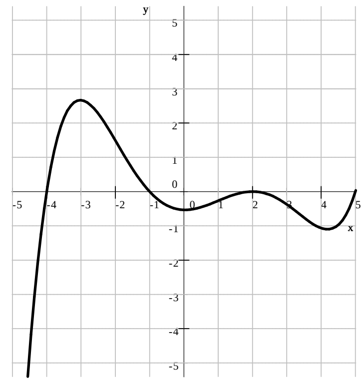
Quiz 6 will cover the following topics: all topics from Quiz 1, 2, 3, 4, and 5, Exam 1, and complete analysis of functions, and differentiation rules 1.

Review Problems

1. The graph below shows the graph of a function $f(x)$. Plot the graph of $g(x) = \dfrac{1}{f(x)}$.



2. Sketch the graph and give a complete analysis for each of the following functions.

   (a) $f(x) = \sqrt{x+1} - 2$

   (b) $f(x) = ||x-1|+1|$

   (c) $f(x) = 10x - x^2 + 11$ where the domain is the closed interval $[3, 8]$.

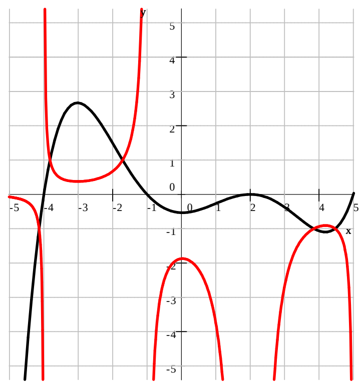
3. Find and prove, using the definition of the derivative as the limit of the differential quotient.

   (a) $f(x) = 5x^2$

   (b) $f(x) = \sqrt{x-3}$

Review Problems - Answers

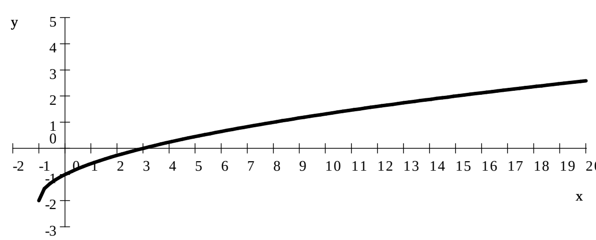
1. $g(x) = \dfrac{1}{f(x)}$.



2. a) $f(x) = \sqrt{x+1} - 2$

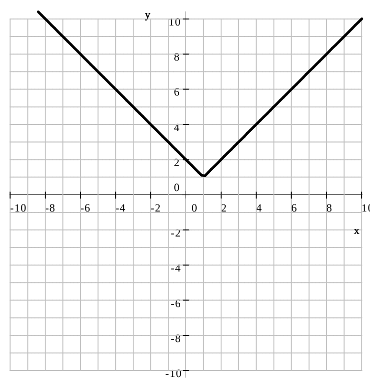
Solution:

domain: $[-1, \infty)$	one-to-one	no point of inflection
range: $[-2, \infty)$	no relative maximum	concave down on $[-1, \infty)$
no asymptotes	no absolute maximum	continuous on $(-1, \infty)$
$y-$intercept: $(0, -1)$	no relative minimum	even/odd: neither
$x-$intercept: $(3, 0)$	absolute minimum: $(-1, -2)$	end-behavior:
bounded below	increasing on $[-1, \infty)$	$\lim\limits_{x \to -\infty} f(x) = \text{undefined}$
		$\lim\limits_{x \to \infty} f(x) = \infty$



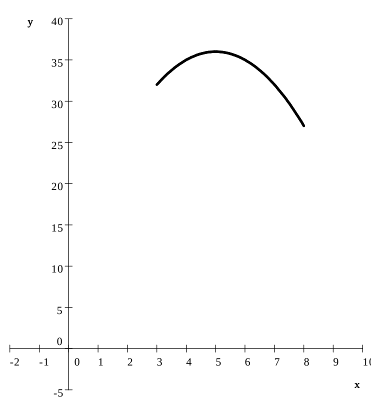
b) $f(x) = ||x-1|+1|$
Solution:

domain: $\mathbb{R}$	not one-to-one	no point of inflection:
range: $[1, \infty)$	no relative maximum	not concave up
no asymptotes	no absolute maximum	not concave down
$y-$intercept: $(0, 2)$	relative minimum: $(1, 1)$	continuous on $\mathbb{R}$
no $x-$intercept	absolute minimum: $(1, 1)$	even/odd: neither
bounded below	increasing on $[1, \infty)$	end-behavior:
	decreasing on $(-\infty, 1]$	$\lim\limits_{x \to -\infty} f(x) = \infty$ and
		$\lim\limits_{x \to \infty} f(x) = \infty$



c) $f(x) = 10x - x^2 + 11$ where the domain is the closed interval $[3, 8]$.
Solution:

domain: $[3, 8]$	not one-to-one	no point of inflection:
range: $[27, 36]$	relative maximum: $(5, 36)$	concave down on $[3, 8]$
no asymptotes	absolute maximum: $(5, 36)$	continuous on $(3, 8)$
no intercepts	no relative minimum	even/odd: neither
bounded	absolute minimum: $(8, 27)$	end-behavior:
	increasing: on $[3, 5]$	$\lim\limits_{x \to -\infty} f(x) =$ undefined
	decreasing: on $[5, 8]$	$\lim\limits_{x \to \infty} f(x) =$ undefined



3. a) If $f\left(x\right)=5x^2$ then $f'\left(x\right)=10x$
Proof:

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{5(x+h)^2-5(x)^2}{h} = \lim_{h\to 0}\frac{5(x^2+2xh+h^2)-5(x)^2}{h} \\ &= \lim_{h\to 0}\frac{5(x^2+2xh+h^2)-5(x)^2}{h} = \lim_{h\to 0}\frac{5x^2+10xh+5h^2-5x^2}{h} = \lim_{h\to 0}\frac{10xh+5h^2}{h} \\ &= \lim_{h\to 0}\frac{10xh+5h^2}{h} = \lim_{h\to 0}\frac{h\left(10x+5h\right)}{h} = \lim_{h\to 0}10x+5h = 10x \ \blacksquare \end{aligned}$$

b) If $f\left(x\right)=\sqrt{x-3}$ then $f'\left(x\right)=\dfrac{1}{2\sqrt{x-3}}$

Proof:

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h} = \lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}\cdot 1 \\ &= \lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}\cdot\frac{\sqrt{x+h-3}+\sqrt{x-3}}{\sqrt{x+h-3}+\sqrt{x-3}} \\ &= \lim_{h\to 0}\frac{\left(\sqrt{x+h-3}-\sqrt{x-3}\right)\left(\sqrt{x+h-3}+\sqrt{x-3}\right)}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)} \\ &= \lim_{h\to 0}\frac{\left(\sqrt{x+h-3}\right)^2-\left(\sqrt{x-3}\right)^2}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)} = \lim_{h\to 0}\frac{(x+h-3)-(x-3)}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)} \\ &= \lim_{h\to 0}\frac{x+h-3-x+3}{h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)} = \lim_{h\to 0}\frac{\not h}{\not h\left(\sqrt{x+h-3}+\sqrt{x-3}\right)} \\ &= \lim_{h\to 0}\frac{1}{\sqrt{x+h-3}+\sqrt{x-3}} = \frac{1}{2\sqrt{x-3}} \ \blacksquare \end{aligned}$$