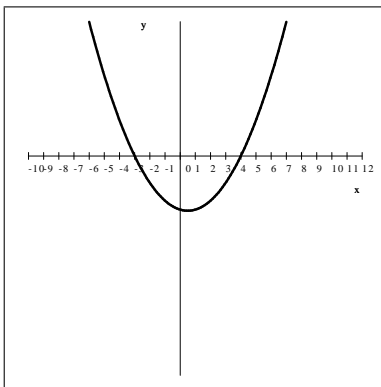


12. The graph of f is given on each of the graphs shown below. Plot the graph of $g(x)$ if $g'(x) = f(x)$ and

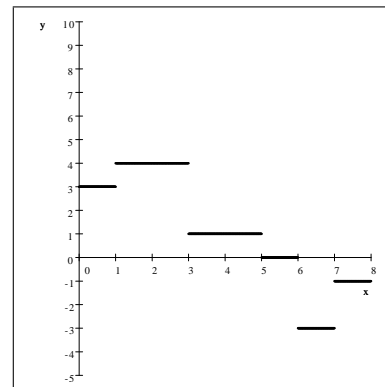
a) $g(0) = 4$



b) $g(0) = -6$



c) $g(0) = -5$



13. Evaluate each of the following integrals.

a) $\int \frac{1}{3x-5} dx$

c) $\int x\sqrt{x-3} dx$

e) $\int_0^5 e^{-2x} dx$

g) $\int_1^2 \frac{(\ln x)^2}{x} dx$

b) $\int_0^1 \frac{6x^2 - 4x + 5}{x+1} dx$

d) $\int 5^x dx$

f) $\int_0^3 xe^{-4x} dx$

h) $\int_1^5 \frac{x}{\sqrt{3x+1}} dx$

14. Compute each of the following improper integrals.

a) $\int_0^{\infty} 2e^{-2x} dx$

b) $\int_1^{\infty} \frac{1}{x^3} dx$

c) $\int_0^{\infty} xe^{-2x} dx$

d) $\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$

15. A manufacturer estimates that when q units of a particular product are produced each month, the total cost will be $C(q) = 5q + 17000$ dollars, and all q units can be sold at a price of $p(q) = 65 - \frac{q}{100}$ dollars per unit.

a) Find the fixed cost.

b) Determine the level of production that results in a maximum profit.

c) What is the maximum profit?

16. It costs 12 dollars each to manufacture and distribute a certain product. If we price it at x dollars each, the number sold is approximated by $n(x) = \frac{60}{x-12} + \frac{1}{5}(100-x)$. What selling price will bring in the maximum profit? How much money is that?

17. Oil is leaking from a container at the rate of $R(t) = 2000e^{-0.2t}$ gallons per hour, where t is time, measured in hours. How much oil has leaked out of the container after 10 hours? Round your answer to the nearest gallon.

18. A virus is spreading through a population in a manner that can be modeled by the function $g(t) = \frac{A}{1+3e^{-t}}$ where A is the total population and $g(t)$ is the number infected at time t . What proportion of the population is infected when the virus is spreading the fastest?

19. In a certain town, the rate of deaths at time t due to a particular disease is modeled by $\frac{10000}{(t+3)^3}$ where $t \geq 0$. What is the total number of deaths from this disease predicted by the model?

20. A company has \$120 000 to spend on the development and promotion of a new product. The company estimates that if x is spent on the development and y is spent on promotion, then approximately $\frac{x^{1/2}y^{3/2}}{400\,000}$ items of new product will be sold. Based on this estimate, what is the maximum number of products that the company can sell?
21. A company can sell 20 products if it charges \$40 per product. For each dollar decrease or increase in the price, the company can sell one more or one less product, respectively. The total cost of producing q products is $C(q) = 32q + 100$. What is the maximum profit that the company can achieve from manufacturing and selling this product?

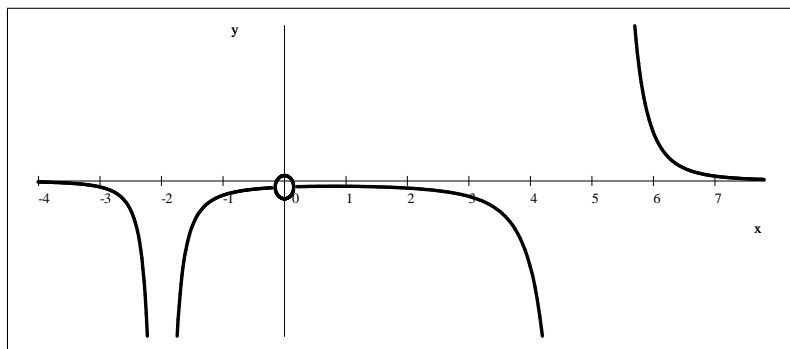
Answers

1. a) $a - 1$ b) $a + b - 2$ c) $\frac{a - 1}{b - 1}$

2. 14

3. a) -2 b) 9 c) $-\infty$ d) $\frac{1}{8}$ e) $\frac{\ln 2}{\ln 3}$ f) ∞

4. $f(x) = \frac{-6(x+2)^3(4x)^2(x-5)}{3(-2-x)^5x^2(10-2x)^4}$ (The hole at $x = 0$ is not at $y = 0$, it is at $y = -\frac{1}{250}$)



