Exam 1 will cover the following topics. These are: all topics covered by Quizzes 1, 2, 3 and 4, exponents and logarithms, limits at infinity, quadratic inequalities, and tangent lines. (Appendix A, Chapter 1, and Sections 4.1 and 4.2)

## Review Problems

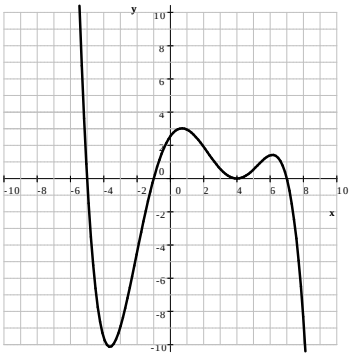
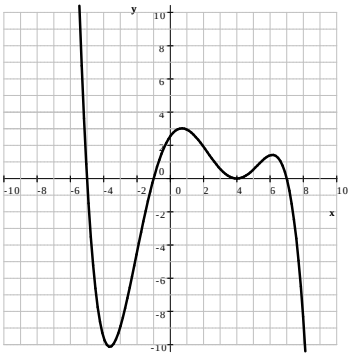
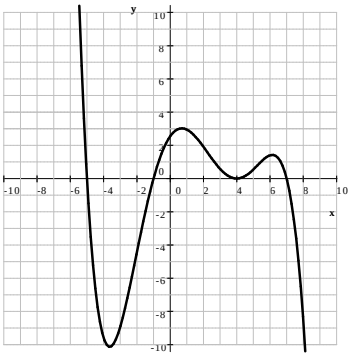
1. Solve each of the following inequalities.
   a) $4x + x^2 < 21$ b) $2x^2 + 2 \geq 5x$
2. Simplify each of the following.
   a) $6^{-2}$ d) $\log_3 3$ g) $\log_a a^{23}$ j) $\log_4\left(\frac{1}{16}\right)$ m) $\log_8\left(\frac{1}{16}\right)$
   b) $32^{3/5}$ e) $\log_3 1$ h) $\log_5 125$ k) $\log_9 \sqrt{27}$ n) $\log_{27}\left(\frac{1}{9}\right)$
   c) $64^{-2/3}$ f) $\log_3 0$ i) $\log_{125} 5$ l) $8^{\log_2 5}$
3. Solve each of the following equations. Make sure to check your solution.
   a) $x^3 = 24x^2 + 217x$ d) $3(x-5) - 5(x-1) = -2x + 1$
   b) $\frac{3-x}{4} - \frac{10-3x}{5} = x + 2$ e) $x^2 + 7 = 6x$
   c) $(x+4)(1-2x) = 3x - 2(x-3)^2$
4. Factor $3x^2 - 4x - 319$ by completing the square.
5. In case of each polynomial given, determine (by completing the square) whether it factors or not. (You do not have to actually factor.)
   a) $20x + 2x^2 + 44$ b) $20x - 5x^2 - 25$
6. Find each of the following limits.
   a) $\lim_{x\to-\infty}\left(-2x^5 + 8x^2\right)$ d) $\lim_{x\to\infty}\left(-2x^5 + 8x^6\right)$ g) $\lim_{x\to\infty}\frac{2x^2+3x+1}{3x^2-5x+2}$
   b) $\lim_{x\to\infty}\left(-2x^5 + 8x^2\right)$ e) $\lim_{x\to-\infty}\frac{3x^2-1}{5x^2-3x+2}$ h) $\lim_{x\to\infty}\frac{-x^3+2x+1}{x-3}$
   c) $\lim_{x\to-\infty}\left(-2x^5 + 8x^6\right)$ f) $\lim_{x\to-\infty}\log_2 x$ i) $\lim_{x\to-\infty}(2^x)$
7. Find an equation for the cubic polynomial function whose graph passes through the points $A(-2,-5)$, $B(0,-1)$, $C(2,-5)$, and $D(4,-65)$.
8. Find an equation for all tangent lines drawn to the graph of $f(x) = 7x - \frac{1}{2}x^2 - 20$ from the point $(4,8)$.
9. Graph the parabola $y = 6 - 2x^2 - 4x$. Clearly label the coordinates of five points on the parabola, including vertex and intercepts.
10. Graph each of the pair of functions given in the same coordinate system.
    a) $f(x) = 2^x$ and $g(x) = \log_2 x$ b) $f(x) = \log_3 x$ and $g(x) = \log_{(1/3)} x$
11. Graph $f(x) = ||x-1| - 2|$

