Please note that this document is designed to give you an idea of the **length** of the exam, not the type of questions on it.

1. Simplify each of the following.

   a) $\dfrac{6^{-2}-2^{-1}}{2^{-2}+3^{-2}}$

   b) $\dfrac{6^{-2}\cdot 2^{-1}}{2^{-2}\cdot 3^{-2}}$

   c) $e^{\ln A}+e^{2\ln B}-e^{\ln C}$

   d) $e^{\ln A+2\ln B-\ln C}$

   e) $\dfrac{\log_2 3\cdot\log_4 5\cdot\log_6 7}{\log_4 3\cdot\log_6 5\cdot\log_8 7}$

   f) $\log 33-\dfrac{1}{2}\log 44-\log 15-\log\sqrt{1100}$

2. Solve each of the following.

   a) $\log_6(x+4)+\log_6(x-1)=2$ b) $2x^2+8<12x$ c) $\tan x=\tan^2 x$

3. Find an equation for the tangent line drawn to the circle $4y-10x+x^2+y^2+10=1$ at the point $P(7,-6)$.

4. Compute each of the following limits.

   a) $\lim\limits_{x\to-\infty}\dfrac{-2x^5+3x^2-8}{x^2+x-4}$

   b) $\lim\limits_{x\to\infty}\dfrac{-2x^5+3x^2-8}{x^2+x-4}$

   c) $\lim\limits_{x\to\infty}\dfrac{\cos x}{x}$

   d) $\lim\limits_{x\to\infty}\log_5\left(1-x^2\right)$

   e) $\lim\limits_{x\to5}\dfrac{5-x}{\sqrt{4+x}-3}$

   f) $\lim\limits_{x\to\infty}\dfrac{2^x+1}{2^x-1}$

   g) $\lim\limits_{x\to0}\dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$

5. Compute the average velocity of an object between $t_1=1\,\text{s}$ and $t_2=5\,\text{s}$ if its location function (measured in meters) is given by $L(t)=t^3-2t^2+3$

6. Find the domain for the function $f(x)=\dfrac{-2x}{3-\sqrt{x+5}}$

7. Prove the following identity: $\dfrac{2}{\sin 2x}=\tan x+\cot x$

8. Give a complete analysis for the function $f(x)=6x-x^2+7$ on $[-2,4]$

Answers

1. a) $-\frac{17}{13}$ b) $\frac{1}{2}$ c) $A+B^2-C$ d) $\frac{AB^2}{C}$ e) 3 f) -2

2. a) 5 b) $\left(-\sqrt{5}+3, \sqrt{5}+3\right)$ c) $x = k\pi$ or $x = \frac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$

3. $\frac{1}{2}(x-7) = y+6$

4. a) ∞ b) $-\infty$ c) 0 d) undefined e) -6 f) 1 g) $-\frac{1}{4}$ $19\frac{\text{m}}{\text{s}}$

5. $[-5, 4) \cup (4, \infty)$

6. $$\text{RHS} = \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x} \cdot \frac{2}{2} = \frac{2}{2 \sin x \cos x} = \frac{2}{\sin 2x} = \text{LHS}$$

7. domain: $[-2, 4]$
   range: $[-9, 16]$
   $x-$intercept: $(-1, 0)$
   $y-$intercept: $(0, 7)$
   minimum: $(-2, -9)$
   maximum: $(3, 16)$