Quiz 9 will cover the following topics: all topics from Quizzes 1-8, Exams 1, 2, 3, and implicit differentiation and integration by substitution.

Review Problems

1. Compute each of the following definite integrals.

a) $\displaystyle\int_{-5}^{5} (4x^3 - x)\ dx$

b) $\displaystyle\int_{1}^{3} \frac{1}{x^3}\ dx$

c) $\displaystyle\int_{1}^{5} (2^x - 3)\ dx$

d) $\displaystyle\int_{6}^{8} \frac{1}{x-5}\ dx$

e) $\displaystyle\int_{0}^{2} e^{-2x}\ dx$

f) $\displaystyle\int_{4}^{8} \frac{1}{(x+2)^2}\ dx$

g) $\displaystyle\int_{0}^{4} \sqrt{x^3}\ dx$

h) $\displaystyle\int_{0}^{100} e^{-x}\ dx$

i) $\displaystyle\int_{0}^{2} e^{-3x}\ dx$

j) $\displaystyle\int_{-5}^{5} \frac{1}{x} dx$

k) $\displaystyle\int_{0}^{\ln 8} e^x - e^{-x}\ dx$

l) $\displaystyle\int_{a}^{7a} \frac{1}{x}\ dx$

m) $\displaystyle\int_{0}^{2} \frac{2x^3 - x}{x^4 - x^2 + 1}\ dx$

n) $\displaystyle\int_{0}^{2} \frac{2x^3 - x}{(x^4 - x^2 + 1)^3}\ dx$

2. Compute each of the following indefinite integrals.

a) $\displaystyle\int x\left(x^2+7\right)^{10}\ dx$

b) $\displaystyle\int \sqrt{2x-1}\ dx$

c) $\displaystyle\int x\sqrt{2x-1}\ dx$

d) $\displaystyle\int \frac{x}{\sqrt{2x-1}}\ dx$

e) $\displaystyle\int \frac{e^x+1}{e^x+x}\ dx$

f) $\displaystyle\int e^{2x}\sqrt{e^{2x}-1}\ dx$

3. Find the slope of the tangent line drawn to the graph of $x^4 - y^4 = 2x^2y + 23$ to the point $(2, -1)$.

4. Find an equation for the tangent line drawn to the graph of $x^3 + y^3 - 5y^2 = 6x^2 + 13x - 42$ at the point $(-3, 5)$.

5. Find the equation of the tangent line drawn to the graph of $-3x^2 - 16xy - 2y^2 + 3y = 178$ at the point $(-3, 5)$.

6. Find an equation for all tangent lines drawn to the graph of $2x^2 + y^2 = 5y - x$ at $x = -2$.

7. Find an equation of all tangent lines drawn to the curve $x^2 - xy + y^2 = 16$ at $x = 0$.

8. Consider the relation determined by the equation $xy^2 - 5x = 2\left(y^2 + x^2y - 16\right)$. Find an equation for all tangent line(s) drawn to the graph of the relation at $x = 3$.

Answers

1.) a) 0 b) $\frac{4}{9}$ c) $\frac{30}{\ln 2} - 12$ d) $\ln 3$ e) $\frac{1}{2} - \frac{1}{2e^4}$ f) $\frac{1}{15}$ g) $\frac{64}{5}$ h) $1 - \frac{1}{e^{100}}$

i) $\frac{1}{3} - \frac{1}{3}e^{-6} \cong 0.332\,5$ j) undefined k) $\frac{49}{8}$ l) $\ln 7$ m) $\frac{1}{2}\ln 13$ n) $\frac{42}{169}$

2.) a) $\int x\left(x^2+7\right)^{10}\ dx = \frac{\left(x^2+7\right)^{11}}{22} + C$

Solution: Let $u = x^2 + 7$. Then $du = 2x dx$ and so $dx = \frac{du}{2x}$.

$$\int x\left(x^2+7\right)^{10}\ dx = \int \not{x} u^{10}\ \frac{du}{2\not{x}} = \frac{1}{2}\int u^{10}\ du = \frac{1}{2}\cdot\frac{u^{11}}{11} + C = \frac{\left(x^2+7\right)^{11}}{22} + C$$

b) $\int \sqrt{2x-1}\ dx = \frac{1}{3}(2x-1)^{\frac{3}{2}} + C$

Solution: Let $u = 2x - 1$. Then $du = 2dx$ and so $dx = \frac{du}{2}$.

