Quiz 2 will cover the following topics: all topics from Quiz 1; radical expressions (A1, handout), functions and their graphs (1.1, 1.2), linear functions and slope (1.3, handout); and graphing a parabola (handout), optimization (1.4, handout), limits at infinity (1.5, handout) and average veloctiy (see this review)

Review Problems

1. Simplify each of the following expressions.

   a) $\sqrt{125} - 3\sqrt{80} + \sqrt{45}$ b) $\left(\sqrt{7} - 2\right)^2$ c) $\left(\sqrt{3} - 1\right)^3$

2. Rationalize the denominator in each of the following expressions.

   a) $\dfrac{3}{\sqrt{5}}$ b) $\dfrac{1}{\sqrt{10} - 3}$ c) $\dfrac{2}{\sqrt{7} + 1}$

3. Compute the exact value of $x^2 - 4x + 6$ if $x = 2 - \sqrt{3}$.

4. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find

   a) $f(g(4))$ b) $g(f(4))$ c) $f(g(x))$ d) $g(f(x))$

5. Find an equation of the straight line that passes through the points $(3, -1)$ and $(1, 5)$.

6. Graph the parabola $y = -8x + x^2 + 15$. Clearly label the coordinates of five points on the parabola, including vertex and intercepts.

7. One side of a rectangle is 4 in shorter than 3 times the other side. Find the sides of the rectangle if its perimeter is 48 in.

8. One side of a rectangle is 18 in shorter than the other side. Find the sides of the rectangle if its area is 360 in^2.

9. Among the rectangles of perimeter 12 m, which one has the largest area?

10. If we set the price of our product to be \$18 per item, then we can sell 300 items. For every dollar we raise the price, we can sell 5 less items.

    a) How much is the total income if we price the product at \$21?

    b) What price woud guarantee an income of \$6625?

    c) What price would guarantee the possible highest income? What is the highest possible income?

11. A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.4q^2 + 3q + 40$ thousand dollars, and all q units can be sold at a price of $p(q) = 22.2 - 1.2q$ thousand dollars per unit. Find the maximal profit.

12. A farmer wishes to enclose a rectangular pasture with 320 ft of fence. What dimensions give the maximum area if the fence is on three sides of the pasture and the fourth side is bounded by a wall?

13. We threw a tennis ball upwards, standing on a roof top 200 meters high, with an initial upward velocity of 30 $\frac{\text{m}}{\text{s}}$. The ball travels upward and then falls to the ground. The height of the ball, measured in meter (counting ground level as zero) is $h(t) = -5t^2 + 30t + 200$. Time is measured in seconds.

a) Complete the following table.

t	0	1	2	3	4	6	9
$h(t)$							

b) How long does it take for the ball to hit the ground?

c) Find the average velocity between

i) $t = 0$ and $t = 2$ seconds

ii) $t = 3$ seconds and $t = 5$ seconds

iii) $t = 4$ seconds and $t = 9$ seconds

d) What is the highest point that the ball reaches?

14. Compute each of the following limits.

a) $\lim_{x \to -\infty} (-5x^2 - 8x^3)$

b) $\lim_{x \to \infty} (-5x^2 - 8x^3)$

c) $\lim_{x \to -\infty} (5x^2 - 2x + 1)$

d) $\lim_{x \to \infty} (5x^2 - 2x + 1)$

e) $\lim_{x \to \infty} \frac{5x^2 - 2x + 1}{3x^2 - 8x + 13}$

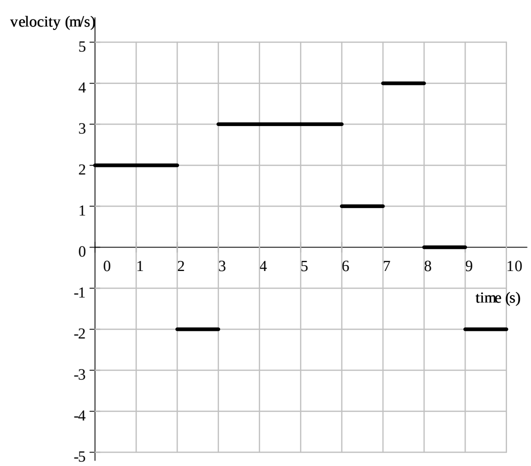
f) $\lim_{x \to -\infty} \frac{5x^5 - 2x + 1}{3x - 8x^3 - 7}$

