1. Let $a = \log_2 6$ and $b = \log_2 10$. Express each of the following in terms of a and b.

   a) $\log_2 3$ b) $\log_2 15$ c) $\log_5 3$

2. Solve the equation $\log_3(2x-1) + \log_3(x-5) = 5$.

3. Find each of the following limits.

   a) $\displaystyle\lim_{x \to 1+} \frac{x^2-1}{x^2-3x+2}$ b) $\displaystyle\lim_{x\to\infty} \left(\log_2\left(40x^3+1\right) - \log_2\left(5x^3+7x-21\right)\right)$ c) $\displaystyle\lim_{x\to\infty} \frac{-x^3+2x+1}{x-3}$

4. Sketch the graph of $f(x) = \dfrac{-6(x+2)^3(4x)^2(x-5)}{3(-2-x)^5 x^2 (10-2x)^4}$.

5. Find the derivative for each of the following functions.

   a) $f(x) = 2x^2 \ln x - x^2$

   b) $f(x) = \dfrac{e^{4x^5-7x}}{e^{x^5}}$

   c) $f(x) = \dfrac{5x^3-1}{x^3}$

   d) $f(x) = -2\ln\left(4e^{-3x}\right)$

   e) $f(x) = 7^x - x^7 + \sqrt{7x} - e^7$

   f) $f(x) = \dfrac{1}{4}xe^{4x} - \dfrac{1}{16}e^{4x}$

   g) $f(x) = \dfrac{8x^2-20x+1}{4x-10}$

6. Find the equation of the tangent line drawn to the graph of $f(x) = x^3 - 7x + 6$ at $x = -1$.

7. Find the equation of the tangent line drawn to the graph of $-3x^2 - 16xy - 2y^2 + 3y = 178$ at the point $(-3, 5)$.

8. Consider the relation determined by the equation $xy^2 - 5x = 2\left(y^2 + x^2y - 16\right)$. Find an equation for all tangent line(s) drawn to the graph of the relation at $x = 3$.

9. Let $f(x) = 3e^{2x}$.

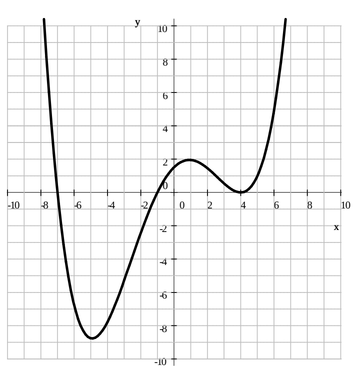
   a) Compute $\displaystyle\int f(x)\,dx$. b) Find the average value of f on the interval $[-2, 2]$.

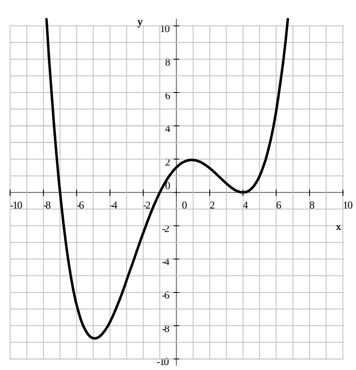
   c) Find the equation for $P(x)$ if P is a polynomial function satisfying the following conditions. $P(x)$ is of degree three, and $P(0) = f(0)$, $P'(0) = f'(0)$, $P''(0) = f''(0)$, and $P'''(0) = f'''(0)$.

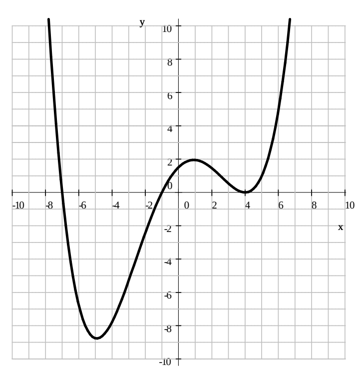
10. We know the following things about a function f. $f'(x) = 20x^3 - 3$ and $f(-1) = 16$. Find f.