5. a) $f'(x) = 4x \ln x$ b) $f'(x) = (15x^4 - 7)e^{3x^5 - 7x}$ c) $f'(x) = \frac{3}{x^4}$ d) $f'(x) = 6$

e) $f'(x) = 7^x \ln 7 - 7x^6 + \frac{\sqrt{7}}{2\sqrt{x}}$ f) $f'(x) = xe^{4x}$ g) $f'(x) = \frac{8x^2 - 40x + 49}{4x^2 - 20x + 25} = 2 - \frac{1}{(2x - 5)^2}$

6. $y = -4x + 8$

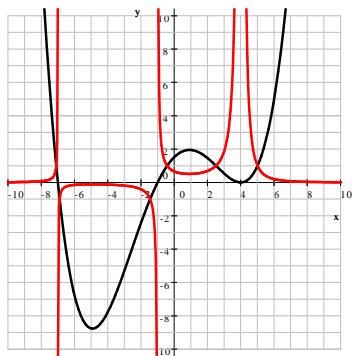
7. $y = 2x + 11$

8. $y = -5x + 32$ and $y = -x + 2$

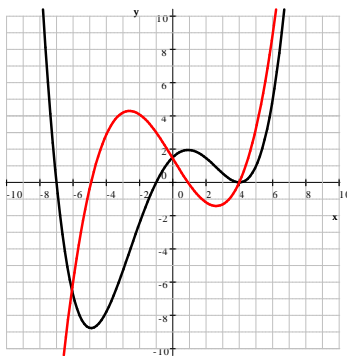
9. a) $\frac{3}{2}e^{2x} + C$ b) $\frac{3}{8}(e^4 - e^{-4}) \cong 20.467438$ c) $4x^3 + 6x^2 + 6x + 3$

10. $f(x) = 5x^4 - 3x + 8$

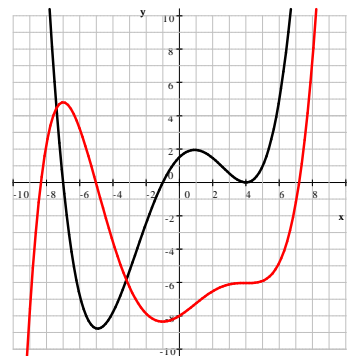
11. a) $g(x) = \frac{1}{f(x)}$



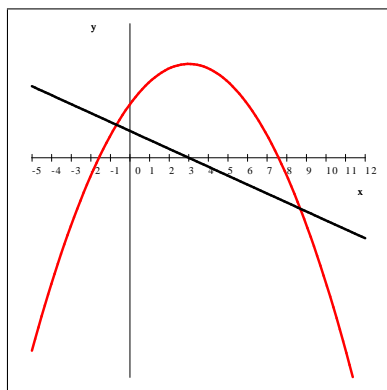
b) $g(x) = f'(x)$



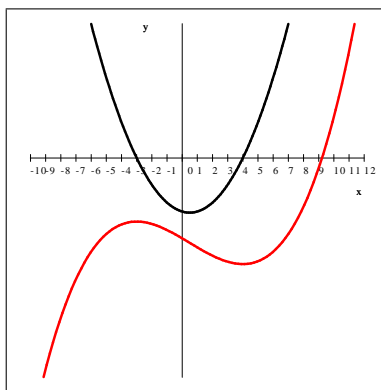
c) $g(x)$ where $g'(x) = f(x)$



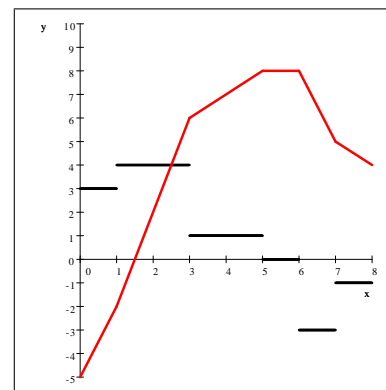
12. a)



b)



c)



13. a) $\frac{1}{3} \ln|3x - 5| + C$ b) $15 \ln 2 - 7$ c) $\frac{2}{5} (x - 3)^{5/2} + 2(x - 3)^{3/2} + C$

d) $\frac{5^x}{\ln 5} + C$ e) $\frac{1}{2} - \frac{1}{2}e^{-10}$ f) $\frac{1}{16} - \frac{13}{16}e^{-12}$ g) $\frac{1}{3} \ln^3 2$ h) $\frac{100}{27}$

14. a) 1 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{\ln 2}$

15. a) \$17 000 b) 3000 c) \$73 000

16. \$440 with a price of \$56

17. 8647 gallons

18. $\frac{1}{2}$

19. 556

20. 11 691

21. \$96