Quiz 3 will cover the following topics: all topics from Quiz 1 and Quiz 2, Functions and their graphs (1.1, 1.2), Linear Functions (1.3, handout) and Optimization 1 (1.4, handout)

Review Problems

1. We are standing on the top of a 720 ft tall building and throw a small object upward. The object's distance, measured in feet, after t seconds is
$$h(t) = -16t^2 + 192t + 720$$
   a) How high is the object after 3 seconds? (In short, find $h(3)$.)
   b) How high is the object after 5 seconds? (In short, find $h(5)$.)
   c) Compute the average velocity of the object between $t = 3$ seconds and $t = 5$ seconds.
   d) Compute the average velocity of the object between $t = 8$ seconds and $t = 10$ seconds.
   e) How long does it take for the object to hit the ground?
   f) What is the highest point that the object reaches?

2. Among the rectangles of perimeter 12 m, which one has the largest area?

3. If we set the price of our product to be \$18 per item, then we can sell 300 items. For every dollar we raise the price, we can sell 5 less items.
   a) How much is the total income if we price the product at \$21?
   b) What price woud guarantee an income of \$6625?
   c) What price would guarantee the possible highest income? What is the highest possible income?

4. A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.4q^2 + 3q + 40$ thousand dollars, and all q units can be sold at a price of $p(q) = 22.2 - 1.2q$ thousand dollars per unit. Find the maximal profit.

5. A farmer wishes to enclose a rectangular pasture with 320 ft of fence. What dimensions give the maximum area if the fence is on three sides of the pasture and the fourth side is bounded by a wall?

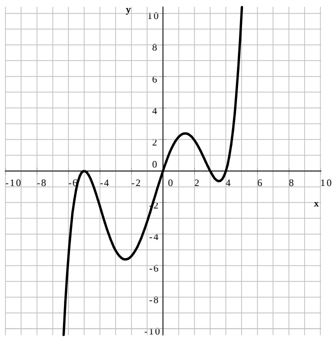
6. The location of an object, measured in meters, is given by $L(t) = 5t^2 - 3t + 1$, where t is measured in seconds. Find the average velocity of the object between
   a) $t = 2$ s and $t = 2.5$ s b) $t = 2$ s and $t = 2.01$ s c) $t = 2$ s and $t = 2.001$ s

7. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find
   a) $f(g(4))$ b) $g(f(4))$ c) $f(g(x))$ d) $g(f(x))$

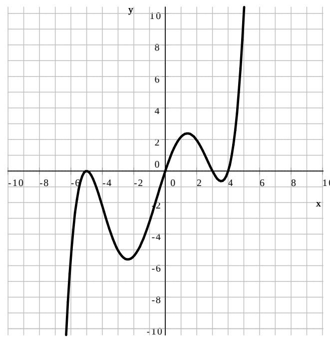
8. Graph each of the following equations. In case of each of these equations, determine whether y is a function of x or not.
   a) $y = x^2 - 6x + 8$ c) $x = y^2$ e) $xy = 1$ g) $x^2y = 1$
   b) $x^2 = y^2$ d) $x^2 + y = 4$ f) $2x + 3y = 4$

9. The figures below all show the graph of $f(x)$. Sketch the graph of $g(x)$ into the same coordinate system if

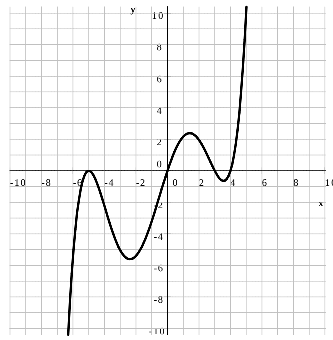
a) $g(x) = f(x) + 2$



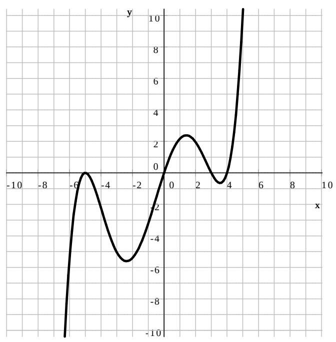
b) $g(x) = f(x+2)$



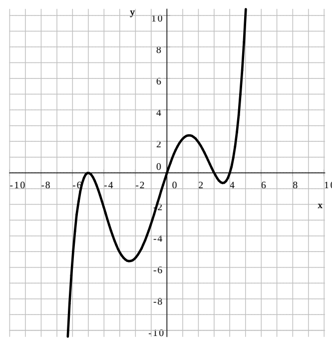
c) $g(x) = -f(x)$



e) $g(x) = \dfrac{f(x) + |f(x)|}{2}$



f) $g(x) = f(-x)$



Review Problems - Answers

1. See Optimization 1

   a) 1152 ft b) 1280 ft c) $64\frac{\text{ft}}{\text{s}}$ d) $-96\frac{\text{ft}}{\text{s}}$ e) 15 s f) 1296 ft