11. The figures below all show the graph of $f(x)$. Sketch the graph of $g(x)$ into the same coordinate system if

    a) $g(x) = \dfrac{1}{f(x)}$ b) $g(x) = f'(x)$ c) $g(x)$ where $g'(x) = f(x)$

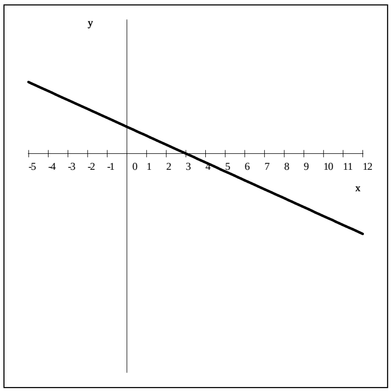




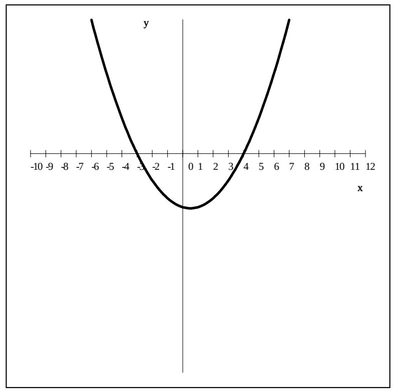


12. The graph of f is given on each of the graphs shown below. Plot the graph of $g(x)$ if $g'(x) = f(x)$ and

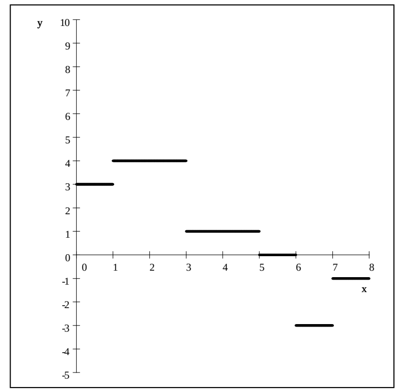
a) $g(0) = 4$



b) $g(0) = -6$



c) $g(0) = -5$



13. Evaluate each of the following integrals.

a) $\displaystyle\int \frac{1}{3x-5}\,dx$ c) $\displaystyle\int x\sqrt{x-3}\,dx$ e) $\displaystyle\int_0^5 e^{-2x}\,dx$

b) $\displaystyle\int_0^1 \frac{6x^2-4x+5}{x+1}\,dx$ d) $\displaystyle\int 5^x\,dx$ f) $\displaystyle\int_0^3 xe^{-4x}\,dx$

14. Compute each of the following improper integrals.

a) $\displaystyle\int_0^\infty 2e^{-2x}\,dx$ b) $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$

15. A manufacturer estimates that when q units of a particular product are produced each month, the total cost will be $C(q) = 5q + 17\,000$ dollars, and all q units can be sold at a price of $p(q) = 65 - \dfrac{q}{100}$ dollars per unit.

a) Find the fixed cost.

b) Determine the level of production that results in a maximum profit.

c) What is the maximum profit?

16. It costs 12 dollars each to manufacture and distribute a certain product. If we price it at x dollars each, the number sold is approximated by $n(x) = \dfrac{60}{x-12} + \dfrac{1}{5}(100-x)$. What selling price will bring in the maximum profit? How much money is that?

17. Oil is leaking from a container at the rate of $R(t) = 2000e^{-0.2t}$ gallons per hour, where t is time, measured in hours. How much oil has leaked out of the container after 10 hours? Round your answer to the nearest gallon.

18. A virus is spreading through a population in a manner that can be modeled by the function $g(t) = \dfrac{A}{1+Be^{-t}}$ where A is the total population, $g(t)$ is the number infected at time t, and B is a constant. What proportion of the population is infected when the virus is spreading the fastest?

19. In a certain town, the rate of deaths at time t due to a particular disease is modeled by $\dfrac{10000}{(t+3)^3}$ where $t \geq 0$. What is the total number of deaths from this desease predicted by the model?

20. A company has \$120 000 to spend on the development and promotion of a new product. The company estimates that if x is spent on the development and y is spent on promotion, then approximately $\dfrac{x^{1/2}y^{3/2}}{400\,000}$ items of new product will be sold. Based on this estimate, what is the maximum number of products that the company can sell?

21. A company can sell 20 products if it charges \$40 per product. For each dollar decrease or increase in the price, the company can sell one more or one less product, respectively. The total cost of producing q products is $C(q) = 32q + 100$. What is the maximum profit that the company can achieve from manufacturing and selling this product?