12. Graph each of the following functions.

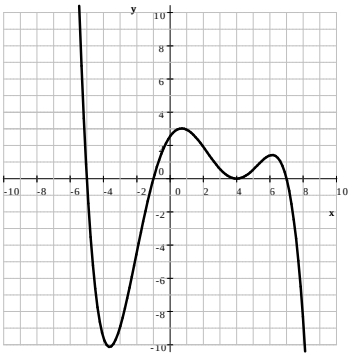
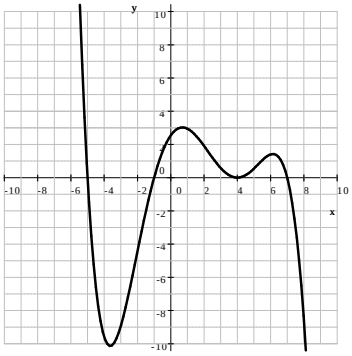
a) $f(x) = \dfrac{1}{x-3}$ b) $g(x) = \dfrac{1}{x^2}$ c) $h(x) = (x-1)^3 + 2$

13. The picture below shows the graph of a function, $f(x)$. Graph each of the following.

a) $g(x) = f(x-2)$ b) $g(x) = f(-x)$ c) $g(x) = -2f(x)$

d) $g(x) = \dfrac{f(x) + |f(x)|}{2}$ e) $g(x) = \dfrac{1}{f(x)}$

14. Graph each of the following

a) $f(x) = (x+4)(x+1)^2(x-3)$ c) $f(x) = \dfrac{(x-3)(x+1)}{(x+4)(x+1)}$

b) $f(x) = \dfrac{(x+4)(x+1)(x-3)}{(x+1)}$ d) $f(x) = \dfrac{1}{(x+4)(x+1)^2(x-3)}$

15. A citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. Find the total number of trees the grower should plant to maximize yield.

16. The cost of manufacturing $q$ units of a product is given by $C(q) = 6q^2 + 10q$. Suppose we can sell all $q$ units for a total of $142q + 1674$ dollars. Find the maximum profit we can achieve.

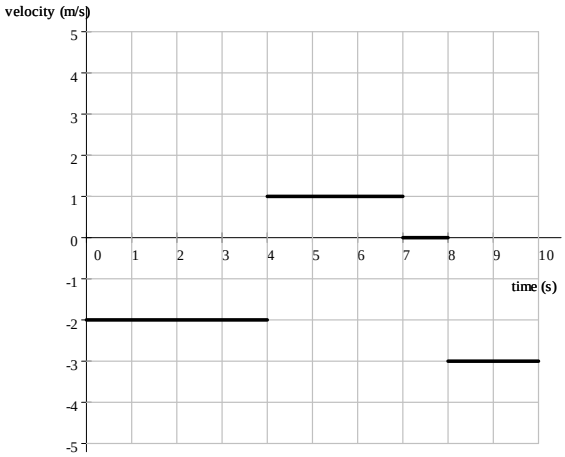
17. An object's height (measured in feet) is defined by $s(t) = t^3 - 12t$ where $t$ is the time, measured in seconds.

a) Find the location of the object at $t = 3$ seconds.

b) Find the average velocity of the object between

i) $t = 0$ and $t = 2$ s ii) $t = 1$ s and $t = 2$ s iii) $t = 1.5$ s and $t = 2$ s

18. The picture below shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward.).




a) Suppose that the object starts at a height of 5 m. Where is it at $t = 10$ seconds?.

b) Suppose that the object starts at a height of 9 m. Where is it at $t = 10$ seconds?

## Review Problems - Answers

1. a) $-7 < x < 3$ b) $x \le \frac{1}{2}$ or $x \ge 2$

2. a) $\frac{1}{36}$ b) 8 c) $\frac{1}{16}$ d) 1 e) 0 f) undefined g) 23 h) 3
   i) $\frac{1}{3}$ j) $-2$ k) $\frac{3}{4}$ l) 125 m) $-\frac{4}{3}$ n) $-\frac{2}{3}$

3. a) $-7, 0, 31$ b) $-5$ c) 1 d) no solution e) $3+\sqrt{2}$ and $3-\sqrt{2}$

4. $3\left(x+\frac{29}{3}\right)(x-11) = (3x+29)(x-11)$ For detailed solutions and practice, see the handout Completing the Square, Part 3. This problem is Example 3.