   Solution: c)

$$v_{\text{average}} = \frac{\text{distance traveled}}{\text{time}} = \frac{h(5\,\text{s}) - h(3\,\text{s})}{5\,\text{s} - 3\,\text{s}} = \frac{1280\,\text{ft} - 1152\,\text{ft}}{2\,\text{s}} = \frac{128\,\text{ft}}{2\,\text{s}} = 64\frac{\text{ft}}{\text{s}}$$

   d) We first compute $h(8)$ and $h(10)$. They are: $h(8) = 1232\,\text{ft}$ and $h(10) = 1040\,\text{ft}$

$$v_{\text{average}} = \frac{\text{distance traveled}}{\text{time}} = \frac{h(10\,\text{s}) - h(8\,\text{s})}{10\,\text{s} - 8\,\text{s}} = \frac{1040\,\text{ft} - 1232\,\text{ft}}{2\,\text{s}} = \frac{-192\,\text{ft}}{2\,\text{s}} = -96\frac{\text{ft}}{\text{s}}$$

   Where the negative sign indicates that the object has traveled downward.

   e) We have to solve the quadratic equation $-16t^2 + 192t + 720 = 0$.

$$\begin{aligned} 0 &= -16t^2 + 192t + 720 \\ 0 &= -16\left(t^2 - 12t - 45\right) \\ 0 &= -16\left(t^2 - 12t + 36 - 36 - 45\right) \\ 0 &= -16\left((t-6)^2 - 81\right) \end{aligned}$$

$$0 = -16(t - 6 + 9)(t - 6 - 9)$$
$$0 = -16(t+3)(t-15)$$
$$t_1 = -3 \qquad t_2 = 15$$

   In this case the negative solution is ruled out, and so the answer is: it takes 15 seconds for the object to hit the ground.

   e) From the previous computation, we have that

$$\begin{aligned} h(t) &= -16\left((t-6)^2 - 81\right) \qquad \text{distribute } -16 \\ h(t) &= -16(t-6)^2 + 1296 \end{aligned}$$

   This last line indicates that the upside down parabola $h(t)$ has its vertex at $(6, 1296)$. Thus the highest point is reached 6 seconds after we threw the object, and it is 1296 feet high.

2. see Optimization 1 the square

3. see Optimization 1 a) \$5985 b) \$25 and \$53 c) \$7605 with a price of \$39

4. see Optimization 1 \$17600

5. see Optimization 1 80 ft by 160 ft

6. a) $19.5\frac{\text{m}}{\text{s}}$ b) $17.05\frac{\text{m}}{\text{s}}$ c) $17.005\ \frac{\text{m}}{\text{s}}$

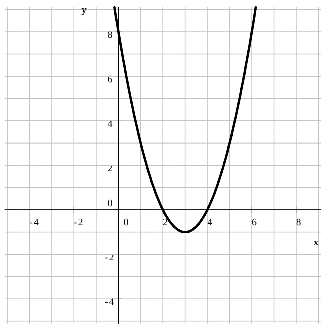
7. a) $\sqrt{15}$ b) 3 c) $\sqrt{x^2-1}$ d) $x-1$

   Solution:

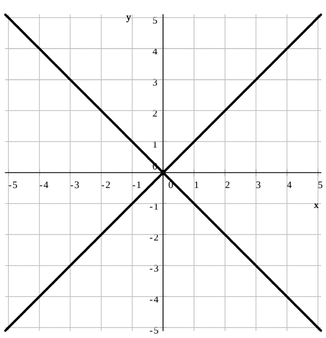
   a) $f(g(4)) = f(4^2 - 1) = f(15) = \sqrt{15}$ b) $g(f(4)) = g\left(\sqrt{4}\right) = g(2) = 2^2 - 1 = 3$

   c) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$ d) $g(f(x)) = g(\sqrt{x}) = \left(\sqrt{x}\right)^2 - 1 = x - 1$

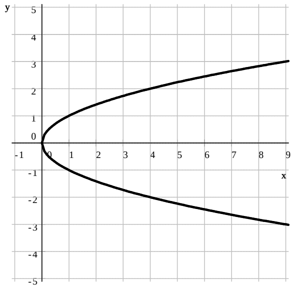
8. a) $y = x^2 - 6x + 8$
Solution: This is a function.
Algebraically: There is a unique formula to compute y in terms of x.
Geometrically: its graph passes the vertical line test.