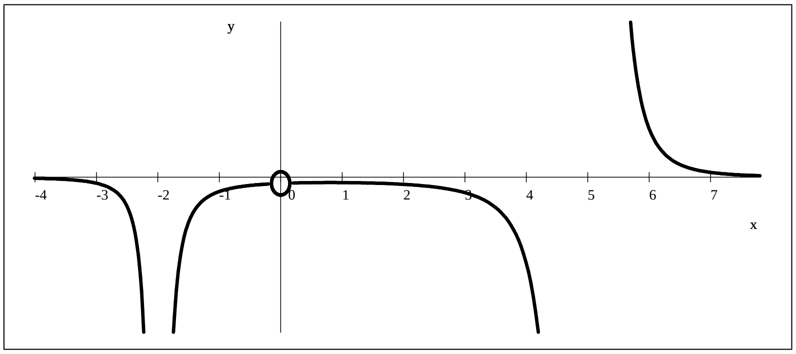
Answers

1. a) $a - 1$ b) $a + b - 2$ c) $\dfrac{a-1}{b-1}$

2. 14

3. a) -2 b) 3 c) $-\infty$

4. $f(x) = \dfrac{-6(x+2)^3(4x)^2(x-5)}{3(-2-x)^5 x^2 (10-2x)^4}$ (The hole at $x = 0$ is not at $y = 0$, it is at $y = -\dfrac{1}{250}$)



5. a) $f'(x) = 4x \ln x$ b) $f'(x) = \left(15x^4 - 7\right) e^{3x^5 - 7x}$ c) $f'(x) = \dfrac{3}{x^4}$ d) $f'(x) = 6$

   e) $f'(x) = 7^x \ln 7 - 7x^6 + \dfrac{\sqrt{7}}{2\sqrt{x}}$ f) $f'(x) = xe^{4x}$ g) $f'(x) = \dfrac{8x^2 - 40x + 49}{4x^2 - 20x + 25} = 2 - \dfrac{1}{(2x-5)^2}$

6. $y = -4x + 8$

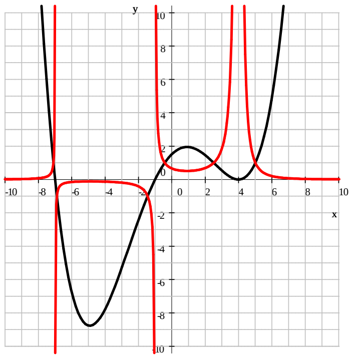
7. $y = 2x + 11$

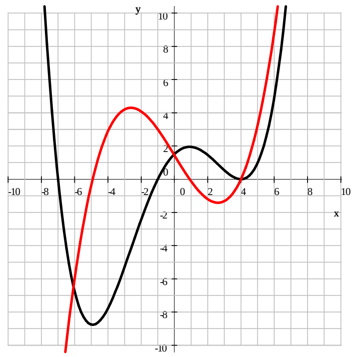
8. $y = -5x + 32$ and $y = -x + 2$

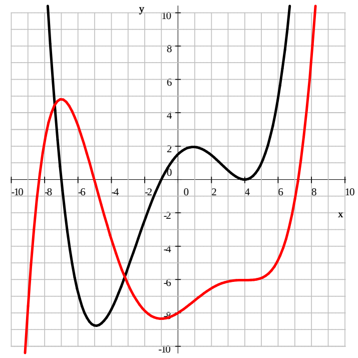
9. a) $\dfrac{3}{2}e^{2x} + C$ b) $\dfrac{3}{8}\left(e^4 - e^{-4}\right) \cong 20.467\,438$ c) $4x^3 + 6x^2 + 6x + 3$

10. $f(x) = 5x^4 - 3x + 8$

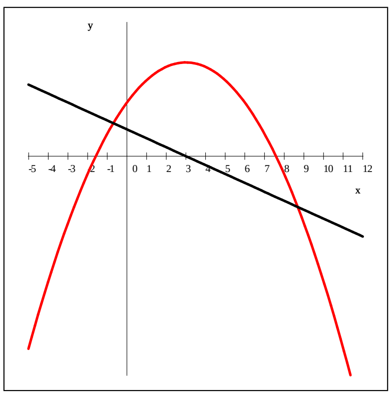
11. a) $g(x) = \dfrac{1}{f(x)}$ b) $g(x) = f'(x)$ c) $g(x)$ where $g'(x) = f(x)$