g) $\lim_{x \to \infty} \frac{2x^3 - 6x^2}{x^7 - 8x - 1}$

h) $\lim_{x \to -\infty} \frac{x^5 - 1}{x^2 + x - 1}$

i) $\lim_{x \to -\infty} \frac{3x^{204} - 1}{7x^{204} - 1}$

15. The picture below shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward).



How far is the object from the starting point at

a) $t = 1\,\text{s}$ b) $t = 3\,\text{s}$ c) $t = 5\,\text{s}$ d) $t = 10\,\text{s}$?

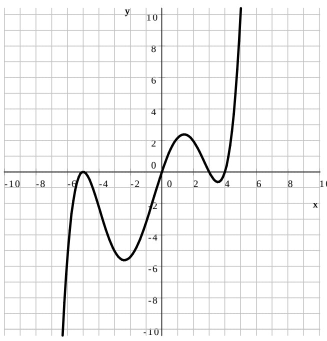
16. Graph each of the following equations. In case of each of these equations, determine whether y is a function of x or not.

a) $y = x^2 - 6x + 8$ c) $x = y^2$ e) $xy = 1$ g) $x^2y = 1$

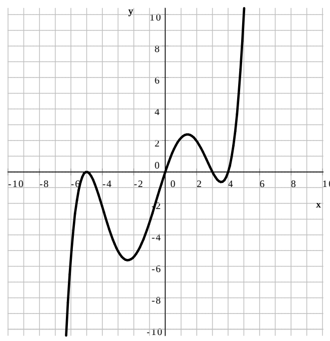
b) $x^2 = y^2$ d) $x^2 + y = 4$ f) $2x + 3y = 4$

17. The figures below all show the graph of $f(x)$. Sketch the graph of $g(x)$ into the same coordinate system if

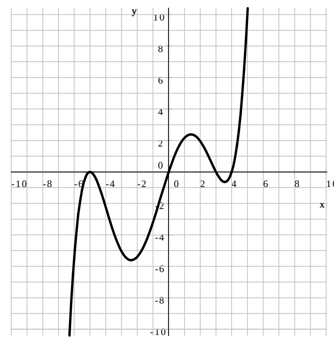
a) $g(x) = f(x) + 2$



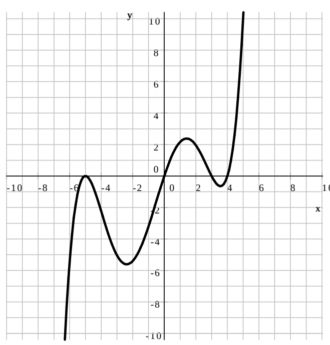
b) $g(x) = f(x + 2)$



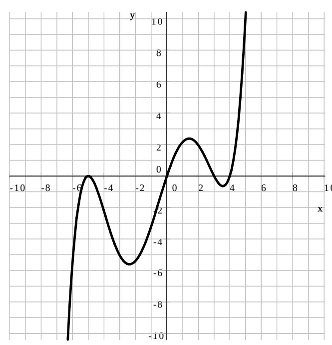
c) $g(x) = -f(x)$



e) $g(x) = \dfrac{f(x) + |f(x)|}{2}$



f) $g(x) = f(-x)$



Review Problems - Answers

1. For more practice and detailed solutions, see handout Radical Expressions.

a) $-4\sqrt{5}$ b) $11 - 4\sqrt{7}$ c) $-10 + 6\sqrt{3}$

2. For more practice and detailed solutions, see handout Radical Expressions.

a) $\dfrac{3\sqrt{5}}{5}$ b) $\sqrt{10} + 3$ c) $\dfrac{\sqrt{7} - 1}{3}$

3. For more practice and detailed solutions, see handout Radical Expressions. 5

4. For more practice and detailed solutions, see 1.1.

   a) $\sqrt{15}$ b) 3 c) $\sqrt{x^2-1}$ d) $x-1$

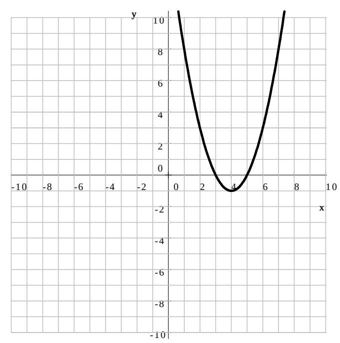
   Solution:

   a) $f(g(4)) = f(4^2-1) = f(15) = \sqrt{15}$ b) $g(f(4)) = g(\sqrt{4}) = g(2) = 2^2-1 = 3$

   c) $f(g(x)) = f(x^2-1) = \sqrt{x^2-1}$ d) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x-1$

5. For more practice and detailed solutions, see the textbook, Section 1.3 and handout: Writing Equations of Lines. $y = -3x+8$

6. For more practice and detailed solutions, see handout Graphing a Parabola.
   $y-$intecept: $(0,15)$; vertex: $(4,-1)$; $x-$intercepts: $(3,0)$ and $(5,0)$. Additional points: $(2,3)$; $(6,3)$



7. For more practice and detailed solutions, see handout Word Problems 2. 7 in and 17 in

8. For more practice and detailed solutions, see handout Word Problems 2. 12 in and 30 in

9. see Optimization 1 the square

10. see Optimization 1 a) \$5985 b) \$25 and \$53 c) \$7605 with a price of \$39

11. see Optimization 1 \$17600

12. see Optimization 1 80 ft by 160 ft

13. For more practice and detailed solutions, see handout Factoring 1 and Optimization 1.

    a) b) 10 seconds c) i) $20\frac{\text{m}}{\text{s}}$ ii) $-10\frac{\text{m}}{\text{s}}$ iii) $-35\frac{\text{m}}{\text{s}}$ d) 245 m

t	0 s	1 s	2 s	3 s	4 s	6 s	9 s
$h(t)$	200 m	225 m	240 m	245 m	240 m	200 m	65 m

Solution:

a) In each case we simply substitute into the formula. For instance, we compute $h(2)$.

$$h(t) = -5t^2 + 30t + 200$$
$$h(2) = -5\cdot 2^2 + 30\cdot 2 + 200 = -5\cdot 4 + 30\cdot 2 + 200 = -20 + 60 + 200 = 240$$

b) see Factoring 1

c) i)

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{h(2\,\text{s}) - h(0\,\text{s})}{2\,\text{s} - 0\,\text{s}} = \frac{240\,\text{m} - 200\,\text{m}}{2\,\text{s}} = 20\frac{\text{m}}{\text{s}}$$

ii)

$$\begin{aligned} v_{\text{av}} &= \frac{\text{distance traveled}}{\text{time}} = \frac{h(5\,\text{s}) - h(3\,\text{s})}{5\,\text{s} - 3\,\text{s}} \\ &= \frac{(-5 \cdot 5^2 + 30 \cdot 5 + 200)\ \text{m} - (-5 \cdot 3^2 + 30 \cdot 3 + 200)\ \text{m}}{2\,\text{s}} = \frac{225\,\text{m} - 245\,\text{m}}{2\,\text{s}} = -10\frac{\text{m}}{\text{s}} \end{aligned}$$

The negative sign indicates that the object has traveled downward.

iii)

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{h(9\,\text{s}) - h(4\,\text{s})}{9\,\text{s} - 4\,\text{s}} = \frac{65\,\text{m} - 240\,\text{m}}{5\,\text{s}} = \frac{-175\,\text{m}}{5\,\text{s}} = -35\frac{\text{m}}{\text{s}}$$

The negative sign indicates that the object has traveled downward.