$$\int \sqrt{2x-1}\ dx = \int u^{1/2}\ \frac{du}{2} = \frac{1}{2}\cdot\frac{u^{3/2}}{\frac{3}{2}} + C = \frac{1}{3}(2x-1)^{\frac{3}{2}} + C$$

c) $\int x\sqrt{2x-1}\ dx = \frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C$

Solution: Let $u = 2x - 1$. Then $du = 2dx$ and so $dx = \frac{du}{2}$. Also we solve for x in $u = 2x - 1$ and obtain $x = \frac{u+1}{2}$.

$$\begin{aligned}
\int x\sqrt{2x-1}\ dx &= \int \left(\frac{u+1}{2}\right)u^{1/2}\ \frac{du}{2} = \frac{1}{4}\int (u+1)\,u^{1/2}\ du = \frac{1}{4}\int u^{3/2} + u^{1/2}\ du \\
&= \frac{1}{4}\left(\frac{u^{5/2}}{\frac{5}{2}} + \frac{u^{3/2}}{\frac{3}{2}}\right) + C = \frac{1}{10}u^{5/2} + \frac{1}{6}u^{3/2} + C \\
&= \frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C
\end{aligned}$$

d) $\displaystyle\int \frac{x}{\sqrt{2x-1}}\,dx = \frac{1}{6}(2x-1)^{3/2} + \frac{1}{2}(2x-1)^{1/2} + C$

Solution: Let $u = 2x-1$. Then $du = 2dx$ and so $dx = \dfrac{du}{2}$. Also we solve for x in $u = 2x-1$ and obtain $x = \dfrac{u+1}{2}$.

$$\begin{aligned}\int \frac{x}{\sqrt{2x-1}}\,dx &= \int \frac{\frac{u+1}{2}}{\sqrt{u}}\,\frac{du}{2} = \frac{1}{4}\int \frac{u+1}{\sqrt{u}}\,du = \frac{1}{4}\int u^{1/2} + u^{-1/2}\,du \\ &= \frac{1}{4}\left(\frac{u^{3/2}}{\frac{3}{2}} + \frac{u^{1/2}}{\frac{1}{2}}\right) + C = \frac{1}{6}u^{3/2} + \frac{1}{2}u^{1/2} + C \\ &= \frac{1}{6}(2x-1)^{3/2} + \frac{1}{2}(2x-1)^{1/2} + C\end{aligned}$$

e) $\displaystyle\int \frac{e^x+1}{e^x+x}\,dx = \ln|e^x+x| + C$

Solution: Let $u = e^x + x$. Then $du = (e^x+1)\,dx$ and so $dx = \dfrac{du}{e^x+1}$

$$\int \frac{e^x+1}{e^x+x}\,dx = \int \frac{e^x+1}{u}\,\frac{du}{e^x+1} = \int \frac{1}{u}\,du = \ln|u| + C = \ln|e^x+x| + C$$

f) $\displaystyle\int e^{2x}\sqrt{e^{2x}-1}\,dx = \frac{1}{3}\left(e^{2x}-1\right)^{\frac{3}{2}} + C$

Solution: Let $u = e^{2x}-1$. Then $du = 2e^{2x}dx$ and so $dx = \dfrac{du}{2e^{2x}}$.

$$\int e^{2x}\sqrt{e^{2x}-1}\,dx = \int e^{2x}\sqrt{u}\,\frac{du}{2e^{2x}} = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\cdot\frac{u^{3/2}}{\frac{3}{2}} + C = \frac{1}{3}\left(e^{2x}-1\right)^{\frac{3}{2}} + C$$

3.) 10 4.) $-2(x+3) = y-5$ 5.) $y = 2x+11$ 6.) $y = -7x-12$ and $y = 7x+17$

7.) $y = \frac{1}{2}x + 4$ and $y = \frac{1}{2}x - 4$ 8.) $y = -5x+32$ and $y = -x+2$