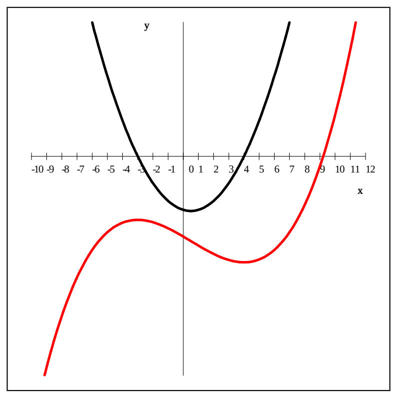


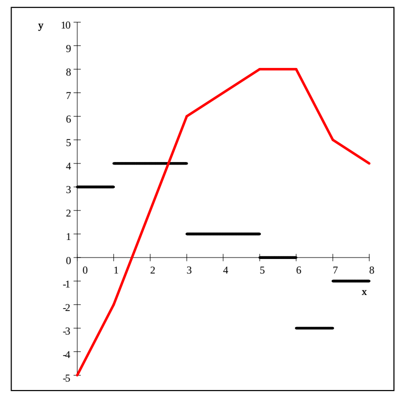




12. a) b) c)







13. a) $\dfrac{1}{3}\ln|3x-5| + C$ b) $15\ln 2 - 7$ c) $\dfrac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + C$

d) $\dfrac{5^x}{\ln 5} + C$ e) $\dfrac{1}{2} - \dfrac{1}{2}e^{-10}$ f) $\dfrac{1}{16} - \dfrac{13}{16}e^{-12}$

14. a) 1 b) $\dfrac{1}{2}$

15. a) \$17 000 b) 3000 c) \$73 000

16. \$440 with a price of \$56

17. 8647 gallons

18. $\dfrac{1}{2}$

19. 556

20. 11 691

21. \$96

Solutions

7.) Find the equation of the tangent line drawn to the graph of $-3x^2 - 16xy - 2y^2 + 3y = 178$ at the point $(-3, 5)$.
Solution: We start with implicit differentiation. We first differentiate both sides: Then we solve for y'.

$$\begin{aligned}
-3x^2 - 16xy - 2y^2 + 3y &= 178 \\
-6x - 16y - 16xy' - 4yy' + 3y' &= 0 \\
-16xy' - 4yy' + 3y' &= 6x + 16y \\
y'(-16x - 4y + 3) &= 6x + 16y \\
y' &= \frac{6x + 16y}{-16x - 4y + 3} \qquad \text{compute } y' \text{ when } x = -3 \text{ and } y = 5 \\
y' &= \frac{6(-3) + 16(5)}{-16(-3) - 4(5) + 3} = 2
\end{aligned}$$

The line must pass through $(-3, 5)$ and have slope 2.

$$\begin{aligned}
y - 5 &= 2(x + 3) \\
y &= 2x + 6 + 5 \\
y &= 2x + 11
\end{aligned}$$

Thus the answer is $y = 2x + 11$.

8.) Consider the relation determined by the equation $xy^2 - 5x = 2\left(y^2 + x^2y - 16\right)$. Find an equation for all tangent line(s) drawn to the graph of the relation at $x = 3$.
Solution: We substitute $x = 3$ into the equation and solve for y.

$$\begin{aligned}
3y^2 - 15 &= 2\left(y^2 + 9y - 16\right) \\
3y^2 - 15 &= 2y^2 + 18y - 32 \\
y^2 - 18y + 17 &= 0 \\
(y - 17)(y - 1) &= 0
\end{aligned}$$

$$y_1 = 17 \qquad y_2 = 1$$

Thus there are two points with tangent lines: $(3, 17)$ and $(3, 1)$.
For the slope of each tangent lines, we differentiate both sides and solve for y'.

$$\begin{aligned}
xy^2 - 5x &= 2\left(y^2 + x^2y - 16\right) \\
y^2 + x\left(2yy'\right) - 5 &= 2\left(2yy' + 2xy + x^2y'\right) \\
y^2 + 2xyy' - 5 &= 4yy' + 4xy + 2x^2y' \\
y^2 - 4xy - 5 &= 4yy' + 2x^2y' - 2xyy' \\
y^2 - 4xy - 5 &= y'\left(4y + 2x^2 - 2xy\right) \\
\frac{y^2 - 4xy - 5}{4y + 2x^2 - 2xy} &= y'
\end{aligned}$$