14. For more practice and detailed solutions, see 1.5 and handout Limits at Infinity.

a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{5}{3}$ f) $-\infty$ g) 0 h) $-\infty$ i) $\frac{3}{7}$

15. a) 2 m b) 2 m c) 8 m d) 14 m

Solution:

a) In the first second, the object had a constant velocity of $2\frac{\text{m}}{\text{s}}$. For one second of traveling, this means a distance of

$$s = vt \quad \Longrightarrow \quad s = 2\frac{\text{m}}{\text{s}} \cdot 1\,\text{s} = 2\,\text{m}$$

b) For the first two seconds, the velocity is $2\frac{\text{m}}{\text{s}}$. Then, for one second, it is $-2\frac{\text{m}}{\text{s}}$. The distance traveled is

$$s = vt \quad \Longrightarrow \quad s = 2\frac{\text{m}}{\text{s}} \cdot 2\,\text{s} + \left(-2\frac{\text{m}}{\text{s}}\right) \cdot 1\,\text{s} = 4\,\text{m} - 2\,\text{m} = 2\,\text{m}$$

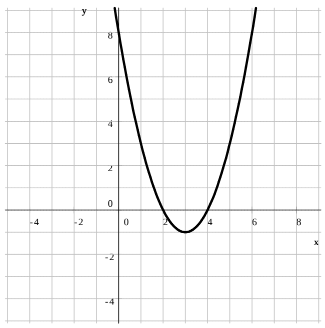
c) For the first two seconds, the velocity is $2\frac{\text{m}}{\text{s}}$. Then, for one second, it is $-2\frac{\text{m}}{\text{s}}$. Then, for two seconds it is $2\frac{\text{m}}{\text{s}}$. The distance traveled is

$$\begin{aligned} s &= vt \\ s &= 2\frac{\text{m}}{\text{s}} \cdot 2\,\text{s} + \left(-2\frac{\text{m}}{\text{s}}\right) \cdot 1\,\text{s} + 3\frac{\text{m}}{\text{s}} \cdot 2\,\text{s} = 4\,\text{m} - 2\,\text{m} + 6\,\text{m} = 8\,\text{m} \end{aligned}$$

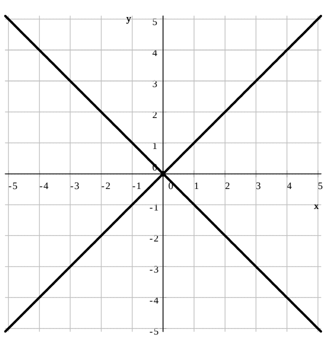
d)

$$\begin{aligned} s &= vt \\ s &= 2\frac{\text{m}}{\text{s}} \cdot 2\,\text{s} + \left(-2\frac{\text{m}}{\text{s}}\right) \cdot 1\,\text{s} + 3\frac{\text{m}}{\text{s}} \cdot 3\,\text{s} + 1\frac{\text{m}}{\text{s}} \cdot 1\,\text{s} + 4\frac{\text{m}}{\text{s}} \cdot 1\,\text{s} + 0\frac{\text{m}}{\text{s}} \cdot 1\,\text{s} + \left(-2\frac{\text{m}}{\text{s}}\right) \cdot 1\,\text{s} \\ &= 4\,\text{m} - 2\,\text{m} + 9\,\text{m} + 1\,\text{m} + 4\,\text{m} - 2\,\text{m} = 14\,\text{m} \end{aligned}$$

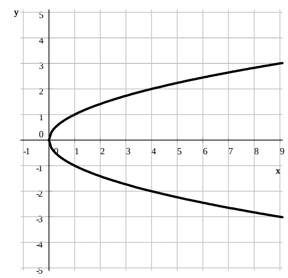
16. a) $y = x^2 - 6x + 8$
Solution: This is a function.
Algebraically: There is a unique formula to compute y in terms of x.
Geometrically: its graph passes the vertical line test.



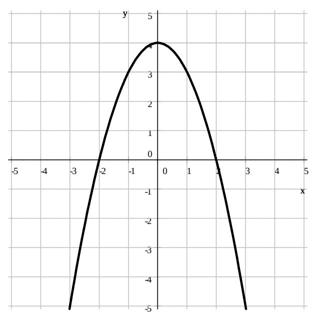
b) $x^2 = y^2$
Solution: Not a function.
Algebraically: We solve for y
$x^2 = y^2$
$0 = y^2 - x^2$
$0 = (y + x)(y - x)$
$y_1 = -x \qquad y_2 = x$
Geometrically: its graph fails the vertical line test.



