

1. Simplify each of the following

a)  $6^{-2}$       c)  $64^{-2/3}$       e)  $8^{\log_2 5}$       g)  $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 - 1$       i)  $e^{-2 \ln A}$   
 b)  $32^{3/5}$       d)  $\log_9 \sqrt{27}$       f)  $3^{\log_9 10}$       h)  $\log_2 (4x^2) - 3 \log_2 \left(\frac{6}{x}\right) + \log_4 (4x^6)$

2. Let  $x = \log_3 2$ . Express each of the following in terms of  $x$ .

a)  $\log_3 72$       b)  $\log_2 3$       c)  $\log_6 72$

3. Factor  $3x^2 - 4x - 319$  by completing the square.

4. In case of each polynomial given, determine (by completing the square) whether it can be factored over the real numbers or not. (You do not have to actually factor.)

a)  $20x + 2x^2 + 44$       b)  $20x - 5x^2 - 25$

5. Solve the equation  $9x^2 - 12x = 11$  and check one of your solution using exact values.

6. Solve each of the following equations. Present all solutions, in radians.

a)  $\sin x + \sin 2x = 0$       b)  $\sin x + \cos 2x = 0$       c)  $\tan^3 x = 3 \tan x$

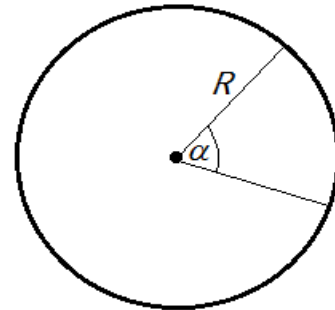
7. Suppose that  $\alpha$  is a central angle (less than  $360^\circ$ ) in a circle with radius  $R$ .

a) Express the length of the arc subtended by the central angle  $\alpha$  in terms of  $\alpha$  and  $R$ . Assume that  $\alpha$  is measured in degrees.

b) Express the area of the sector determined by the central angle  $\alpha$  in terms of  $\alpha$  and  $R$ . Assume that  $\alpha$  is measured in degrees.

c) Express the length of the arc subtended by the central angle  $\alpha$  in terms of  $\alpha$  and  $R$ . Assume that  $\alpha$  is measured in radians.

d) Express the area of the sector determined by the central angle  $\alpha$  in terms of  $\alpha$  and  $R$ . Assume that  $\alpha$  is measured in radians.



8. Solve each of the following equations.

a)  $2x^3 = 6x$       d)  $\log_3 (7 - x) + \log_3 (1 - x) = 3$   
 b)  $2x^2 - 3x - 1 = 0$       e)  $\log_6 (-8 - x) + \log_6 (8 - x) = 2$   
 c)  $\log_2 (x + 5) - \log_2 (x - 7) = -1$       f)  $\frac{2x - 1}{3} - \frac{x - 1}{2} = x - 4$

9. Solve each of the following inequalities.

a)  $x^2 \geq 4x$       b)  $8x + x^2 < 33$       c)  $x^2 < -2x + 2$       d)  $4x^2 \leq 4x - 1$       e)  $x^2 - 6x > -10$

10. Find the domain for each of the following functions.

a)  $f(x) = \ln(x^2 - 10x + 29)$       c)  $f(x) = \log_5(x^2 - 10x + 21)$   
 b)  $g(x) = \frac{1}{\log_2(4 - x)}$       d)  $k(x) = \frac{1}{\log_5(x^2 - 10x + 21)}$

