Quiz 9 Information

Quiz 9 will cover the following topics: all topics from Quiz 1-8, Exams 1, 2, and differentiation rules: product rule, quotient rule (2.3, handout), chain rule (2.4, handout), and differentiating exponential functions (4.3, handout).

Review Problems

1. Differentiate each of the following functions.

   a) $f(x) = \dfrac{1}{(2x-7)^5}$

   b) $f(x) = \sqrt{x^7 - 3\ln x - 1}$

   c) $f(x) = 7^x - x^7 + \sqrt{7x} - e^7$

   d) $f(x) = \dfrac{e^{10x} - e^{-10x}}{2}$

   e) $f(x) = -\dfrac{1}{4}e^{-2x} - \dfrac{1}{2}xe^{-2x}$

   f) $f(x) = 5^{x^2-4x+11}$

   g) $f(x) = \dfrac{6x^2 - 14x + 1}{3x - 7}$

   h) $f(x) = \log_3(7x^2)$

   i) $f(x) = \dfrac{\ln(x^3 - 5x^2 + 6)}{x^2 + 1}$

2. Suppose that g is a function satisfying the following conditions $g(2) = 3$ and $g'(2) = -4$. Find $f'(2)$ if

   a) $f(x) = x^2 - 4g(x)$

   b) $f(x) = x^2 g(x)$

3. Let $f(x) = x^3 + ax^2 + bx + c$. Find values of a, b and c such that

   a) f has a relative maximum at $x = 1$ and a relative minimum at $x = 2$.

   b) f has a point of inflection at $x = 1$ and a relative minimum at $x = 2$.

4. The velocity of an object is given by $v(t) = -16t + 8$. Suppose that $s(0) = 64$. Find the location function $s(t)$ of the object.

5. Consider the function $f(x) = (x-1)^{10}(4-x)^5$.

   a) Find all values of x for which $f(x)$ has a relative maximum.

   b) Find all values of x for which $f(x)$ has a relative minimum.

   c) Find all values of $f(x)$ for which $f'(x) = 0$ and f has a point of inflection.

6. Find the second derivative for the function $f(x) = \dfrac{5x-1}{5x+1}$. (Hint: although you may use the quotient rule, it can be avoided with a little algebraic transformation before the calculus).

Review Problems - Answers

1. a) $f'(x) = -\dfrac{10}{(2x-7)^6}$ b) $f'(x) = \dfrac{7x^6 - \dfrac{3}{x}}{2\sqrt{x^7 - 3\ln x - 1}}$ c) $f'(x) = 7^x \ln 7 - 7x^6 + \dfrac{7}{2\sqrt{7x}}$

   d) $f'(x) = 5e^{10x} + 5e^{-10x}$ e) $f'(x) = xe^{-2x}$ f) $f'(x) = (\ln 5)(2x-4)\,5^{x^2-4x+11}$

   g) $f'(x) = \dfrac{18x^2 - 84x + 95}{9x^2 - 42x + 49} = 2 - \dfrac{3}{(3x-7)^2}$ h) $f'(x) = \dfrac{2}{x\ln 3}$

   i) $f'(x) = \dfrac{\dfrac{3x^2 - 10x}{x^3 - 5x^2 + 6} - 2x\ln\left(x^3 - 5x^2 + 6\right)}{\left(x^2+1\right)^2}$

2. a) 20 b) -4
   Solution:
   a) We first find the derivative f' in general, and then evaluate it at $x = 2$.

$$\begin{aligned} f(x) &= x^2 - 4g(x) && \text{differentiate} \\ f'(x) &= 2x - 4g'(x) && \text{and so} \\ f'(2) &= 2(2) - 4(-4) = 4 + 16 = 20 \end{aligned}$$

   b) We apply the product rule.

$$\begin{aligned} f(x) &= x^2 g(x) \\ f'(x) &= \left(x^2\right)' g(x) + x^2 (g(x))' = 2xg(x) + x^2 g'(x) \\ f'(2) &= 2(2)(3) + (2)^2(-4) = 12 - 16 = -4 \end{aligned}$$

3. a) $a = -\dfrac{9}{2}$, $b = 6$, and c can be any number b) $a = -3$, $b = 0$, and c can be any number

4. $s(t) = -8t^2 + 8t + 64$

5. a) 3 b) 1 c) 4

6. $f''(x) = -\dfrac{100}{(5x+1)^3}$
   Solution:

$$f(x) = \frac{5x-1}{5x+1} = \frac{5x+1-2}{5x+1} = 1 - \frac{2}{5x+1} = 1 - 2(5x+1)^{-1}$$

$$\begin{aligned} f'(x) &= -2(-1)(5x+1)^{-2}(5) = 10(5x+1)^{-2} \\ f''(x) &= 10(-2)(5x+1)^{-3}(5) = -100(5x+1)^3 = -\frac{100}{(5x+1)^3} \end{aligned}$$