The slope of the tangent line drawn to $(3, 17)$

$$m_1 = \frac{y^2 - 4xy - 5}{4y + 2x^2 - 2xy} = \frac{17^2 - 4(3)(17) - 5}{4(17) + 2(3)^2 - 2(3)(17)} = \frac{80}{-16} = -5$$

We can easily find the equation of the line with slope -5, passing through $(3,17)$: it is $y=-5x+32$.
The other tangent line, passing through $(3,1)$ has slope

$$m_2 = \frac{y^2-4xy-5}{4y+2x^2-2xy} = \frac{1^2-4(3)(1)-5}{4(1)+2(3)^2-2(3)(1)} = \frac{-16}{16} = -1$$

Thus the slope is -1 and the equation of this line is $y=-x+2$.

9. b) Let $f(x)=3e^{2x}$. Find the average value of f on the interval $[-2,2]$.
Solution:

$$\mathrm{Av} = \frac{\int_{-2}^{2} f(x)\,dx}{2-(-2)} = \frac{1}{4}\int_{-2}^{2} f(x)\,dx = \frac{1}{4}\cdot\frac{3}{2}e^{2x}\Big|_{-2}^{2} = \frac{3}{8}\left(e^4-e^{-4}\right) \cong 20.467\,438$$

c) Find the equation for $P(x)$ if P is a polynomial function satisfying the following conditions. $P(x)$ is of degree three, and $P(0)=f(0)$, $P'(0)=f'(0)$, $P''(0)=f''(0)$, and $P'''(0)=f'''(0)$.
Solution: We first find the first three order derivatives of f.

$$f(x)=3e^{2x} \qquad f'(x)=6e^{2x} \qquad f''(x)=12e^{2x} \qquad f'''(x)=24e^{2x}$$

We now evaluate these at $x=0$.

$$f(0)=3 \qquad f'(0)=6 \qquad f''(0)=12 \qquad f'''(0)=24$$

Now we want to find the formula for the third degree polynomial with

$$P(0)=3, \quad P'(0)=6, \quad P''(0)=12, \quad \text{and} \quad P'''(0)=24$$

We write $P(x)=a_3x^3+a_2x^2+a_1x+a_0$. Then clearly

$$\begin{aligned} P(x) &= a_3x^3+a_2x^2+a_1x+a_0 & P(0)&=a_0 \\ P'(x) &= 3a_3x^2+2a_2x+a_1 & P'(0)&=a_1 \\ P''(x) &= 6a_3x+2a_2 & P''(0)&=2a_2 \\ P'''(x) &= 6a_3 & P'''(0)&=6a_3 \end{aligned}$$

We solve the system

$$\begin{aligned} P(0) &= a_0=3 \\ P'(0) &= a_1=6 \\ P''(0) &= 2a_2=12 \\ P'''(0) &= 6a_3=24 \end{aligned}$$

and obtain the values $a_0=3$, $a_1=6$, $a_2=6$, and $a_3=4$. Thus $P(x)=4x^3+6x^2+6x+3$.

13.) a) $\displaystyle\int \frac{1}{3x-5}dx$

Solution: Let us substitute $u=3x-5$. Then $du=3dx$ and so $dx=\dfrac{du}{3}$

$$\int \frac{1}{3x-5}dx = \int \frac{1}{u}\frac{du}{3} = \frac{1}{3}\int \frac{1}{u}du = \frac{1}{3}\ln|u|+C = \frac{1}{3}\ln|3x-5|+C$$

b) $\displaystyle\int_0^1 \frac{6x^2-4x+5}{x+1}dx$

Solution: We will transform the function into a form that is easier to integrate.