11. a) Find the center and radius of a circle with equation  $2y - 6x + x^2 + y^2 = 10$   
 b) Find an equation for the tangent line drawn to the circle  $(x + 2)^2 + (y - 4)^2 = 10$  to the point  $(-5, 3)$
12. Find an equation for the curve that consists of points  $P(x, y)$  with the following property: they are twice as far from point  $A(2, -5)$  as from point  $B(-1, 1)$ . Describe the curve.
13. a) Solve the formula  $V = 2\pi r^3 + \frac{1}{2}\pi r^2 h$  for  $h$ .  
 b) A right pyramid has a square base with sides 30 meters long. The pyramid is 24 meters tall. At what height is the perpendicular cross section a square with sides 10 meters?
14. Graph each of the following pairs of functions in the same coordinate system.  
 a)  $f(x) = 2^x$  and  $g(x) = \log_2 x$                       b)  $f(x) = 2^x$  and  $g(x) = \left(\frac{1}{2}\right)^x$   
 c)  $f(x) = \log_2 x$  and  $g(x) = \log_{1/2} x$                       d)  $f(x) = \left(\frac{1}{2}\right)^x$  and  $g(x) = \log_{1/2} x$
15. For each of the following functions given, give a complete analysis and sketch its graph.  
 a)  $f(x) = 9 - 3x^2 - 6x$  on  $[-2, 3]$                       c)  $f(x) = \log_3 x$                       e)  $f(x) = 1 + \sqrt{16 - x^2}$   
 b)  $f(x) = \sqrt{x + 3}$                       d)  $f(x) = 0.7^x$
16. An object's height (measured in feet) is defined by  $s(t) = 0.4t + 12$  where  $t$  is the time, measured in seconds.  
 a) Find the location of the object at  $t = 10$  seconds.  
 b) Find the average velocity of the object between  $t = 0$  and  $t = 3$  seconds  
 c) Find the average velocity of the object between  $t = 5$  and  $t = 10$  seconds
17. An object's height (measured in feet) is defined by  $s(t) = t^3 - 12t$  where  $t$  is the time, measured in seconds.  
 a) Find the location of the object at  $t = 3$  seconds.  
 b) Find the average velocity of the object between  
     i)  $t = 0$  and  $t = 2$  seconds      ii)  $t = 1$  s and  $t = 2$  s      iii)  $t = 1.5$  s and  $t = 2$
18. A bus travels between cities A and B. The distance between these cities is 60 miles. It takes the bus 2 hours to get from A to B. On its way back, the traveling time was only 1.5 hours. Find the average speed of the bus for  
 a) the trip from A to B                      b) the trip from B to A                      c) for the roundtrip.  
 d\*) A bus travels between cities A and B. From A to B, the bus has an average speed of  $v_1$ . On its way back, the average speed is  $v_2$ . Express the average speed of the bus in terms of  $v_1$  and  $v_2$ .
19. Find the coordinates of all points where the graphs of  $f(x) = x^2 - 2x - 26$  and  $g(x) = 2x - 5$  intersect each other.
20. Compute each of the following limits.  
 a)  $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^2)$                       d)  $\lim_{x \rightarrow \infty} (-2x^5 + 8x^6)$                       g)  $\lim_{x \rightarrow -\infty} \log_2 x$   
 b)  $\lim_{x \rightarrow \infty} (-2x^5 + 8x^2)$                       e)  $\lim_{x \rightarrow -\infty} \frac{3x^2 - 1}{5x^2 - 3x + 2}$                       h)  $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$   
 c)  $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^6)$                       f)  $\lim_{x \rightarrow -\infty} \frac{100x - 1}{5x^2 - 3x + 2}$                       i)  $\lim_{x \rightarrow \infty} \frac{-x^3 + 2x + 1}{x - 3}$

j)  $\lim_{x \rightarrow -\infty} 2^x$

n)  $\lim_{x \rightarrow \infty} \frac{3^{x+1} \cdot \left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

q)  $\lim_{x \rightarrow -\infty} \frac{\cos x - 2}{x^3 + 1}$

k)  $\lim_{x \rightarrow \infty} (\log_2 (x^2 - 5x + 17))$

r)  $\lim_{x \rightarrow \infty} \tan x$

l)  $\lim_{x \rightarrow \infty} \frac{12 + \log_7 3x}{15 + \log_7 x}$

o)  $\lim_{x \rightarrow \infty} x \left( \frac{1}{3} - \frac{1}{3 - \frac{1}{x}} \right)$

