Math 204 Extra Credit Assignment 1

1. (still pending) We proved in the first class that terminating and repeating decimals always represent rational numbers. Prove the converse: that every rational number, when written as a decimal, is either terminating or repeating.

2. (still pending) Suppose that towns A and B are s distance away from each other. If we travel from A to B with an average speed of v_1 and from B to A with an average speed of v_2, compute the average speed for the entire roundtrip in terms of s, v_1 and v_2.

3. The **Fibonacci sequence**, (F_n) is $1, 1, 2, 3, 5, 8, 13, 21, ...$ Every element is obtained by adding the previous two elements. The recursive definition is

$$\begin{aligned} F_1 &= 1 \\ F_2 &= 1 \\ F_{n+2} &= F_n + F_{n+1} \quad \text{for all natural number } n, \text{ with } n \geq 1 \end{aligned}$$

Consider now another sequence, (q_n) that is formed by taking the ratios of consecutive elements of the Fibonacci sequence. That is, $\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, ...$

$$q_n = \frac{F_n}{F_{n+1}} \quad \text{for all natural number } n.$$

When we compute the decimal presentation of the first few elements of (q_n), we notice some interesting behavior. The elements of q_n appear to appraoch a fixed number. Find the exact value of this number! In short, find the exact value of

$$\lim_{n \to \infty} q_n$$

4. Let f be any function given. Find a formula for a new function, g, in terms of f, with the following properties:

$$g(x) = \begin{cases} f(x) & \text{if } f(x) \leq 0 \\ -f(x) & \text{if } f(x) > 0 \end{cases}$$