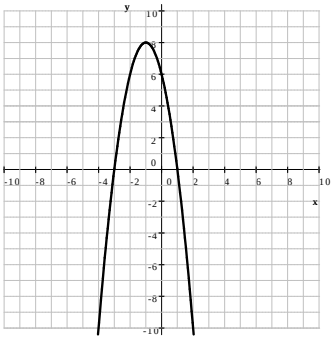
5. a) factors b) does not factor

6. a) $\infty$ b) $-\infty$ c) $\infty$ d) $\infty$ e) $\frac{3}{5}$ f) N/A, the function is not defined on negative numbers
   g) $\frac{2}{3}$ h) $-\infty$ i) 0

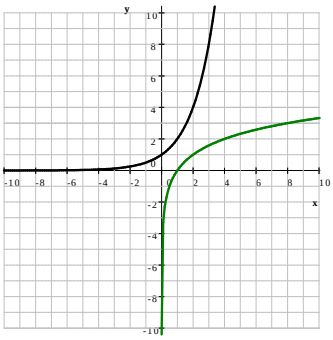
7. $y = -x^3 - x^2 + 4x - 1$

8. $y = -x + 12$ and $y = 7x - 20$ see handout Tangent Lines

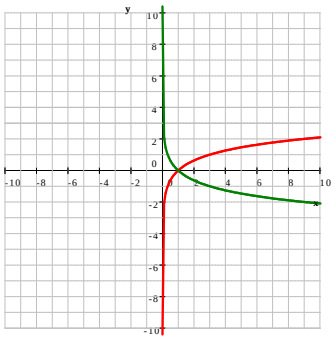
9. vertex: $(-1, 8)$ $x-$intercepts: $(-3, 0)$ and $(1, 0)$ $y-$intecept: $(0, 6)$



10. a) $f(x) = 2^x$ and $g(x) = \log_2 x$



b) $f(x) = \log_3 x$ and $g(x) = \log_{(1/3)} x$

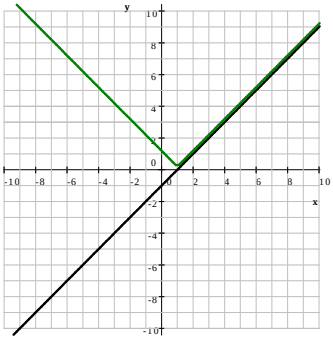


11. Graph $f(x) = ||x - 1| - 2|$

Solution:

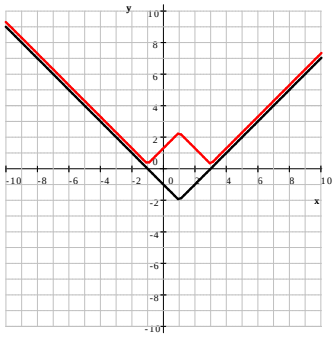
Step 1. Graph $y = x - 1$. (black graph)

Step 2. Graph now $y = |x - 1|$ (green graph)

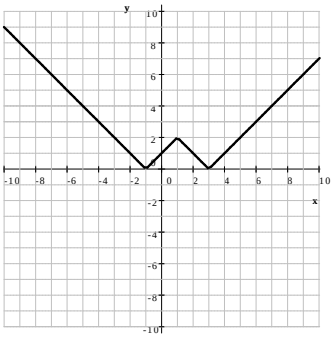


Step 3. We graph $y = |x - 1| - 2$ (black graph)

Step 4. We graph $y = ||x - 1| - 2|$ (red graph)

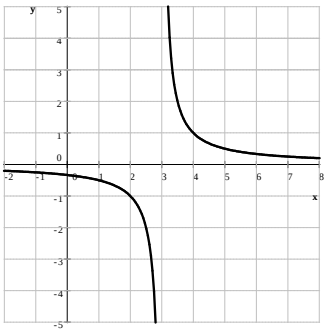


Thus the final graph is:



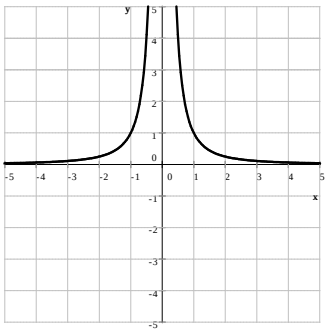
12. a) $f(x) = \dfrac{1}{x-3}$

the graph of $\dfrac{1}{x}$ is shifted to the right by 3 units



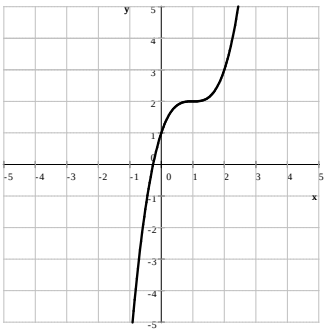
b) $g(x) = \dfrac{1}{x^2}$

take the reciprocal of the graph of $x^2$

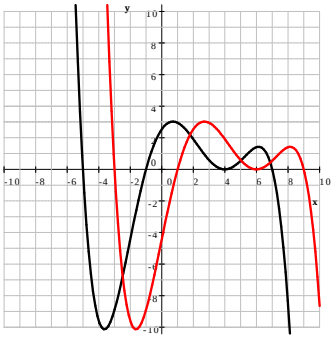


c) $h(x) = (x-1)^3 + 2$

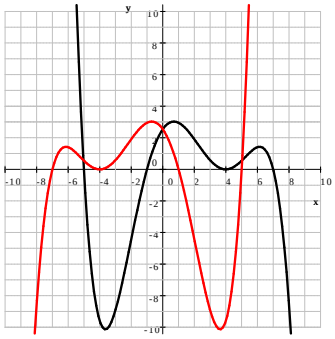
the graph of $x^3$ is shifted to the right by 1 unit and up by 2 units