s)  $\lim_{x \rightarrow \infty} \frac{\ln 2x}{\ln 3x}$

m)  $\lim_{x \rightarrow \infty} \frac{2^{x+5}}{4^{x-1}}$

p)  $\lim_{x \rightarrow \infty} \frac{\sqrt{4 - \frac{1}{x}} - 2}{\frac{1}{x}}$

t)  $\lim_{\theta \rightarrow \infty} (\sin^2 \theta + \cos^2 \theta)$

u)  $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 - 1}}{x}$

21. Based on the graph of a function  $f$  shown on the picture, determine each of the following limits.

a)  $\lim_{x \rightarrow 1^-} f(x)$

d)  $\lim_{x \rightarrow 2^-} f(x)$

g)  $\lim_{x \rightarrow 4^-} f(x)$

j)  $\lim_{x \rightarrow 6^-} f(x)$

m)  $\lim_{x \rightarrow 10^-} f(x)$

b)  $\lim_{x \rightarrow 1^+} f(x)$

e)  $\lim_{x \rightarrow 2^+} f(x)$

h)  $\lim_{x \rightarrow 4^+} f(x)$

k)  $\lim_{x \rightarrow 6^+} f(x)$

n)  $\lim_{x \rightarrow 10^+} f(x)$

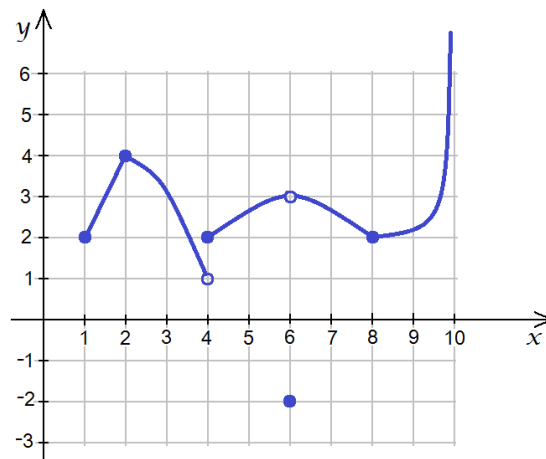
c)  $\lim_{x \rightarrow 1} f(x)$

f)  $\lim_{x \rightarrow 2} f(x)$

i)  $\lim_{x \rightarrow 4} f(x)$

l)  $\lim_{x \rightarrow 6} f(x)$

o)  $\lim_{x \rightarrow 10} f(x)$



22. Compute each of the following limits.

a)  $\lim_{x \rightarrow -\infty} \frac{x^2 - 9}{2x^2 - 8x + 6}$

e)  $\lim_{x \rightarrow 1^-} \frac{x^2 - 9}{2x^2 - 8x + 6}$

i)  $\lim_{x \rightarrow 3^+} \frac{x^2 - 9}{2x^2 - 8x + 6}$

b)  $\lim_{x \rightarrow -3^-} \frac{x^2 - 9}{2x^2 - 8x + 6}$

f)  $\lim_{x \rightarrow 1^+} \frac{x^2 - 9}{2x^2 - 8x + 6}$

j)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{2x^2 - 8x + 6}$

c)  $\lim_{x \rightarrow -3^+} \frac{x^2 - 9}{2x^2 - 8x + 6}$

g)  $\lim_{x \rightarrow 1} \frac{x^2 - 9}{2x^2 - 8x + 6}$

k)  $\lim_{x \rightarrow \infty} \frac{x^2 - 9}{2x^2 - 8x + 6}$

d)  $\lim_{x \rightarrow -3} \frac{x^2 - 9}{2x^2 - 8x + 6}$

h)  $\lim_{x \rightarrow 3^-} \frac{x^2 - 9}{2x^2 - 8x + 6}$

23. Based on your results in the previous problem, sketch the graph of  $f(x) = \frac{x^2 - 9}{2x^2 - 8x + 6}$ . (Plot a few additional points if needed.)

24. Compute each of the following limits.

$$\text{a) } \lim_{x \rightarrow 2} \frac{2-x}{\frac{1}{2} - \frac{1}{x}}$$

$$\text{f) } \lim_{x \rightarrow 1^+} \frac{x^2 - 9}{2x^2 - 8x + 6}$$

$$\text{l) } \lim_{a \rightarrow 0} \frac{\sqrt{5a+4} - 2}{a}$$

$$\text{b) } \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{2 - \sqrt{3+x}}$$

$$\text{g) } \lim_{x \rightarrow 0} \frac{\sqrt{5+x} - \sqrt{5}}{x}$$

$$\text{m) } \lim_{m \rightarrow 1^-} \frac{m^2 - 1}{m^4 - 1}$$

$$\text{c) } \lim_{x \rightarrow 3} \frac{|2x-4|}{x-8}$$

$$\text{h) } \lim_{x \rightarrow 3^-} \frac{x^2 - 9}{2x^2 - 8x + 6}$$

$$\text{n) } \lim_{x \rightarrow 0} \frac{1+x+\sin x}{\cos x}$$

$$\text{d) } \lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

$$\text{i) } \lim_{x \rightarrow \pi} \frac{\cos x}{\sin x}$$

$$\text{o) } \lim_{x \rightarrow \pi/3} (\tan^2 x - 1)$$

$$\text{e) } \lim_{x \rightarrow 0} \frac{|x|}{x}$$

$$\text{j) } \lim_{x \rightarrow \pi^+} \frac{\cos x}{\sin x}$$

$$\text{p) } \lim_{h \rightarrow 0} \frac{\sqrt{h^2+100} - 10}{h^2}$$

$$\text{k) } \lim_{x \rightarrow \pi/4} \tan x$$

$$\text{q) } \lim_{x \rightarrow 0^+} \ln x$$

25. Consider the limit  $\lim_{x \rightarrow \pi/2} \frac{1 - \tan^2 x}{1 + \tan^2 x}$ .

a) Explain why we can not evaluate the limit in its current form.

b) Prove that  $\frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x$  and use this to evaluate the limit.

