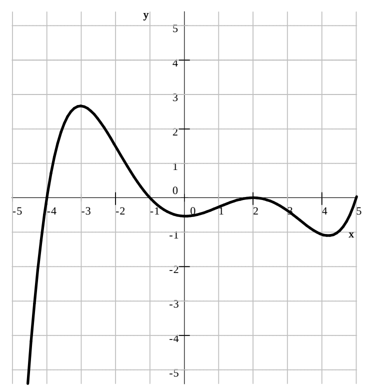
Quiz 4 will cover the following topics: all topics from Quiz 1, 2, and 3, Exam 1, logarithms, complete analysis of functions, and differentiation rules 1.

Review Problems

1. Find the domain of each of the following expressions.

   a) $\log_5\left(-x^2+10x-20\right)$ b) $\log_2\left(x^2-6x+8\right)$

2. The graph below shows the graph of a function $f(x)$. Plot the graph of $g(x)=\dfrac{1}{f(x)}$.



3. Sketch the graph and give a complete analysis for the function $f(x)=10x-x^2+11$ where the domain is the closed interval $[3,8]$.

4. Find and prove, using the definition of the derivative as the limit of the differential quotient.

   a) $f(x)=5x^2$ b) $f(x)=\sqrt{x-3}$

5. Find an equation for the tangent line drawn to the graph of $f(x)=x^4+2x^3-10x+4$ at $x=-2$.

6. Let $L(t)=-t^3+21t^2-99t+450$ be the location function of an object, where $t\geq 0$.

   a) Find all values of t where the object is moving upward.

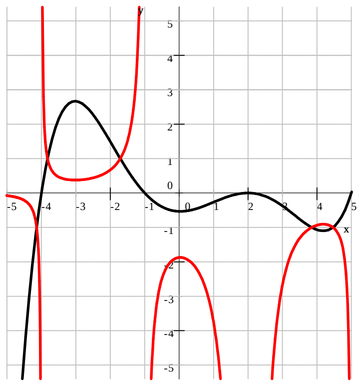
   b) When is the object moving upward with the greatest velocity?

7. Let $L(t)=\log_3 t$ be the location function of an object. Find an approximation, accurate to at least 3 decimal places, for the velocity of the object at $t=2$.

Review Problems - Answers

1. a) $\left(5-\sqrt{5}, 5+\sqrt{5}\right)$ b) $(-\infty, 2) \cup (4, \infty)$

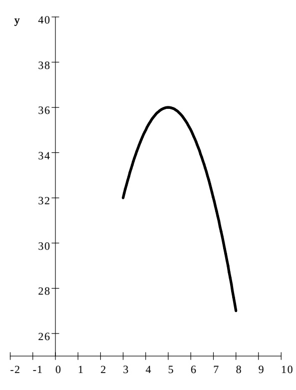
2. $g(x) = \dfrac{1}{f(x)}$



3. $f(x) = 10x - x^2 + 11$ on $[3, 8]$

Solution:

1. Is it a function? yes
2. domain: $[3, 8)$
3. range: $[27, 36)$
4. horizontal asymptote: none
   vertical asymptote: none
5. $y-$intercept: none
6. $x-$intercept: none
7. boundedness: bounded
8. one-to-one: no
9. relative maximum: $(5, 36)$
   absolute maximum: $(5, 36)$
10. relative minimum: none
    absolute minimum: $(8, 27)$
11. increasing: on $[3, 5]$
    decreasing: on $[5, 8]$
12. point of inflection: none
13. concave up: nowhere
    concave down: on $[3, 8]$
14. continuous: on $[3, 8]$
15. even/odd: neither
16. $\lim\limits_{x\to-\infty} f(x) =$ undefined and $\lim\limits_{x\to\infty} f(x) =$ undefined
17. Graph:



4. a) $f(x) = 5x^2 \qquad f'(x) = 10x$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{5(x+h)^2-5(x)^2}{h}\\
&= \lim_{h\to 0}\frac{5(x^2+2xh+h^2)-5(x)^2}{h} = \lim_{h\to 0}\frac{5(x^2+2xh+h^2)-5(x)^2}{h}\\
&= \lim_{h\to 0}\frac{5x^2+10xh+5h^2-5x^2}{h} = \lim_{h\to 0}\frac{10xh+5h^2}{h}\\
&= \lim_{h\to 0}\frac{10xh+5h^2}{h} = \lim_{h\to 0}\frac{h(10x+5h)}{h} = \lim_{h\to 0} 10x+5h = 10x
\end{aligned}$$

b) $f(x) = \sqrt{x-3} \qquad f'(x) = \dfrac{1}{2\sqrt{x-3}}$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}\\
&= \lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}\cdot 1\\
&= \lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}\cdot\frac{\sqrt{x+h-3}+\sqrt{x-3}}{\sqrt{x+h-3}+\sqrt{x-3}}\\
&= \lim_{h\to 0}\frac{\left(\sqrt{x+h-3}-\sqrt{x-3}\right)\left(\sqrt{x+h-3}+\sqrt{x-3}\right)}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)}\\
&= \lim_{h\to 0}\frac{\left(\sqrt{x+h-3}\right)^2-\left(\sqrt{x-3}\right)^2}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)} = \lim_{h\to 0}\frac{(x+h-3)-(x-3)}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)}\\
&= \lim_{h\to 0}\frac{x+h-3-x+3}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)} = \lim_{h\to 0}\frac{\not h}{\not h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)}\\
&= \lim_{h\to 0}\frac{1}{\sqrt{x+h-3}+\sqrt{x-3}} = \frac{1}{2\sqrt{x-3}}
\end{aligned}$$

5. $y = -18x - 12$ or $y - 24 = -18(x+2)$

6. a) $(3, 11)$ b) $v(7) = 48$

7. 0.455 12