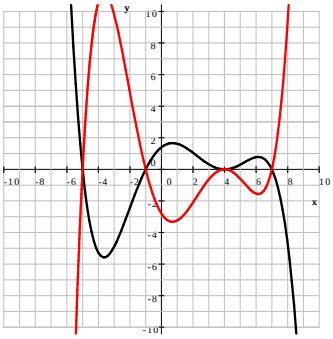
13. a) $g(x) = f(x-2)$



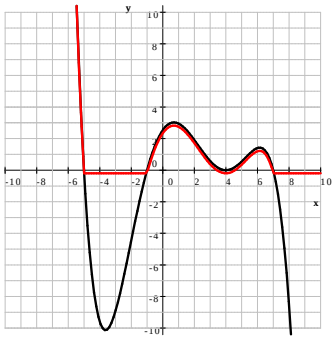
b) $g(x) = f(-x)$



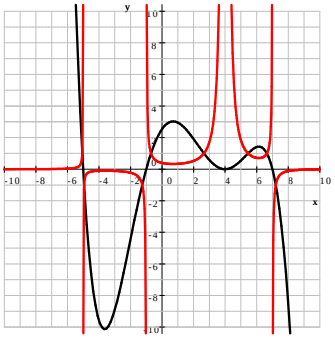
c) $g(x) = -2f(x)$



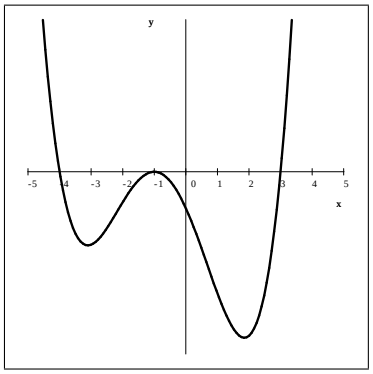
d) $g(x) = \dfrac{f(x) + |f(x)|}{2}$

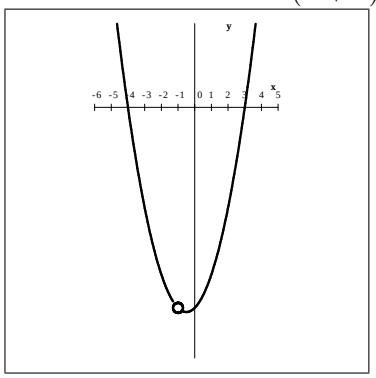


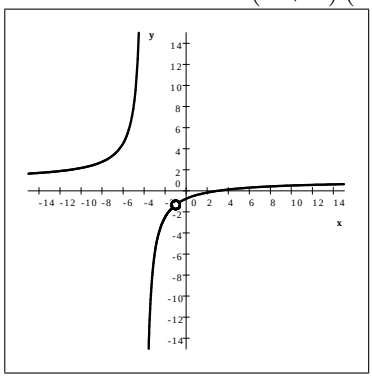
e) $g(x) = \dfrac{1}{f(x)}$



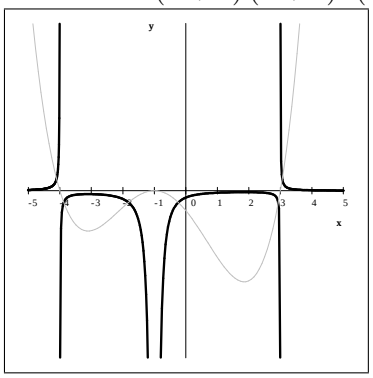
14. a) $f(x) = (x+4)(x+1)^2(x-3)$ b) $f(x) = \dfrac{(x+4)(x+1)(x-3)}{(x+1)}$ c) $f(x) = \dfrac{(x-3)(x+1)}{(x+4)(x+1)}$




d) $f(x) = \dfrac{1}{(x+4)(x+1)^2(x-3)}$




15. 80 trees (for 25600 oranges) See Optimization 1

16. \$2400. See Optimization 1

17. a) $-9\,\text{ft}$ b) i) $-8\dfrac{\text{ft}}{\text{s}}$ ii) $-5\dfrac{\text{ft}}{\text{s}}$ iii) $-2.75\dfrac{\text{ft}}{\text{s}}$

18. a) $-6\,\text{m}$ b) $-2\,\text{m}$