Long division $\left(x^2-4x+5\right) \div (x+1) = 6x - 10 + \dfrac{15}{x+1}$

The indefinite integral is then

$$\int \frac{6x^2-4x+5}{x+1}dx = \int 6x - 10 + \frac{15}{x+1}dx = 3x^2 - 10x + 15\ln|x+1| + C$$

And so the definite integral is

$$\begin{aligned}\int_0^1 \frac{6x^2-4x+5}{x+1}dx &= 3x^2 - 10x + 15\ln|x+1|\Big|_0^1 \\ &= \left(3(1)^2 - 10(1) + 15\ln|1+1|\right) - \left(3(0)^2 - 10(0) + 15\ln|0+1|\right) \\ &= (-7 + 15\ln 2) - 0 = 15\ln 2 - 7\end{aligned}$$

c) $\displaystyle\int x\sqrt{x-3}dx$

Solution: We will substitute $u = x - 3$. Then of course $du = dx$, and also $u + 3 = x$.

$$\begin{aligned}u &= x - 3 \\ du &= dx\end{aligned}$$

$$\begin{aligned}\int x\sqrt{x-3}dx &= \int (u+3)\sqrt{u}du = \int \left(u^{3/2} + 3u^{1/2}\right)du = \int u^{3/2}du + 3\int u^{1/2}du \\ &= \frac{2}{5}u^{5/2} + 3\left(\frac{2}{3}\right)u^{3/2} + C = \frac{2}{5}u^{5/2} + 2u^{3/2} + C \\ &= \frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + C\end{aligned}$$

e) $\displaystyle\int_0^5 e^{-2x}dx$

Solution: We first use substitution to find the indefinite integral. Let $u = -2x$ and then $du = -2dx$. We solve for $dx = -\dfrac{du}{2}$

$$\int e^{-2x}dx = \int e^u\left(-\frac{du}{2}\right) = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-2x} + C$$

Then the definite integral is

$$\begin{aligned}\int_0^5 e^{-2x}dx &= -\frac{1}{2}e^{-2x}\Big|_0^5 = -\frac{1}{2}e^{-2x}\Big|_0^5 = -\frac{1}{2}\frac{1}{e^{2x}}\Big|_0^5 = -\frac{1}{2}\left(\frac{1}{e^{2\cdot 5}} - \frac{1}{e^{2\cdot 0}}\right) \\ &= -\frac{1}{2}\left(\frac{1}{e^{10}} - \frac{1}{e^0}\right) = -\frac{1}{2}\left(\frac{1}{e^{10}} - 1\right) = -\frac{1}{2}e^{-10} + \frac{1}{2}\end{aligned}$$

f) $\displaystyle\int_0^3 xe^{-4x}dx = \frac{1}{16} - \frac{13}{16}e^{-12}$

Solution: Let us compute first the indefinite integral, $\int xe^{-4x}dx$. Let $g(x) = x$ and $f'(x) = e^{-4x}$ Then we obtain g' and f by differentiation and integration. To compute $f(x)$, we will use substitution. Let $u = -4x$ then $du = -4dx$ and so $dx = \dfrac{du}{-4}$.

$$f(x) = \int e^{-4x}\,dx = \int e^{u}\,\frac{du}{-4} = -\frac{1}{4}\int e^{u}\,du = -\frac{1}{4}e^{u} + C = -\frac{1}{4}e^{-4x} + C$$

We will choose $C = 0$ and so $f(x) = -\dfrac{1}{4}e^{-4x}$.

$f(x) = -\frac{1}{4}e^{-4x}$	$g(x) = x$
$f'(x) = e^{-4x}$	$g'(x) = 1$

$$\int f'g = fg - \int fg' \quad \text{becomes}$$

$$\int xe^{-4x}\,dx = -\frac{1}{4}xe^{-4x} - \int -\frac{1}{4}e^{-4x}\,dx = -\frac{1}{4}xe^{-4x} + \frac{1}{4}\int e^{-4x}\,dx = -\frac{1}{4}xe^{-4x} + \frac{1}{4}\left(-\frac{1}{4}e^{-4x}\right) + C$$

$$= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x} + C$$

We check our result by differentiating the answer.

$$\left(-\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x} + C\right)' =$$

$$= -\frac{1}{4}\left(xe^{-4x}\right)' - \frac{1}{16}\left(e^{-4x}\right)' = -\frac{1}{4}\left(e^{-4x} + x\left(-4e^{-4x}\right)\right) - \frac{1}{16}\left(-4e^{-4x}\right)$$

$$= -\frac{1}{4}e^{-4x} + xe^{-4x} + \frac{1}{4}e^{-4x} = xe^{-4x}$$

We can now compute the definite integral

$$\int_0^3 xe^{-4x}dx = -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}\Big|_0^3 = \left(-\frac{1}{4}(3)\,e^{-4(3)} - \frac{1}{16}e^{-4(3)}\right) - \left(-\frac{1}{4}(0)\,e^{-4(0)} - \frac{1}{16}e^{-4(0)}\right)$$

$$= \left(-\frac{3}{4}e^{-12} - \frac{1}{16}e^{-12}\right) - \left(-\frac{1}{16}\right) = e^{-12}\left(-\frac{3}{4} - \frac{1}{16}\right) + \frac{1}{16} = \frac{1}{16} - \frac{13}{16}e^{-12}$$