26\*. Consider the function  $f(x) = 2mx - 8m + mx^2 + 3$  where  $m$  is a fixed real number.

a) Find all values of  $m$  for which the graph of  $f$  is NOT a parabola.

b) Graph  $y = f(x)$  in the same coordinate system when  $m = 0, 1$ , and  $-1$ .

c) Prove that there exist two points  $P$  and  $Q$  that are on the graph of  $f(x)$  for all values of  $m$ .

## Answers

$$1. \text{ a) } \frac{1}{36} \quad \text{b) } 8 \quad \text{c) } \frac{1}{16} \quad \text{d) } \frac{3}{4} \quad \text{e) } 125 \quad \text{f) } \sqrt{10} \quad \text{g) } \log_3 2 = \frac{\ln 2}{\ln 3} \quad \text{h) } \log_2 \left( \frac{x^8}{27} \right) \quad \text{i) } \frac{1}{A^2}$$

$$2. \text{ a) } 3x + 2 \quad \text{b) } \frac{1}{x} \quad \text{c) } \frac{3x + 2}{x + 1}$$

$$3. 3 \left( x + \frac{29}{3} \right) (x - 11) = (3x + 29)(x - 11)$$

4. a) factors      b) can not be factored

$$5. \frac{2 \pm \sqrt{15}}{3} \quad \text{Check: if } x = \frac{2 - \sqrt{15}}{3}, \text{ then the left-hand side of the equation is}$$

$$\begin{aligned} \text{LHS} &= 9x^2 - 12x = 9 \left( \frac{2 - \sqrt{15}}{3} \right)^2 - 12 \left( \frac{2 - \sqrt{15}}{3} \right) = 9 \cdot \frac{(2 - \sqrt{15})^2}{9} - 12 \cdot \frac{2 - \sqrt{15}}{3} \\ &= (2 - \sqrt{15})^2 - 4(2 - \sqrt{15}) = 4 + 15 - 4\sqrt{15} - 8 + 4\sqrt{15} = 19 - 8 = 11 = \text{RHS} \end{aligned}$$

6. a)  $x = k\pi$  or  $x = \pm \frac{2\pi}{3} + 2k\pi$  where  $k \in \mathbb{Z}$

b)  $x = \frac{\pi}{2} + 2k\pi$  or  $x = -\frac{\pi}{6} + 2k\pi$  or  $x = -\frac{5\pi}{6} + 2k\pi$  where  $k \in \mathbb{Z}$       c)  $x = k\pi$  or  $x = \pm \frac{\pi}{3} + k\pi$  where  $k \in \mathbb{Z}$

7. a)  $s = \frac{2\pi R\alpha}{360^\circ}$       b)  $A = \frac{\pi R^2\alpha}{360^\circ}$       c)  $s = R\alpha$       d)  $A = \frac{1}{2}R^2\alpha$

8. a)  $0, -\sqrt{3}, \sqrt{3}$       b)  $\frac{3 \pm \sqrt{17}}{4}$       c) no solution      d)  $-2$       e)  $-10$       f)  $5$

9. a)  $(-\infty, 0] \cup [4, \infty)$       b)  $(-11, 3)$       c)  $(-\sqrt{3}-1, \sqrt{3}-1)$       d)  $x = \frac{1}{2}$       e)  $\mathbb{R}$