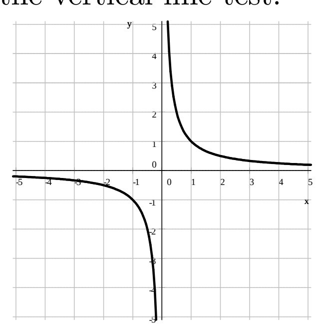
c) $x = y^2$
Solution: Not a function
Algebraically: We solve for y :
$x = y^2$
$0 = y^2 - x$
$0 = y^2 - (\sqrt{x})^2$
$0 = (y + \sqrt{x})(y - \sqrt{x})$
$y_1 = -\sqrt{x} \qquad y_2 = \sqrt{x}$
Geometrically: the graph fails the vertical line test.



d) $x^2 + y = 4$
Solution: Function. $y = -x^2 + 4$



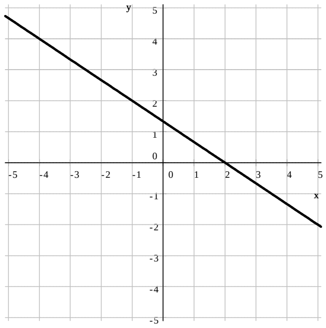
e) $xy = 1$
Solution: Function.
Algebraically: We solve for y and see that it is a unique formula. $y = \dfrac{1}{x}$
Geometrically: Its graph passes the vertical line test.



f) $2x + 3y = 4$
Solution: Function. We solve for y and see that it is a unique formula.

$$\begin{aligned} 2x + 3y &= 4 && \text{subtract } 2x \\ 3y &= -2x + 4 && \text{divide by } 3 \\ y &= \frac{-2x+4}{3} \\ y &= -\frac{2}{3}x + \frac{4}{3} \end{aligned}$$

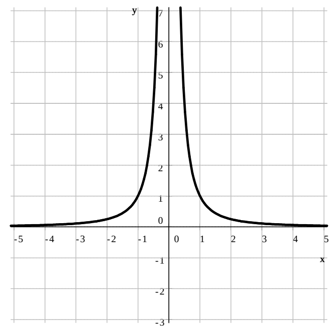
Its graph passes the vertical line test.



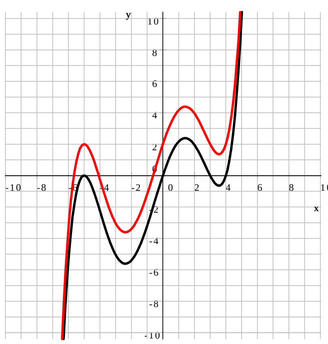
g) $x^2 y = 1$
Solution: Function. We solve for y and see that it is a unique formula.

$$\begin{aligned} x^2 y &= 1 && \text{divide by } x^2 \\ y &= \frac{1}{x^2} \end{aligned}$$

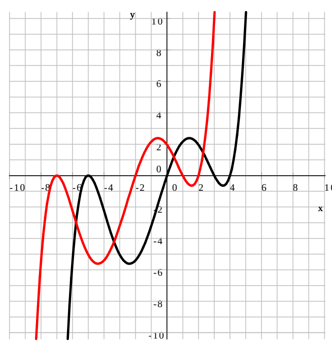
Its graph passes the vertical line test.



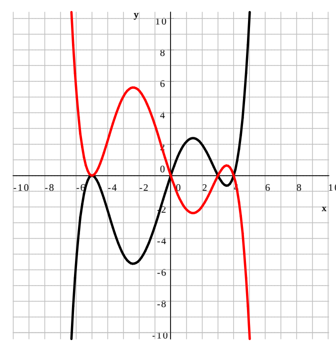
17. a) vertical shift upward by 2 units



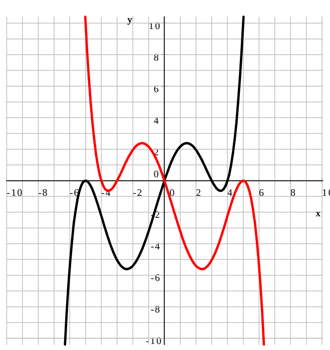
b) horizontal shift to the left by 2 units



c) reflection to the $x-$axis



d) reflection to the $y-$axis



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