14.) a) $\displaystyle\int_0^\infty 2e^{-2x}dx$

Solution: We first use substitution to find the indefinite integral. (See previous problem.) Clearly,

$$\int 2e^{-2x}dx = -e^{-2x} + C$$

Then the improper integral becomes the following limit.

$$\int_0^\infty 2e^{-2x}dx = \lim_{N\to\infty}\int_0^N 2e^{-2x}dx = \lim_{N\to\infty}\left[-e^{-2x}\right]_0^N = \lim_{N\to\infty}\left(1 - \frac{1}{e^{2N}}\right) = 1$$

b) $\displaystyle\int_1^\infty \frac{1}{x^3}dx$

Solution:

$$\begin{aligned}\int_1^\infty \frac{1}{x^3}dx &= \lim_{N\to\infty}\int_1^N x^{-3}dx = \lim_{N\to\infty}\left.\frac{x^{-2}}{-2}\right|_1^N = -\frac{1}{2}\lim_{N\to\infty}\left.x^{-2}\right|_1^N = -\frac{1}{2}\lim_{N\to\infty}\left.\frac{1}{x^2}\right|_1^N \\ &= -\frac{1}{2}\lim_{N\to\infty}\left(\frac{1}{N^2}-\frac{1}{1^2}\right) = \frac{1}{2}\lim_{N\to\infty}\left(1-\frac{1}{N^2}\right) = \frac{1}{2}\end{aligned}$$

16.) Let $P(x)$ denote the profit as a functon of f. We need to find the maximum for P.

$$\begin{aligned}\text{Profit} &= \text{Revenue} - \text{Cost} \\ P(x) &= n(x)\cdot x - n(x)\cdot 12 = n(x)\cdot(x-12) \\ &= \left(\frac{60}{x-12}+\frac{1}{5}(100-x)\right)(x-12) \\ &= 60+\frac{1}{5}(100-x)(x-12)\end{aligned}$$

This is an upside down parabola, thus its vertex indeed is a maximum. The x-intercepts of P are at $x_1 = 100$ and $x_2 = 12$. The vertex is at their average, $\frac{100+12}{2} = 56$. This means that the maximal profit occurs if we set the price at \$56. Then we can sell

$$n(56) = \frac{60}{56-12}+\frac{1}{5}(100-56) = 10.1636 \qquad \text{we can sell 10 items}$$

$$\text{Profit} = \text{Income} - \text{Cost} = 10(56) - 10(12) = 440$$

Thus the maximal profit, \$440 occurs when the price is \$56. Notice that $P(56) = 60+\frac{1}{5}(100-56)(56-12) =$ \$447.20 would be the wrong answer because it computes the profit in terms of a fractional number of items sold.

17.) Compute $\displaystyle\int_0^{10} 2000e^{-0.2t}dt = 10\,000\left(1-e^{-2}\right) = 8646.647\,17$ and then round to 8647

18.) "Spreading the fastest" means greatest velocity, thus we need to find the zeroes of $g''(t)$.

$$\begin{aligned}g(t) &= A\left(1+Be^{-t}\right)^{-1} \\ g'(t) &= A(-1)\left(1+Be^{-t}\right)^{-2}\left(Be^{-t}\right)(-1) = \frac{ABe^{-t}}{\left(1+Be^{-t}\right)^2} \\ g''(t) &= AB\frac{-e^{-t}\left(1+Be^{-t}\right)^2 - e^{-t}(2)\left(1+Be^{-t}\right)\left(-Be^{-t}\right)}{\left(1+Be^{-t}\right)^4} = ABe^{-t}\frac{-\left(1+Be^{-t}\right)+2Be^{-t}}{\left(1+Be^{-t}\right)^3} \\ &= ABe^{-t}\frac{-1-Be^{-t}+2Be^{-t}}{\left(1+Be^{-t}\right)^3} = ABe^{-t}\frac{Be^{-t}-1}{\left(1+Be^{-t}\right)^3}\end{aligned}$$

$$\begin{aligned}g''(t) &= 0 \\ ABe^{-t}\frac{Be^{-t}-1}{\left(1+Be^{-t}\right)^3} &= 0\end{aligned}$$