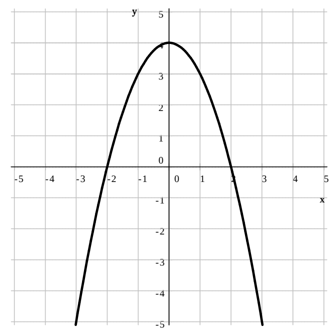
b) $x^2 = y^2$
Solution: Not a function.
Algebraically: If we solve for y, we get $y = x$ or $y = -x$.
$x^2 = y^2$
$0 = y^2 - x^2$
$0 = (y + x)(y - x)$
$y_1 = -x \qquad y_2 = x$
Geometrically: its graph fails the vertical line test,



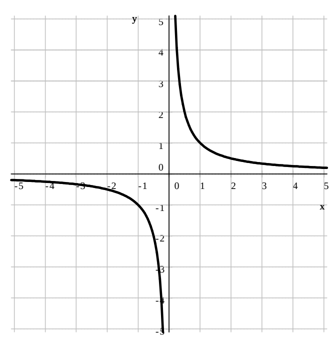
c) $x = y^2$
Solution: Not a function
Algebraically: when we solve for y, we get that $y = \pm\sqrt{x}$.
$x = y^2$
$0 = y^2 - x$
$0 = y^2 - \left(\sqrt{x}\right)^2$
$0 = \left(y + \sqrt{x}\right)\left(y - \sqrt{x}\right)$
$y_1 = -\sqrt{x} \qquad y_2 = \sqrt{x}$
Geometrically: the graph fails the vertical line test.



d) $x^2 + y = 4$
Solution: Function. We solve for y and see that it is a unique formula. $y = -x^2 + 4$ Its graph passes the vertical line test.



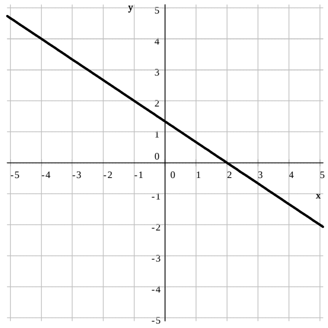
e) $xy = 1$
Solution: Function.
Algebraically: We solve for y and see that it is a unique formula. $y = \dfrac{1}{x}$
Geometrically: Its graph passes the vertical line test.



f) $2x + 3y = 4$
Solution: Function. We solve for y and see that it is a unique formula.

$$\begin{aligned} 2x + 3y &= 4 && \text{subtract } 2x \\ 3y &= -2x + 4 && \text{divide by } 3 \\ y &= \frac{-2x+4}{3} \\ y &= -\frac{2}{3}x + \frac{4}{3} \end{aligned}$$

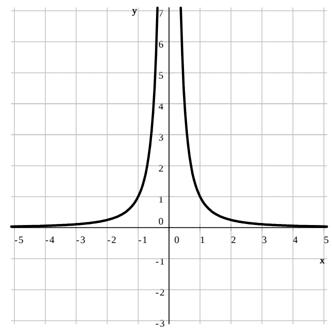
Its graph passes the vertical line test.



g) $x^2 y = 1$
Solution: Function. We solve for y and see that it is a unique formula.

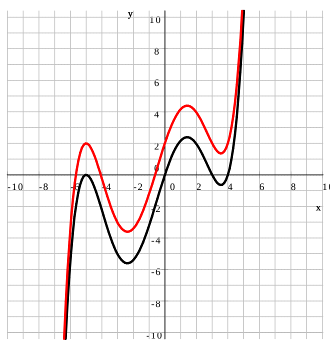
$$\begin{aligned} x^2 y &= 1 && \text{divide by } x^2 \\ y &= \frac{1}{x^2} \end{aligned}$$

Its graph passes the vertical line test.

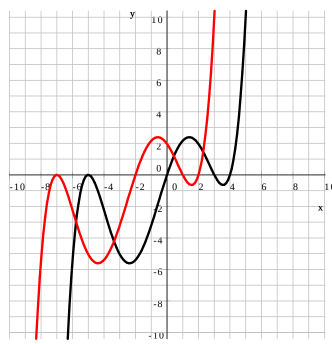


9. The figures below all show the graph of $f(x)$. Sketch the graph of $g(x)$ into the same coordinate system if

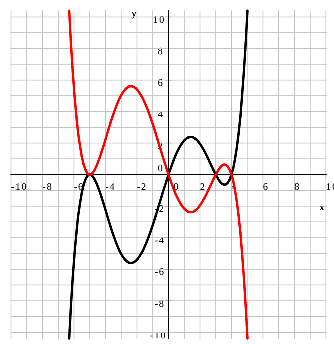
a) vertical shift upward by 2 units



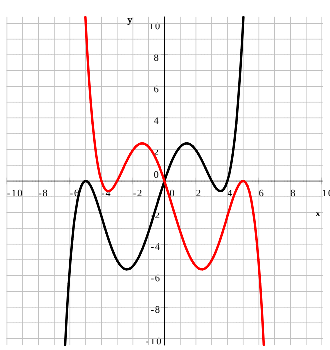
b) horizontal shift to the left by 2 units



c) reflection to the $x-$axis



d) reflection to the $y-$axis



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