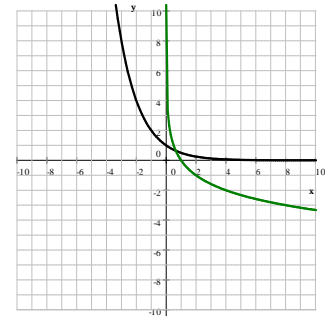
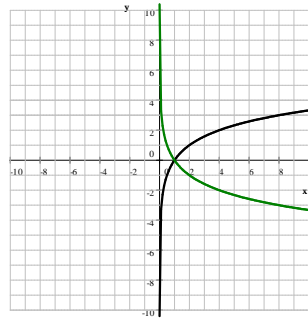
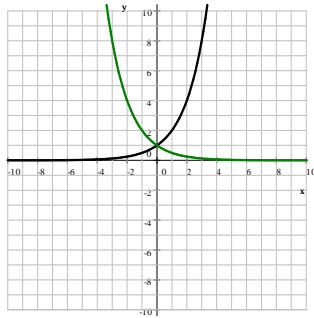
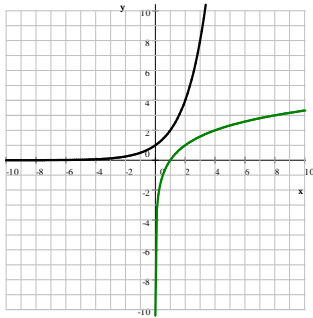
10. a)  $\mathbb{R}$       b)  $x < 4$  but  $x \neq 3$       c)  $x < 3$  or  $x > 7$   
d)  $x < 3$  but  $x \neq 5 - \sqrt{5}$  or  $x > 7$  but  $x \neq 5 + \sqrt{5}$

11. a) center:  $(3, -1)$  radius:  $2\sqrt{5}$       b)  $y = -3x - 12$

12.  $(x+2)^2 + (y-3)^2 = 20$       a circle, centered at  $(-2, 3)$  with radius  $\sqrt{20}$

13. a)  $h = \frac{2(V - 2\pi r^3)}{\pi r^2}$  or  $h = \frac{2V}{\pi r^2} - 4r$       b) At a height of 16 meters

14.      a)      b)      c)      d)



15. a)  $f(x) = 9 - 3x^2 - 6x$  on  $[-2, 3]$

domain:  $[-2, 3]$

range:  $[-36, 12]$

$x$ -intercept:  $(1, 0)$

$y$ -intercept:  $(0, 9)$

maximum:  $(-1, 12)$

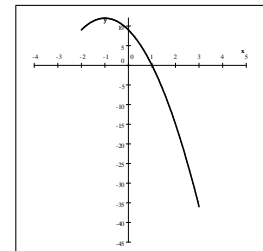
minimum:  $(3, -36)$

and decreasing on  $(-1, 3)$

one-to-one: no

end-behavior: none

i.e.  $\lim_{x \rightarrow \pm\infty} f(x) = \text{undefined}$



b)  $f(x) = \sqrt{x+3}$

domain:  $[-3, \infty)$

range:  $[-3, \infty)$

$x$ -intercept:  $(-3, 0)$

$y$ -intercept:  $(0, \sqrt{3})$

maximum: none

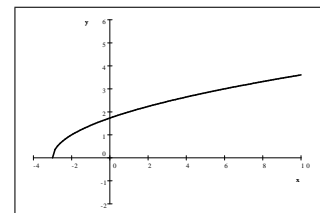
minimum:  $(-3, 0)$

one-to-one: yes

end-behavior:

$\lim_{x \rightarrow -\infty} f(x) = \text{undefined}$

and  $\lim_{x \rightarrow \infty} f(x) = \infty$



c)  $f(x) = \log_3 x$

domain:  $(0, \infty)$ range:  $\mathbb{R}$  $x$ -intercept:  $(1, 0)$  $y$ -intercept: none

maximum: none

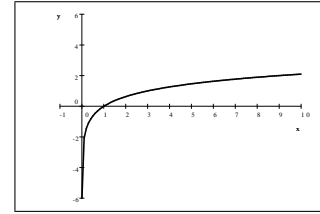
minimum: none

one-to-one: yes

end-behavior:

$$\lim_{x \rightarrow -\infty} f(x) = \text{undefined}$$

and 
$$\lim_{x \rightarrow \infty} f(x) = \infty$$



d)  $f(x) = 0.7^x$

domain:  $\mathbb{R}$ range:  $(0, \infty)$  $x$ -intercept: none $y$ -intercept:  $(0, 1)$ 

maximum: none

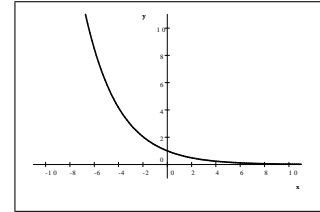
minimum: none

one-to-one: yes

end-behavior:

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

and 
$$\lim_{x \rightarrow \infty} f(x) = 0$$



e)  $f(x) = 1 + \sqrt{16 - x^2}$

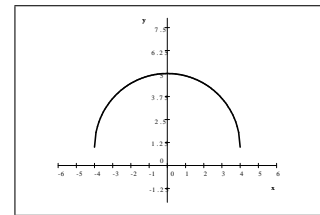
domain:  $[-4, 4]$ range:  $[1, 5]$  $x$ -intercept: none $y$ -intercept:  $(0, 5)$ maximum:  $(0, 5)$ minimum:  $(-4, 1)$  and  $(4, 1)$ 

one-to-one: no

end-behavior:

$$\lim_{x \rightarrow -\infty} f(x) = \text{undefined}$$

and 
$$\lim_{x \rightarrow \infty} f(x) = \text{undefined}$$



16. a) 16 ft    b)  $0.4 \frac{\text{ft}}{\text{s}}$     c)  $0.4 \frac{\text{ft}}{\text{s}}$

17. a) -9 ft    b) i)  $-8 \frac{\text{ft}}{\text{s}}$     ii)  $-5 \frac{\text{ft}}{\text{s}}$     iii)  $-2.75 \frac{\text{ft}}{\text{s}}$

18. a)  $30 \frac{\text{mi}}{\text{h}}$     b)  $40 \frac{\text{mi}}{\text{h}}$     c)  $\frac{240}{7} \frac{\text{mi}}{\text{h}} \approx 34.2857143 \frac{\text{mi}}{\text{h}}$     d)  $\frac{2v_1v_2}{v_1 + v_2}$

19.  $(7, 9)$  and  $(-3, -11)$

20. a)  $\infty$     b)  $-\infty$     c)  $\infty$     d)  $\infty$     e)  $\frac{3}{5}$     f) 0    g) undefined    h)  $\frac{2}{3}$     i)  $-\infty$     j) 0

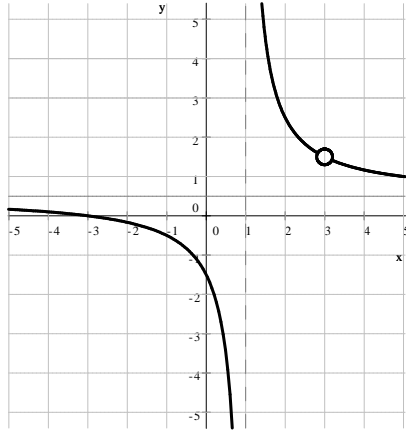
k)  $\infty$     l) 1    m) 0    n) 3    o)  $-\frac{1}{9}$     p)  $-\frac{1}{4}$     q) 0    r) undefined    s) 1    t) 1    u)  $\sqrt{2}$

21. a) undefined    b) 2    c) undefined    d) 4    e) 4    f) 4    g) 1    h) 2    i) undefined

j) 3    k) 3    l) 3    m)  $\infty$     n) undefined    o) undefined

22. a)  $\frac{1}{2}$     b) 0    c) 0    d) 0    e)  $-\infty$     f)  $\infty$     g) undefined    h)  $\frac{3}{2}$     i)  $\frac{3}{2}$     j)  $\frac{3}{2}$     k)  $\frac{1}{2}$

23.



24. a)  $-4$    b)  $-8$    c)  $-\frac{2}{5}$    d)  $-1$    e) undefined   f)  $\infty$    g)  $\frac{\sqrt{5}}{10}$    h)  $\frac{3}{2}$    i) undefined  
 j)  $\infty$    k)  $1$    l)  $\frac{5}{4}$    m)  $\frac{1}{2}$    n)  $1$    o)  $2$    p)  $\frac{1}{20}$    q)  $-\infty$

25. a)  $\tan \frac{\pi}{2}$  is undefined and the tangent function approaches infinity (and negative infinity) as  $x$  approaches  $\frac{\pi}{2}$ . In short,  $\lim_{x \rightarrow \pi/2^-} \tan x = \infty$  and  $\lim_{x \rightarrow \pi/2^+} \tan x = -\infty$ . So the limit as stated is a  $\frac{-\infty}{\infty}$  type of an indeterminate.

b)  $\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$  we multiply numerator and denominator by  $\cos^2 x$ .

$$\frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos 2x}{1} = \cos 2x$$

Thus the limit is  $\lim_{x \rightarrow \pi/2} \frac{1 - \tan^2 x}{1 + \tan^2 x} = \lim_{x \rightarrow \pi/2} \cos 2x = \cos \pi = -1$

26. a)  $0$    b) see below   c)  $y = m(x - 2)(x + 4) + 3$  No matter what  $m$  is,  $(2, 3)$  and  $(-4, 3)$  will be on the graph