This equation is not so difficult once we realize that A, B, e^{-t} , and $\left(1+Be^{-t}\right)^3$ are all non-zero. Thus we really only have

$$Be^{-t}-1=0 \quad \Longrightarrow \quad Be^{-t}=1$$

Now we just substitute this into the formula of $g(t)=\dfrac{A}{1+Be^{-t}}$

$$g(t)=\frac{A}{1+Be^{-t}}=g(t)=\frac{A}{1+1}=\frac{A}{2}$$

This means that half of the population is infected when the virus spreads the fastest.

19.) Compute the improper integral $\displaystyle\int_0^\infty \frac{10\,000}{(t+3)^3}dt$

$$\begin{aligned}\int_0^\infty \frac{10\,000}{(t+3)^3}dt &= \lim_{N\to\infty} 10\,000\int_0^N \frac{1}{(t+3)^3}dt = 10\,000\lim_{N\to\infty}\left(\left.\frac{-1}{2(t+3)^2}\right|_0^N\right)= \\ &= 10\,000\lim_{N\to\infty}\left(\frac{-1}{2(N+3)^2}-\frac{-1}{2(0+3)^2}\right)=10\,000\left(\frac{1}{18}\right)=\frac{5000}{9}\simeq 555.\overline{5}\end{aligned}$$

and round to 556.

20.) $x+y=120\,000 \quad \Longrightarrow y=120\,000-x$

$N(x)=\dfrac{x^{1/2}y^{3/2}}{400\,000}=\dfrac{x^{1/2}(120\,000-x)^{3/2}}{400\,000}$. We now find the zeroes of $N'(x)$.

$$\begin{aligned}N'(x) &= \frac{1}{400\,000}\left(x^{1/2}(120\,000-x)^{3/2}\right)' \\ &= \frac{1}{400\,000}\left(\frac{1}{2}x^{-1/2}(120\,000-x)^{3/2}+x^{1/2}\left(\frac{3}{2}\right)(120\,000-x)^{1/2}(-1)\right) \\ &= \frac{1}{400\,000}\left(\frac{(120\,000-x)^{3/2}}{2x^{1/2}}-\frac{3}{2}x^{1/2}(120\,000-x)^{1/2}\right)\end{aligned}$$

We solve for the zeroes:

$$\begin{aligned}\frac{1}{400\,000}\left(\frac{(120\,000-x)^{3/2}}{2x^{1/2}}-\frac{3}{2}x^{1/2}(120\,000-x)^{1/2}\right) &= 0 \quad \text{multiply by } 800\,000 \\ \frac{(120\,000-x)^{3/2}}{x^{1/2}}-3x^{1/2}(120\,000-x)^{1/2} &= 0 \quad \text{add } 3x^{1/2}(120\,000-x)^{1/2} \\ \frac{(120\,000-x)^{3/2}}{x^{1/2}} &= 3x^{1/2}(120\,000-x)^{1/2} \quad \text{multiply by } x^{1/2} \\ (120\,000-x)^{3/2} &= 3x(120\,000-x)^{1/2} \quad \text{divide by } (120\,000-x)^{1/2} \\ 120\,000-x &= 3x \quad \text{add } x \\ 120\,000 &= 4x \quad \text{divide by } 4 \\ 30\,000 &= x\end{aligned}$$

We substitute $x=30\,000$ into $N(x)$.

$$N(30\,000)=\frac{30\,000^{1/2}90\,000^{3/2}}{400\,000}\simeq 11691.342\,951\,1$$

21.) Let x denote the raise from the price \$40. Then we can sell $q = 20 - x$ many products.

$$\begin{aligned} \text{Profit} &= \text{Revenue} - \text{cost} \\ P(x) &= (40 + x)(20 - x) - (32(20 - x) + 100) = -x^2 + 12x + 60 \end{aligned}$$

This upside down parabola has a maximum: $(6, 96)$